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On variety effects and linear demand systems

Radostina Shopova
Economic Research Institute, Bulgarian Academy of Sciences

Abstract

This short paper complements Choné and Linnemer's (2020) study on linear demand systems for differentiated goods by addressing how these frameworks handle variation in the number of products 'n' and the associated variety effects. While Choné and Linnemer briefly discuss varying 'n' with Spence (1976) and Levitan and Shubik (1980), this note expands on their differences. The note highlights why the "shutting-down-variety-effects" approach of Levitan and Shubik can be problematic when 'n' varies, and argues that frameworks with parameters constant in 'n', such as Spence's, align better with rational consumer choice. The note acknowledges and complements a point made by Höffler (2008) on modeling product availability.

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Contact: Radostina Shopova - radostina.shopova@yahoo.com.
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1 Introduction

Modeling demand for differentiated products is key in Industrial Organization literature, with many researchers focusing on linear demand systems due to their ease of providing closed-form solutions (see Choné and Linnemer (2020)). Studies often examine not only product differentiation but also firms' market entry, introduction of new varieties, and potential for competitor foreclosure.[1] These analyses raise essential questions about how changes in variety and competition affect consumers, firms, and the market overall, requiring a model flexible enough to account for shifts in the number of available products n.

The two main concurrent quasi-linear quadratic utility (QQU) function specifications used to derive a linear demand system are the one by Spence (1976) and the one by Shubik and Levitan (1980). In Section 4.2 of their paper, Choné and Linnemer (2020) vaguely touch on the topic and show how these two main specifications differ in terms of their comparative statics with respect to the number of products n. Specifically, the model of Shubik and Levitan (1980) is recommended at the beginning of Section 4 due to its property of "shutting down *variety effects*" when firms join the market and keeping market size fixed. Some recent work has taken on this recommendation and derived results that crucially depend on this assumption. For example, Han et al. (2022) and Basak et al. (2022) construct a weighted average QQU between the Spence (1976) and Shubik and Levitan (1980) formulations to measure the "extent of market expansion" and its implication on optimal differentiation or wage bargaining. Kittaka and Pan (2023) uses the absence of *variety effects* to model exclusion and obtains a monopolization result in their Proposition 5, which won't hold if the new entrant were to expand the market. Chung (2023) finds that an inefficient entrant may increase prices for consumers - a result absent in the analysis with the *variety effects* framework of Spence (1976).[2]

The contribution of this short communication is to highlight that, when the objective is to examine changes in the number of products in the market n, frameworks with constant parameters that introduce or remove products via additive terms — such as Spence (1976) — are consistent with rational consumer choice. By contrast, while the framework of Shubik and Levitan (1980) has advantages when n is fixed, it may lead to inconsistencies when n varies, thus requiring careful treatment. This paper builds on and extends the critique by Höffler (2008) concerning product availability in the Shubik and Levitan (1980) model in three ways. First, it compares the Spence (1976) and Shubik and Levitan (1980) frameworks in terms of *competition* and, more importantly, *variety effects* on consumer utility, market size, and total demand as n changes. Second, it shows how the approach of "shutting down variety effects" in Shubik and Levitan (1980) leads to inconsistencies in consumer choice when n varies, violating both the Independence of Irrelevant Alternatives and Houthakker's Axiom of Revealed Preference. Third, it identifies further issues stemming from the model's assumptions. Finally, it suggests potential remedies for researchers seeking to fix market size while isolating *variety effects*.

[1] For example, recent attention has focused on "hybrid platforms" introducing their own products; see Etro (2024) for an overview.

[2] See Appendix B for the short argument.

2 Quasi-linear quadratic utility functions, variety, and competition

Competition and variety effects of new products. Let us start with the Spence (1976) formulation. There is a representative consumer with a QQU function of the form

$$U(q_1,...,q_n,\widetilde{q}) = \sum_i aq_i - s\sum_i\sum_{j>i} q_iq_j - \frac{b}{2}\sum_i q_i^2 + \widetilde{q} \tag{1}$$

where q_i is the amount consumed of product $i = 1,..,n$ and $\widetilde{q}$ is a a numéraire good with a price normalized to 1. All products are symmetric with the marginal quality index of each product equal to a, inverse demand slope $b > 0$, and substitution parameter $s \in [0,b]$.[3] When $s = 0$, products are independent, while when $s = b$, products are homogeneous.[4] Thus, any $s \in (0,b)$ corresponds to products being imperfect substitutes. A consumer maximizes its utility in "(1)" by choosing an optimal consumption level $(q_1,...,q_n)$ taking the budget constraint $\widetilde{q} + \sum_i p_iq_i \leq m$ as given, where p_i is the price of product i and m is the income level. Assuming that m is large enough and eliminating $\widetilde{q}$ simplifies to the following optimization problem:[5]

$$\max_q \mathscr{L} = \sum_i (a-p_i)q_i - s\sum_i\sum_{j>i} q_iq_j - \frac{b}{2}\sum_i q_i^2$$

