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Conduct parameter estimation in homogeneous goods markets with equilibrium existence and uniqueness conditions: the case of log-linear specification

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Abstract

We propose a constrained generalized method of moments (GMM) estimator with some equilibrium uniqueness conditions for estimating the conduct parameter in a log-linear model with homogeneous goods markets. Monte Carlo simulations demonstrate that merely imposing parameter restrictions leads to not just inaccurate estimations but also some numerical issues, and adding the equilibrium uniqueness conditions resolves them. We also suggest a formulation of the GMM estimation to further avoid the numerical issues.

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1 Introduction

Measuring competitiveness is central in empirical IO, and the conduct parameter is a widely used proxy. Since marginal cost is typically unobserved, researchers identify and estimate the conduct parameter indirectly.

Bresnahan (1982) develops an identification strategy under linear demand and marginal cost, and Matsumura and Otani (2023) provide detailed conditions for this case. Yet, empirical work often employs log-linear models (Okazaki et al. 2022, Mérel 2009), which tend to produce implausibly low or negative estimates, even though identification conditions for general models are available from Lau (1982) and Matsumura and Otani (2025). This casts doubt on the methodology and complicates model selection.

We address this by proposing a constrained GMM estimator that incorporates theoretical conditions for the uniqueness of equilibrium. First, we derive new conditions guaranteeing a unique equilibrium. Second, Monte Carlo simulations show that parameter restrictions alone yield inaccurate estimates and numerical errors, while adding equilibrium conditions resolves them. We also propose a modified GMM formulation that further mitigates these issues.

2 Model

Consider data with T markets with homogeneous products. Assume there are N firms in each market. Let $t = 1, \dots, T$ be the index of markets. Then, we obtain the supply equation:

$$P_t + \theta Q_t P_t'(Q_t) = MC_t(Q_t), \tag{1}$$

where Q_t is the aggregate quantity, $P_t(Q_t)$ is the inverse demand function, $MC_t(Q_t)$ is the marginal cost function, and $\theta \in [0, 1]$ is the conduct parameter. The equation nests perfect competition ($\theta = 0$), Cournot competition ($\theta = 1/N$), and perfect collusion ($\theta = 1$). See Bresnahan (1982) for the details.

Consider an econometric model. Assume that the inverse demand and the marginal cost functions are given as

$$P_t = f(Q_t, X_t^d, \varepsilon_t^d, \alpha),$$
$$MC_t = g(Q_t, X_t^c, \varepsilon_t^c, \gamma),$$

where X_t^d and X_t^c are the vector of exogenous variables, ε_t^d and ε_t^c are the error terms, and α and γ are the vector of parameters. We allow X_t^d and X_t^c to have common variables, but assume that there is at least one demand variable and one cost variable that are mutually excluded. We also have the demand- and supply-side instruments, Z_t^d and Z_t^c, and assume that the error terms satisfy the mean independence condition, $E[\varepsilon_t^d \mid X_t^d, Z_t^d] = E[\varepsilon_t^c \mid X_t^c, Z_t^c] = 0$.

The identification of the conduct parameter is indirectly characterized by Lau (1982):

Theorem 1. *Under the assumption that the industry inverse demand and cost functions are twice continuously differentiable, the index of competitiveness θ cannot be identified from data on industry price and output and other exogenous variables alone if and only if the industry inverse demand function is separable in X^d, that is, $f(Q, r(X^d))$, but not take the form $P = Q^{-1/\theta} r(X^d) + s(Q)$.*

This theorem implies that the conduct parameter is identified if the inverse demand function is not separable. A demand rotation instrument (Bresnahan 1982) achieves this. See Appendix A.1 for the details of the definition of separability.

