Economics Bulletin

Volume 44, Issue 3

Discrete pricing and consumer utility

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Abstract

We introduce and present a simple treatment of discrete pricing, a nonlinear pricing strategy that entails successive discrete price adjustments, triggered by corresponding purchase quantity thresholds. When the price adjustments are discounts, consumer utility may not necessarily increase with discrete pricing as compared to conventional pricing, depending upon the positions of the quantity thresholds and magnitudes of price discounts. We introduce the concepts of material and economic waste and highlight how the positions of the quantity thresholds affect consumer choices, utility, and the potential level of waste. Discrete pricing results in economic waste – unrealized utility – whenever consumers find it optimal to choose the binding minimum quantity required to obtain the price discount.

Author acknowledges comments of participants at 2017 AEA Annual Meeting Poster Session in San Francisco.
Citation: Edward Osei, (2024) "Discrete pricing and consumer utility", *Economics Bulletin*, Volume 44, Issue 3, pages 1123-1131
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Submitted: June 12, 2024. **Published:** September 30, 2024.

1. Introduction

Nonlinear pricing mechanisms in general have received some attention in the economic literature (e.g., Hausmann, 1985; Moffitt, 1990; Deaton and Muellbauer, 1989; Varian, 1989 section 2.3; Tirole, 1995, section 3.5; Varian, 1980; Ordover and Panzar, 1982; Viswanathan and Wang, 2003; Ye and Zhang, 2017; Heidhues and Koszegi, 2014; Fang et al., 2021). Notwithstanding, the implications to consumers of a common nonlinear pricing phenomenon that entails successive price discounts is hardly ever addressed. We address this specific pricing strategy here, which we refer to as discrete pricing.

We define discrete pricing as a nonlinear pricing strategy that entails successive price adjustments triggered by corresponding purchase quantity thresholds wherein the adjusted prices apply to ***all purchased units*** of the good, not only the amounts above the quantity thresholds. The good in question is sold in discrete units that are multiples of a base unit, c, such that buyers can only purchase the good in multiples of c units. In general, the price adjustments could be discounts or increases, but we focus attention here on discounts, as these are far more common. Examples include "buy 2 get 1 free" or similar volume discount offers. While we can highlight both the producer and consumer aspects of this scenario, we seek to highlight the consumer's utility maximization problem, and will thus focus solely on the consumer aspect, without any loss of generality. We are able to do this because the consumer's utility maximization problem in this context is decoupled from the specific underlying pricing scheme of the producer.

The discrete pricing model bears substantial similarity to the regular demand model with a few notable differences in detail. Like the regular model, we assume that consumers are fully aware of the price of the good. With discrete pricing, this means we require the consumer to have full knowledge of the entire price schedule – each quantity threshold and corresponding price discount. Furthermore, as shown below, with discrete pricing, the consumer faces a discontinuous budget set. Ignoring these salient differences would result in a demand model that often incorrectly estimates quantities of the discretely priced good that the consumer purchases.

Consider such a pricing strategy for a good q, involving multistep, specifically, k-step, discounting such that, $\forall i = 1,2, \dots k$, $p = p_i$ for **all purchased units** of the good when $q \in [q_i^{min}, q_i^{max}]$, with $q_1^{min} = 0$ and $q_k^{max} = \infty$. Furthermore, we have $p_i > p_{i+1}$, reflecting subsequent levels of discounted prices of the good, while $q_{i+1}^{min} = q_i^{max} + c$, represent the quantity breaks (thresholds) that trigger price discounts.