A consumer takes all prices as given and chooses to consume non-negative amounts of each variety $q_i \geq 0$. This leads to the Karush-Kuhn-Tucker (KKT) conditions, which are a system of $i = 1,..,n$ equalities, namely:

$$q_i\frac{\partial \mathscr{L}}{\partial q_i}{=}0 \Leftrightarrow q_i(a - p_i - bq_i - s\sum_{j\neq i} q_j){=}0 \tag{2}$$

If a product i is unavailable, then $q_i = 0$. If a price p_i is prohibitively high, then the expression in the brackets becomes negative and commands that $q_i = 0$.

If prices are not prohibitively high, so that all $q_i > 0$ $\forall i = 1,..,n$ one derives a system of indirect demands

$$p_i = a - bq_i - s\sum_{j\neq i} q_j \quad \forall i = 1,...,n \tag{3}$$

which can be inverted to direct demand functions $\forall i = 1,...,n$

$$q_i = \frac{a}{(b+(n-1)s)} - \frac{b+(n-2)s}{(b-s)(b+(n-1)s)}p_i + \frac{s}{(b-s)(b+(n-1)s)}\sum_{j\neq i} p_j \tag{4}$$

Note that if a price for some product k is prohibitively high, such that $q_k = 0$, the problem becomes equivalent to a case where one assumes that there were initially only $n-1$ products in the market and completely ignores product k. Ergo, whether a firm with a product exits (enters) the market or sets a too high (low enough) price for consumers leads to the same qualitative and quantitative results in the Spence (1976) framework.

[3] As demonstrated by Choné and Linnemer (2020) expanding to asymmetric firms is straightforward.

[4] The argument in this note can be expanded to account for complements where $s < 0$, but one needs to be cautious that the lower bound on s varies with the number of products n as pointed out by Amir et al. (2017).

[5] Note that this assumption is mild only for the case of products being substitutes, as emphasized by Amir et al. (2017).

If one assumes that each product is sold by a separate firm at price p, both the total market size $Q^{Sp} = \sum_n q_i = \dfrac{n(a-p)}{b+(n-1)s}$ and the utility of the representative consumer $U_n^{Sp}(p) = \dfrac{n(a-p)^2}{2(b+(n-1)s)}$ increase in the number of products n. Thus, a new entrant or product expands the market through the so-called *variety effects.* When firms with symmetric marginal costs c compete à la Bertrand to maximize profits $\pi_i = (p_i - c)q_i$ for $i = 1,...,n,$, the optimal price in the market is given by $p^{*Sp} = \dfrac{a(b-s)+(b+(n-2)s)c}{2b+(n-3)s}$. The equilibrium price decreases in n and converges to marginal cost as $n \to \infty$. This *competition effect* leads to a further expansion in total market demand and consumer utility. Thus, the Spence (1976) formulation, as emphasized by Choné and Linnemer (2020), captures both *variety* and *competition effects* of market entry or exit.

"Shutting down variety effects". In their seminal book Shubik and Levitan (1980) suggest a QQU function that has the useful property of keeping the market size fixed. In particular, for any number of products n priced at p, total demand in the market is constant and given by $Q^{LS} = a - p$, and so is utility $U^{LS} = \dfrac{(a-p)^2}{2}$.[6] This is achieved by setting $b = n-(n-1)s$ so that $s \in [0,1]$ in "(1)". Choné and Linnemer (2020) call this "shutting down variety effects", as more products priced at p in the market split but do not grow the market. The only source of growth is through the *competition effect* that pushes the equilibrium price $p^{*LS} = \dfrac{an(1-s)+c(n-s)}{2n-(n+1)s}$ down as in the Spence (1976) specification. For this reason, Choné and Linnemer recommend this model to researchers who want to increase the number of products n in their analysis without causing a market expansion at a given equilibrium price.