2.1 Log-linear demand and log-linear marginal cost

Consider a log-linear model, which is a typical specification. The inverse demand and marginal cost functions are specified as

$$\log P_t = \alpha_0 - (\alpha_1 + \alpha_2 Z_t^R)\log Q_t + \alpha_3 \log Y_t + \varepsilon_t^d, \tag{2}$$
$$\log MC_t = \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c, \tag{3}$$

where Y_t and Z_t^R are excluded demand shifters and W_t and R_t are excluded cost shifters. When Y_t and Z_t^R vary without changing the equilibrium quantity, they work as the demand rotation instrument. Then, (1) is written as

$$P_t = \theta(\alpha_1 + \alpha_2 Z_t^R)P_t + MC_t. \tag{4}$$

By taking logarithm of (4) and substituting (3), we obtain

$$\log P_t = -\log(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) + \gamma_0 + \gamma_1 \log Q_t + \gamma_2 \log W_t + \gamma_3 \log R_t + \varepsilon_t^c. \tag{5}$$

The intersection of (2) and (5) determines the equilibrium, but there could be multiple equilibria. Although this model is widely known, no paper has examined the multiple equilibria problem to our knowledge. The next proposition provides the conditions for uniqueness. The proof is in the online appendix A.2.

Proposition 1. *Assume that* $\alpha_1 + \alpha_2 Z^R \neq 0$. *Let* $\Xi = \gamma_0 + \gamma_1 \frac{\alpha_0 + \alpha_3 \log Y + \varepsilon^d}{\alpha_1 + \alpha_2 Z^R} + \gamma_2 \log W + \gamma_3 \log R + \varepsilon^c$. *The number of equilibria is determined as follows:*

- *When* $1 - \theta(\alpha_1 + \alpha_2 Z^R) \leq 0$, *there is no equilibrium,*
- *When* $1 - \theta(\alpha_1 + \alpha_2 Z^R) > 0$,
  - *If* $\gamma_1 + \alpha_1 + \alpha_2 Z^R \neq 0$, *there is a unique equilibrium,*
  - *If* $\gamma_1 + \alpha_1 + \alpha_2 Z^R = 0$, *there are infinitely many equilibria when* $\exp(\Xi) = 1 - \theta(\alpha_1 + \alpha_2 Z_t^R)$, *but there is no equilibrium otherwise.*

The condition $1 - \theta(\alpha_1 + \alpha_2 Z^R) > 0$ rules out the region where the log transformation leaves its domain, which corresponds to implausibly elastic demand combined with large conduct. The assumption $\gamma_1 + \alpha_1 + \alpha_2 Z^R \neq 0$ excludes the knife-edge case in which the (pseudo) supply and demand are exactly parallel so that every price could be an equilibrium; both restrictions are technical and do not bind in regular empirical settings.

3 Estimation

Let $\xi = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, \gamma_0, \gamma_1, \gamma_2, \gamma_3, \theta)$ be the vector of the parameters in the model. We use the GMM for the estimation. Among GMM estimators, we apply the nonlinear system two-stage-least-squares (N2SLS) using (2) and (5). We rewrite the demand equation (2) and the supply equation (5) as

$$\varepsilon_t^d(\xi) = \log P_t - \alpha_0 + (\alpha_1 + \alpha_2 Z_t^R)\log Q_t - \alpha_3 \log Y_t, \tag{6}$$
$$\varepsilon_t^c(\xi) = \log P_t + \log(1 - \theta(\alpha_1 + \alpha_2 Z_t^R)) - \gamma_0 - \gamma_1 \log Q_t - \gamma_2 \log W_t - \gamma_3 \log R_t. \tag{7}$$

To estimate the parameters, we convert the conditional moments, $E[\varepsilon_t^d \mid Z_t^d] = E[\varepsilon_t^c \mid Z_t^c] = 0$, into unconditional moments, $E[\varepsilon_t^d Z_t^d] = E[\varepsilon_t^c Z_t^c] = 0$. Using Equations (6) and (7), we construct the sample analog of the unconditional moments:

$$g(\xi) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^T \varepsilon_t^d(\xi) Z_t^d \\ \frac{1}{T}\sum_{t=1}^T \varepsilon_t^c(\xi) Z_t^c \end{bmatrix}.$$

We define the N2SLS estimator as the solution to the problem,

$$\xi^* = \arg\min_{\xi} \ g(\xi)^\top W g(\xi) \tag{8}$$

where the weight matrix W is defined as

$$W = \left[\frac{1}{T}\sum_{t=1}^{T} Z_t^\top Z_t\right]^{-1} \text{ where } Z_t = \begin{bmatrix} Z_t^{d\top} & 0 \\ 0 & Z_t^{c\top} \end{bmatrix}.$$