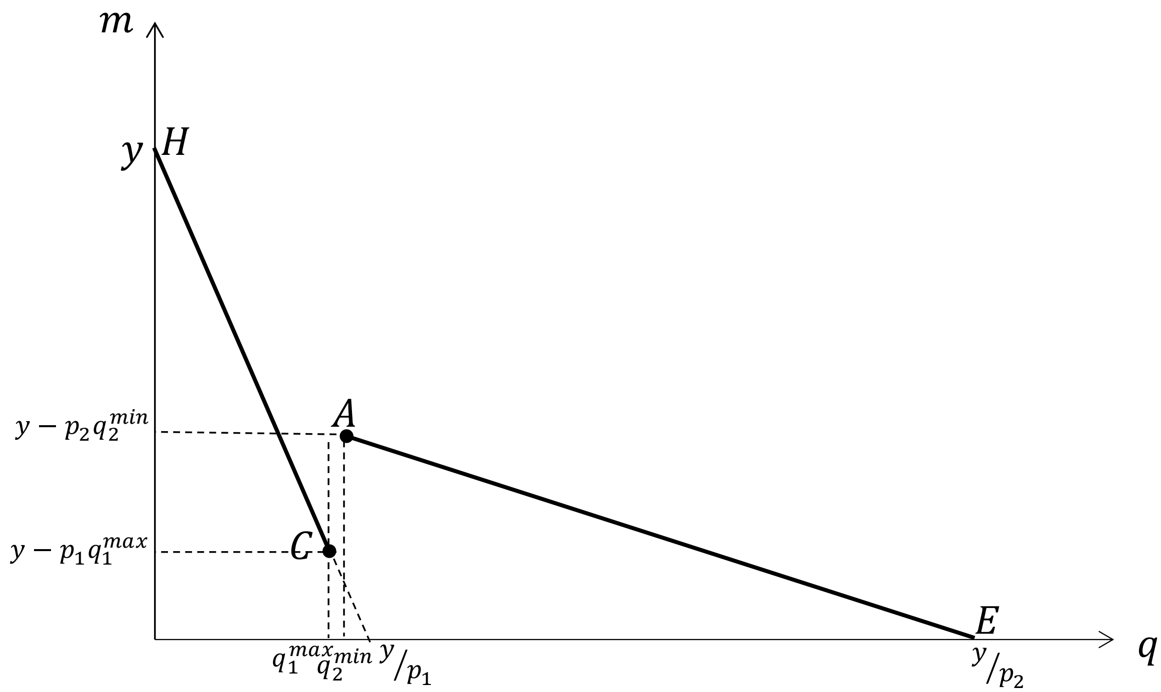
Given the foregoing, we can delineate three possible discrete pricing outcomes the consumer would face, each with interesting ramifications for choice, utility maximization, and waste. For the first outcome, $p_i q_i^{max} < p_{i+1} q_{i+1}^{min}$ – that is, more is more expensive that less, even with the discount. In the second case, $p_i q_i^{max} = p_{i+1} q_{i+1}^{min}$ – i.e., more is just as expensive as less. And for the final case, $p_i q_i^{max} > p_{i+1} q_{i+1}^{min}$ – i.e., more is less expensive than less. It is informative to note that it is rather common to observe price schedules that fit into one or the other of the latter two more interesting cases. The most interesting situation is obviously the third case where a greater quantity can be purchased at lower total cost to the consumer, a scenario that is also quite common in practice.

2. Budget Set and Consumer Equilibrium

The above discrete pricing scheme results in a discontinuous budget set to the consumer. To illustrate this graphically, we assume momentarily, for simplicity, that there are only two goods of interest to the consumer. The first good, q, is subjected to the discrete pricing scheme, while the second, m, will be considered for the moment as the numeraire. Furthermore, for added simplicity, we assume for now that $k = 2$, i.e., there are only two price levels for q with $q_2^{max} = \infty$. The budget set, B, in this case (see Figure 1) is the union of two disjoint sets:

$$B = \{q, m | q, m \in \mathbb{R}_+^2, p_1 q + m \leq y, q \leq q_1^{max}\} \cup \{q, m | q, m \in \mathbb{R}_+^2, p_2 q + m \leq y, q \geq q_2^{min}\}$$

Recall that $q_2^{min} = q_1^{max} + c$. We note that though each of the individual disjoint sets comprising B is convex, B itself is not. In Figure 1, consumer equilibrium will occur at one of three locations: at some point along line segment HC, exactly at point A, or at some point along line segment AE. We are aware that there are interesting rationales underlying the seller's choice of the pricing scheme (see for instance, Varian, 1989). However, once again, for this paper, we focus attention on the consumer problem, without loss of generality.



$y - p_2 q_2^{min}$, $y - p_1 q_1^{max}$, q_1^{max}, q_2^{min}, y/p_1, y/p_2

Figure 1. Discontinuous budget set under discrete pricing with a single quantity break

3. Generalization to many goods, many price breaks

We are interested in assessing the utility maximizing bundle that solves the consumer's conventional problem, but with the added features imposed by discrete pricing. For this purpose, we will generalize to n conventionally priced goods. That is, we assume that instead of only one good beside q, $\boldsymbol{m} \in \mathbb{R}_+^n$ is a vector of n goods that is of interest to the consumer and that these n goods are all conventionally priced – that is, not subject to discrete pricing – with a price vector represented by $\boldsymbol{s}$. We also for completeness, revert to a k-step discrete pricing scheme for q.