Limitations of changing n in Shubik and Levitan (1980). The first to point out an inconsistency of the utility function and preferences of Shubik and Levitan (1980) was Höffler (2008), who focused on the problem of one (or more) products becoming unavailable. To rephrase his argument, suppose there are two products $n = 2$ with the price of product 2 being prohibitively high so that the consumer chooses $q_2 = 0$. In such a case, the KKT conditions in "(2)" dictate that the demand for product 1 should be $q_1 = \dfrac{a-p_1}{2-s}$. We have seen that in the Spence (1976) formulation, whether one assumes that p_2 is set so that non-negative demand conditions bind or that $n = 1$ from the beginning leads to equivalent outcomes. However, if $n = 1$ in Shubik and Levitan (1980) the optimization problem of the consumer yields demand $q_1 = a - p_1 > \dfrac{a-p_1}{2-s}$! As pointed out by Höffler (2008), this leaves any firm that might have the possibility to exclude a competitor with an extraordinary incentive to do so. Thus, any model analyzing not only horizontal but also vertical foreclosure is bound to overpredict such an event.[7]

The main issue with Shubik and Levitan (1980) lies in the assumption that the parameter b varies with n, which implies that the utility function changes with n. Höffler (2008) states that changing n in Shubik and Levitan (1980) is like deriving demand from different consumers.

[6] Outside the scope of this discussion but worth noting, demand and utility remain constant in the substitution parameter s, a different source of *variety effects*. This can be an advantage of this formulation over Spence (1976) as long as n remains fixed.

[7] E.g., as in Kittaka and Pan (2023)'s Proposition 5.

Alternatively, one can rephrase that because the change of b in n changes the utility function of a representative consumer, it also changes their optimal choices.

Let us expand on the theoretical challenges this parametric choice poses to the theory of consumer choice. First, varying n in the QQU of Shubik and Levitan (1980) violates the axiom of Independence of Irrelevant Alternatives (IIA). Consider again having one product priced at marginal cost. For $n = 1$, the consumer chooses to consume $q_1 = a - c$ over the other affordable option $q_1 = \dfrac{a-c}{2-s}$. Suppose a second alternative is introduced at a prohibitively high price $p_2 >> a$ so that the alternative is not a relevant (feasible) consideration and so $q_2 = 0$. The choice of the consumer now, however, has changed to buying $q_1 = \dfrac{a-c}{2-s}$ over $q_1 = a - c$, even though the latter is still affordable.

Further, it can be argued that there does not exist a rational, i.e., complete and transitive, preference relation $\succsim$ that rationalizes the choices generated by the Shubik and Levitan (1980) specification when the number of products n is varied. To see this, suppose that "(1)" with $b = n - (n-1)s,\ s \in [0,1]$ represents the preference relation $\succsim'$ and denote it by $u_n(\mathbf{q})$, where the subscript denotes the specification for the given number of products n in the market. By Proposition 1.B.2 from Mas-Colell et al. (1995) $\succsim'$ should be rational. If that is the case, then one can express the maximization problem as a choice rule

$$C(B_n, \succsim') = \{\mathbf{x} \mid \mathbf{x} = \underset{\mathbf{q} \in B_n}{argmax}\ u_n(\mathbf{q})\}$$

where $B_n = \{\mathbf{q} \mid \mathscr{L}_n(\mathbf{q}) \geq 0\}$ is the set of all non-negative consumption vectors given the prices in the market with $\mathscr{L}_n$ the respective Lagrangian/objective function. Because by definition "(1)" is a concave function, this means that $C(B_n, \succsim')$ is single-valued and non-empty.

We know that a choice rule $C(\cdot)$ can be rationalized by complete and transitive preferences if and only if it satisfies the Houthakker's Axiom of Revealed Preference (HARP), also known as the Strong Axiom of Revealed Preference (see, e.g., Levin and Milgrom (2004) or Proposition 3.J.1 in Mas-Colell et al. (1995)). Let us go back to the example with changing the number of products from one to two, with the second product when introduced priced so that the KKT condition dictates $q_2 = 0$. For $n = 1$ and $p_1 = c$ we have that both $\mathbf{q_1} = (a-c, 0)$ and $\mathbf{q_2} = \left(\dfrac{a-c}{2-s}, 0\right)$ are such that $\mathscr{L} \geq 0$, or $\mathbf{q_1}, \mathbf{q_2} \in B_1$.[8] For $n = 2$ and $p_1 = c$ it holds again that $\mathbf{q_1}, \mathbf{q_2} \in B_2$. From "(3)" we have that $C(B_1, \succsim') = \mathbf{q_1}$ and $C(B_2, \succsim') = \mathbf{q_2}$. But then, by HARP it must be that $\mathbf{q_1} \in C(B_2, \succsim')$ and $\mathbf{q_2} \in C(B_1, \succsim')$. Thus, $\mathbf{q_1} \sim \mathbf{q_2} \Leftrightarrow u_n(\mathbf{q_1}) = u_n(\mathbf{q_2})$ for $n = 1, 2$, but $u_1(\mathbf{q_1}) = \dfrac{(a-c)^2}{2} > u_1(\mathbf{q_2}) = \dfrac{(a-c)^2(3-2s)}{2(2-s)^2}$ and $u_2(\mathbf{q_1}) = \dfrac{s(a-c)^2}{2} < u_2(\mathbf{q_2}) = \dfrac{(a-c)^2}{2(2-s)}$, a contradiction.

In general, consumer choice based on a QQU function with parameters a, b, and s independent of the number n of all possible products in the market, can be rationalized and does not violate the IIA axiom and HARP. This is the case for the Spence (1976) formulation in "(1)" which exhibits both *variety* and *competition effects* in the number of affordable products n. A QQU specification that relies on changing the parameters a, b, and s with n is bound to violate the IIA axiom and HARP, as is the case with Shubik and Levitan (1980) with $b = n + (n-1)s$.[9]

[8] Adding terms in the utility function multiplied by 0 is trivial for any assumed n.

[9] A more formal discussion is provided in Appendix A.

Some other peculiarities of varying n within Shubik and Levitan (1980). We have seen that in the Spence (1976) formulation, strategic price competition converges to marginal cost pricing as the number of firms and products n increases indefinitely. A different price pattern occurs if one takes the Shubik and Levitan (1980) specification. The optimal price a firm charges is $p^{*LS} = \dfrac{an(1-s)+c(n-s)}{2n-(n+1)s}$ and converges to a constant markup $\dfrac{a(1-s)+c}{2-s}$ as the number of firms n converges to infinity. Only when $s \to 1$ does the price converge to marginal cost. Thus, the model predicts that the market power of sellers persists even if they are infinitely small and concentration in the market is practically zero.

Following from this observation is that the cost pass-through of equilibrium prices $\dfrac{\partial p^{*LS}}{\partial c} = \dfrac{n-s}{n(2-s)-s}$ does not converge to one, but converges to $\dfrac{1}{2-s} < 1$ as the number of competitors n converges to infinity. Thus, any theoretical analysis on the effects of an increase in cost could lead to an understatement of the effect on the economy, while any empirical evidence of lower pass-through than anticipated may lead to an interpretation that understates the effect of concentration.

Fixing market size consistently restores variety effects. As pointed out by Motta (2004) and further by Choné and Linnemer (2020), a key appeal of the Shubik and Levitan (1980) model is its ability to fix the maximum market size at level a. A way to keep the total market size fixed and still operate with a QQU specification that does not violate IIA or HARP is to fix the parameter $b = N-(N-1)s$, where N is an exogenous parameter representing the maximum number of products admissible by the market. Then, entry with a new product with a price $p >> a$ changes the number of available products from n to $n+1$, without affecting the optimal choice of the consumer and its utility. In particular, if there are $n < N$ products in the market priced at $p < a$ consumer's utility is $U_n^{LS(N)} = \dfrac{n(a-p)^2}{2(N-(N-n)s)}$ and total market demand is $Q_n^{LS(N)} = \dfrac{n(a-p)}{N-(N-n)s}$. However, if there is an entry with a new product also priced at p, the number of products increases to $n+1$, which, as in the Spence (1976) framework, leads to an increase in both utility and total demand through a *variety effect!* The equilibrium price in the market then decreases from the introduced competition, leading to a further increase in utility and total demand through the *competition effect.* As a result, most qualitative findings align with Spence (1976), though with more involved formulas and an upper limit on n.

Is there a way to model entry without variety effects? When the QQU model is consistently applied to derive a linear demand system, the entry or exit of firms in a differentiated oligopoly always impacts prices due to both *competition* and *variety effects*. To eliminate *variety effects*, one potential approach is to use address models like the Hotelling line or Salop's circular city. Under certain regularity conditions, total demand remains constant and, as in Shubik and Levitan (1980), is independent of the number of firms when all charge the same price, p. However, if a new firm enters at a different location but keeps the same price, nearby consumers switch to save on transportation costs, increasing their surplus via a *variety effect*. Allowing price adjustments further lowers prices through the *competition effect*. The only way to isolate the *variety effect* is to introduce new firms at already occupied locations, ensuring that consumer surplus and prices change solely through competitive pressure.