Throughout this paper, we also, for simplicity, assume a one-period model and that the consumer does not anticipate any form of waste when making purchase decisions. Furthermore, each good is homogeneous; there is no difference in appearance or quality between units of the same commodity. We consider multiperiod models, durable goods, and other aspects in a later study. With k quantity breaks and price discounts, the budget set generalizes to:

$$B = \bigcup_{i=1}^{k} B_i$$

where the disjoint sets B_i, $i = 1,2, \dots k$ are defined as

$$B_i = \{q, \boldsymbol{m} | q \in \mathbb{R}_+^1, \boldsymbol{m} \in \mathbb{R}_+^n, p_i q + \boldsymbol{s} \cdot \boldsymbol{m} \leq y, q \in [q_i^{min}, q_i^{max}]\}$$

The consumer's utility maximization problem will then be subject to B. Note that B already incorporates the stipulation that $p = p_i$ when $q \in [q_i^{min}, q_i^{max}]\ \forall\ i$ with $p_i > p_{i+1}\ \forall i = 1,2, \dots k$. To characterize more formally the nature of the optimal solution, we assume, for simplicity, that the utility function is of the Cobb-Douglas form:

$$U(q, \boldsymbol{m}) = A q^{\alpha} \prod_{j=1}^{n} m_j^{\beta_j}$$

where $\alpha \in (0,1)$, $\beta_j \in (0,1)\ \forall j$, and $A > 0$.

The above formulation results in the following solution set, reflecting $2k - 1$ possible sets of optimal values for the $n + 1$ choice variables $(q^*, \boldsymbol{m}^*)$:

$$(q^*, \boldsymbol{m}^*) = \begin{bmatrix} \frac{\alpha y}{(\alpha + \sum_{j=1}^{n} \beta_j) p_k} & \frac{\beta_1 y}{(\alpha + \sum_{j=1}^{n} \beta_j) s_1} & \cdots & \frac{\beta_n y}{(\alpha + \sum_{j=1}^{n} \beta_j) s_n} \\ q_k^{min} & \frac{\beta_1 (y - p_k q_k^{min})}{\sum_{j=1}^{n} \beta_j} & \cdots & \frac{\beta_n (y - p_k q_k^{min})}{\sum_{j=1}^{n} \beta_j} \\ \frac{\alpha y}{(\alpha + \sum_{j=1}^{n} \beta_j) p_{k-1}} & \frac{\beta_1 y}{(\alpha + \sum_{j=1}^{n} \beta_j) s_1} & \cdots & \frac{\beta_n y}{(\alpha + \sum_{j=1}^{n} \beta_j) s_n} \\ q_{k-1}^{min} & \frac{\beta_1 (y - p_{k-1} q_{k-1}^{min})}{\sum_{j=1}^{n} \beta_j} & \cdots & \frac{\beta_n (y - p_{k-1} q_{k-1}^{min})}{\sum_{j=1}^{n} \beta_j} \\ \vdots & \vdots & \ddots & \vdots \\ q_2^{min} & \frac{\beta_1 (y - p_2 q_2^{min})}{\sum_{j=1}^{n} \beta_j} & \cdots & \frac{\beta_n (y - p_2 q_2^{min})}{\sum_{j=1}^{n} \beta_j} \\ \frac{\alpha y}{(\alpha + \sum_{j=1}^{n} \beta_j) p_1} & \frac{\beta_1 y}{(\alpha + \sum_{j=1}^{n} \beta_j) s_1} & \cdots & \frac{\beta_n y}{(\alpha + \sum_{j=1}^{n} \beta_j) s_n} \end{bmatrix}$$

The quantity breaks and price discounts clearly factor into the potential optimal solutions; $k - 1$ potential optimal solutions occur at the quantity breaks, while k outcomes are interior solutions. The last row (solution set) is equivalent to the optimal interior solution when q is conventionally priced at p_1 – i.e., no price discounts. The remaining $2k - 2$ outcomes represent the differences between the results from the standard demand model and the discrete pricing model. Note that with the Cobb-Douglas formulation chosen, the (interior) solution vector of the conventionally priced goods $\boldsymbol{m}^*$ does not change regardless of the price discounts for q; the optimal values of

$\boldsymbol{m}^*$ change only at the quantity breaks. It can be shown quite readily that all $2k - 1$ possible optimal solutions for the n+1 goods satisfy the standard aggregation, homogeneity, and symmetry properties of Walrasian demands (Mas Colell et al., 1995; Deaton and Muellbauer, 1989).

4. Impacts of Quantity Thresholds

The levels of the quantity breaks and price discounts are of interest in terms of their impacts on the choice variables, the maximum value function – the indirect utility function, and the potential for economic waste. In Figure 2, we depict the impacts of the quantity break on the choice variables (in the top panel) and the indirect utility function (in the bottom panel), holding the price discounts constant. We present for simplicity of exposition only the r^{th} element of $\boldsymbol{m}^*$. We also assume the case of two price levels and hence one relevant quantity break, $q_2^{min} = q_1^{max} + c$ (we assume again for the moment that $q_2^{max} = \infty$). Our interest is thus in delineating the impact of varying levels of q_2^{min} on the consumer's choices – $q^*, \boldsymbol{m}^*$ – as well as indirect utility, again with the price levels unchanged.

Recall that from the solution set, as q_2^{min} rises, the rate at which the optimal value of each of the conventionally priced goods changes at the quantity breaks of q is given by $\frac{\partial m_r^*}{\partial q_2^{min}} = -\frac{\beta_r P_2}{\sum \beta_j}$.

Since by construction, the consumer was maximizing utility before the binding q_2^{min} constraint, utility falls as $q^* = q_2^{min}$ rises and m_r (and for that matter, each m_j) falls proportionately. This loss in utility is a key component of what we refer to as economic waste, and we define this more explicitly in the next section.

The illustration in Figure 2 holds for the case where $\alpha/p_1 < \beta_r/s_r < \alpha/p_2$. For context, we first note that the solution to the conventional utility maximization problem – $(q^*, m_r{}^*) = (\frac{\alpha y}{(\alpha+\sum \beta_j)p_1}, \frac{\beta_r y}{(\alpha+\sum \beta_j)s_r})$ – corresponds to maximized utility level V_c. At low levels of q_2^{min}, the consumer finds it advantageous to purchase a higher level of q ($> q_2^{min}$), specifically $\alpha y/(\alpha + \sum \beta_j)p_2$, than under the conventional price regime ($\frac{\alpha y}{(\alpha+\sum \beta_j)p_1}$). This is an interior solution on the second segment of the budget set in Figure 1, the segment corresponding to the lower price, p_2. The level of m_r is equivalent to that under the conventional price system of q, $\frac{\beta_r y}{(\alpha+\sum \beta_j)s_r}$ – it is the same solution for either $p = p_1$ or $p = p_2$, and is also an interior solution on the budget line. The corresponding utility level under discrete pricing with very low q_2^{min} levels is V_d. Under discrete pricing of q with very low q_2^{min}, the consumer essentially purchases the same amount of m_r but a higher amount of q than under conventional pricing of q.

As q_2^{min} rises, q^* and $\boldsymbol{m}^*$ remain unchanged until $q_2^{min} = \underline{q_2^{min}} = \frac{\alpha y}{(\alpha+\sum \beta_j)p_2}$. Beyond that point, the quantity break at q_2^{min} becomes a binding constraint, and the consumer is forced to purchase and consume more units of q and fewer units of m_r than they would normally have in order to receive the price discount. They are still better off than under conventional pricing of q, but not as well off as when q_2^{min} was very low. As q_2^{min} rises further, the consumer continues to purchase greater quantities of q, seeking to take advantage of the price discount by purchasing the required minimum – q_2^{min} – units of the good. Correspondingly, their purchase of $\boldsymbol{m}$ reduces, resulting in a steady decline in the maximum value function. Finally, at $q_2^{min} = \overline{q_2^{min}}$, when the value of indirect utility under discrete pricing is equivalent to V_c, the consumer foregoes the price discount and falls back to purchasing the optimal quantity $(q^*, \boldsymbol{m}^*)$ that would prevail under conventional pricing of q. These entail the same quantity $\boldsymbol{m}^*$ of the conventionally priced goods as is the case with very low quantity breaks, but a lower optimal quantity q^* of the good that is subject to discrete pricing, and consequently, the lower utility level, V_c

At a specific level of q_2^{min} displayed in Figure 2 as $q_2^{min'}$, the loss in utility due to the binding quantity break can be calculated as $V_d - V_d'$. This is a component of economic waste that we introduce in the next section as economic loss due to a binding quantity break. We now turn to this discussion of material and economic waste.