3 Conclusion

This short communication has examined the use of quasi-linear quadratic utility functions to derive linear demand systems in settings where the number of products n varies in the context of *competition* and *variety* effects. While QQU functions are useful for modeling entry and exit in differentiated oligopoly markets, care must be taken when specifying the utility function parameters. The note shows that the framework of Spence (1976) is quite suitable for such purposes, as it maintains consistency with rational consumer choice when n changes. Alternatively, the "shutting-down-variety-effects" Shubik and Levitan (1980) framework can be adapted by fixing parameters through an exogenous upper bound N, and focusing on cases where $n \leq N$ — a correction that, however, reinstates *variety effects*. The overall conclusion is that any linear demand system derived from a QQU stemming from a rational representative consumer would account for both *variety* and *competition effects* when modeling product entry or exit. Future research is needed to assess whether there are other ways to derive a linear demand system for differentiated products, where entry or exit lead solely to *competition effects*.

A IIA and HARP when varying parameters with n

Suppose we have a utility function as in "(1)" with parameters $a_n,\ b_n,\ s_n$ depending on n. From the KKT conditions, if $q_i > 0$ for all i, we have $q_i(a_n - p_i - b_n q_i - \sum_{j\neq i} q_j) = 0$, such that there is a system of indirect demands $p_i = a_n - b_n q_i - \sum_{j\neq i} q_j,\ i, j = 1,\ldots,n$. This is equivalent to an optimal choice function $C_n(\cdot)$ as discussed in the main text. Adding m irrelevant alternatives with $p_k > a,\ k = 1,..,m$ and associated $q_k = 0$ does not affect the system, provided parameters remain constant: $a_n = a_{n+m} = a,\ b_n = b_{n+m} = b$, and $s_n = s_{n+m} = s$. Hence, under the specification of Spence (1976), choice remains unchanged, satisfying both IIA and HARP.

However, if parameters vary with n, the optimality conditions — and hence the choice — change. Let $\mathbf{q}_n = (q_1^n,\ldots,q_n^n,0_1,\ldots,0_m) = \arg\max u_n(\cdot)$ be the unique solution (by strict concavity when $b_n > s_n$).[10] Suppose some vector $\mathbf{q}_{n+m} = (q_1^{n+m},\ldots,q_n^{n+m},0_1,\ldots,0_m) \neq \mathbf{q}_n$. Then, $u_n(\mathbf{q}_{n+m}) < u_n(\mathbf{q}_n)$ as the optimal solution is preferred. As $\mathbf{q}_{n+m}$ is chosen randomly, then we can find m such that $\mathbf{q}_{n+m} = argmax\ u_{n+m}(\cdot)$ and $u_{n+m}(\mathbf{q_n}) > 0$. By strict concavity $u_{n+m}(\mathbf{q}_{n+m}) > u_{n+m}(\mathbf{q}_n)$, which implies a shift in preference due to the added irrelevant alternatives - violating both IIA and HARP.

B Chung (2023) with a Spence (1976) formulation

Consider two firms $i = 1,2$ each with a marginal cost of production $c = 0$, that face an indirect demand as in "(3)". They maximize $\pi_i = p_i q_i$ by choosing production q_i. Optimally, $q_i^* = \dfrac{a}{2b+s}$ and the prices for consumers are symmetric at $p_i^* = \dfrac{ab}{2b+s}$.

If a third firm enters the market and has cost $c > 0$, the equilibrium quantities of the first two firms change to $q_1^* = q_2^* = \dfrac{(2b-s)a+sc}{2b^2+2bs-2s^2}$ and the entrant sells $q_3^* = \dfrac{(2b-s)a-(2b+s)c}{4b^2+2bs-2s^2} > 0$ as long as $c < \dfrac{2b-s}{2b+s}a$. The respective prices are $p_1^* = p_2^* = \dfrac{b((2b-s)a+sc)}{4b^2+2bs-2s^2}$ and $p_3^* = \dfrac{ab(2b-s)+c(2b^2+bs-2s^2)}{4b^2+2bs-2s^2}$.

[10] As before, adding 0 terms to the utility function is trivial.

The prices of the incumbent two firms increase if $\frac{ab}{2b+s} < \frac{b((2b-s)a+sc)}{4b^2+2bs-2s^2}$ or when $c > \frac{2b-s}{2b+s}a$, which is possible only if $q_3^* < 0$. Thus, in Chung (2023) the driver of the result is that when $b = n-(n-1)s$ and $s = d/(1+d)$, the author compares the prices before entry with parameter $b = n-(n-1)s$ and after with parameter $b = n+1-ns$.

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