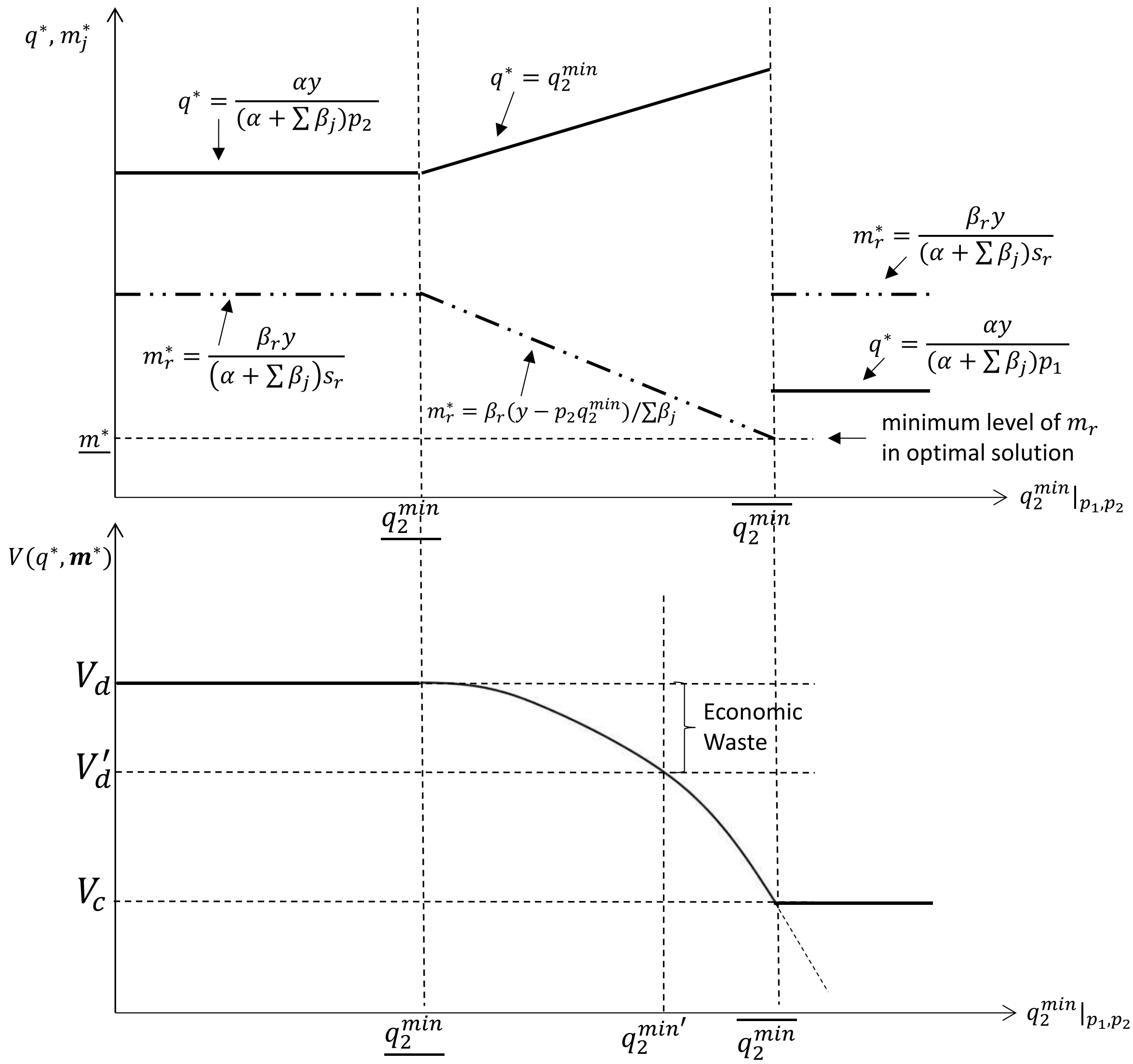


Figure 2: Impacts of quantity break on optimal solution and consumer utility

5. Material waste and economic waste

When a good is subject to discrete pricing, the consumer is often inclined to purchase at least the required minimum quantity in order to receive a given price discount. This phenomenon creates the potential for waste. Within this context, we distinguish two broad classes of waste, which we define presently: material waste and economic waste.

5.1. Material Waste (MW):

We define material waste, MW, as the amount of the good that is not consumed or sold due to deterioration or sheer waste. Given the above assumptions,

$$MW = q^* - q_c$$

where q^* is the amount purchased and q_c is the amount actually consumed. In the one-period model we are utilizing, we envision a scenario where the consumer does not anticipate wasting any component of the good. Thus, the utility maximization problem does not factor in a waste component. However, in reality, only a portion of the quantity purchased is actually consumed. Material waste is consequently an ex-post assessment of the consumer's choices. We address intertemporal issues for durable goods in a separate study, where material waste is captured within the utility maximization problem.

5.2. Economic Waste (EW):

We introduce two components of economic waste (EW): one directly tied to material waste, and the other due to the binding quantity break in the case where the consumer is forced to choose $q^* = q_i^{min}$ for some i.

A. Economic loss from material waste (EWM): This is the loss in utility due to material waste. Assuming that the consumer consumes all of $\boldsymbol{m}^*$, we evaluate this waste at the optimal level of $\boldsymbol{m}^*$. Thus, EWM is defined as

$$EWM = V(q^*, \boldsymbol{m}^*) - V(q_c, \boldsymbol{m}^*).$$

EWM essentially converts material waste into economic units.

B. Economic loss due to binding quantity break constraint (EWC): This corresponds to the standard concept of economic waste. Here, it refers to the economic loss due to the consumer being forced to choose a higher level of q in order to obtain the price discount. To quantify this measure of waste, we first use the information from the optimal solution set in Section 3 to define the interior solution of q for each level of price discount, p_i, as

$$\widehat{q_i} = \begin{cases} \dfrac{(y - \boldsymbol{s} \cdot \widehat{\boldsymbol{m}})}{p_i} = \dfrac{\alpha y}{(\alpha + \sum \beta_j) p_i} & if\ (y - \boldsymbol{s} \cdot \widehat{\boldsymbol{m}})/p_i \in [q_i^{min}, q_i^{max}] \\ 0 & otherwise \end{cases}$$

where $\widehat{\boldsymbol{m}} = \{\widehat{m_r}\}$ with $\widehat{m_r} = \frac{\beta_r y}{(\alpha + \sum \beta_j) s_r}\ \forall r$, and set $\hat{q} = \max_i(\widehat{q_i})$, what the consumer would have purchased under the price discount if they had not been "forced" to buy q^* due to the binding quantity breaks. Then EWC is defined as:

$$EWC = V(\hat{q}, \widehat{\boldsymbol{m}}) - V(q^*, \boldsymbol{m}^*)$$

Recall that with the binding quantity break, $q^* > \hat{q}$ and $\boldsymbol{m}^* < \widehat{\boldsymbol{m}}$. Total economic waste ($EW$) is the sum of EWM and EWC:

$$EW = EWM + EWC = V(\hat{q}, \widehat{\boldsymbol{m}}) - V(q_c, \boldsymbol{m}^*)$$

Since EWM and EWC are both nonnegative, EW is also nonnegative.

6. Summary

We summarize the results of this study as follows:

a) Discrete pricing produces nonconvex budget sets and provides potential for higher consumer utility when associated with price discounts
b) Optimal solution with k-step discounting involves $2k - 1$ possible solution sets, all of which are functions of the discount prices, and $k - 1$ of which are functions of quantity breaks (thresholds).
c) To an extent depending on the position of the quantity breaks and price discounts, higher quantity breaks may force consumers to purchase higher quantities of the good than anticipated, resulting in greater potential for material and economic waste.

7. Conclusions

Discrete pricing with multistep discounting produces consumer behaviors with interesting ramifications. While consumer utility is likely to increase, this hinges on the positions of the quantity breaks and magnitudes of price discounts. In a one-period model, this pricing scheme incentivizes consumers to purchase a greater quantity of the good resulting in higher potential for material and economic waste.

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