We also add the following constraints based on Proposition 1 to (8):

$$0 \leq \theta \leq 1, \tag{9}$$

$$\alpha_1 + \alpha_2 Z_t^R > 0, \quad \gamma_1 > 0, \quad t = 1, \dots, T \tag{10}$$

$$1 - \theta(\alpha_1 + \alpha_2 Z_t^R) > 0, \quad t = 1, \dots, T. \tag{11}$$

Constraint (9) is a standard assumption on the conduct parameter. Constraint (10) implies the downward-sloping demand and upward-sloping marginal cost, which guarantees that $\gamma_1 + \alpha_1 + \alpha_2 Z^R \neq 0$. Constraint (11) relates to the uniqueness of equilibrium. See the detailed simulation setting in the online appendix A.3.

4 Simulation results

We compare N2SLS estimations with and without constraints in Table 1. Panel (a) shows that, without constraints, the estimator fails to recover γ_0 and θ, replicating known issues due to the flat objective function and invalid search regions without equilibrium (Appendix A.4). Panel (b), which imposes Constraint (9), improves estimation in large samples via the domain restriction.[1] However, in small samples, demand parameter estimates degrade and convergence declines. When convergence fails, α_1 becomes large, rendering $1 - \theta(\alpha_1 + \alpha_2 Z_t^R) < 0$ and causing numerical errors inside the log term in (1). Adding constraints (10) and (11) in Panel (c) improves small-sample convergence and demand accuracy, though convergence is not guaranteed. In large samples, performance surpasses that of Panel (b) for some parameters.

To address convergence failure, we propose an alternative formulation (Table 2) that computes ε_t^c via (3) and enforces Equation (4) as a constraint, along with Constraints (9)–(11).[2] This avoids log terms in both objective and constraints, achieving 100% convergence and reducing θ's bias and RMSE to 0.014 and 0.217, though not dominating Panel (c) across all parameters. In sum, incorporating equilibrium uniqueness conditions and eliminating log terms greatly improves conduct parameter estimation. Additional experiments appear in Appendix A.5.

5 Discussion

Two concerns surround the conduct parameter approach: the difficulty of interpreting intermediate or extreme values, and the critique by Corts (1999) that it may understate market power under collusion.[3] We show that implausible estimates in log-linear models often stem from numerical issues—especially when equilibrium conditions are omitted—rather than conceptual flaws. Addressing these issues yields more stable and interpretable results.

This distinction matters: misattributing numerical artifacts to theoretical limits risks dismissing a useful tool. Our findings aim to encourage more constructive use of the conduct parameter approach.

[1] Constraints (10) and (11) alone yield severe bias; see Table 9 and Appendix A.5.

[2] See Appendix A.3 for details.

[3] As Magnolfi and Sullivan (2022) note, this critique does not apply when the data stem from a static model.

Table 1: Performance comparison

(a) N2SLS without Constraints (9), (10), and (11)

	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
α_0	-1.070	7.012	-0.021	5.110	0.365	2.207	0.400	2.030
α_1	-0.164	1.060	-0.001	0.782	0.073	0.458	0.096	0.574
α_2	-0.011	0.104	-0.006	0.071	0.002	0.033	0.005	0.043
α_3	-0.101	0.619	-0.005	0.474	0.021	0.198	0.029	0.187
γ_0	9.735	15.743	9.636	10.870	13.173	13.269	13.294	13.351
γ_1	-0.070	1.624	-0.177	0.469	-0.184	0.248	-0.177	0.220
γ_2	-0.034	0.939	-0.098	0.317	-0.090	0.152	-0.080	0.127
γ_3	-0.047	0.750	-0.091	0.311	-0.098	0.156	-0.085	0.133
θ	-3e+05	3e+06	-2e+05	2e+06	-8e+04	9e+04	-9e+04	1e+05
Runs converged (%)		99.500		99.800		98.600		98.400
Sample size (T)		100		200		1000		1500

(b) N2SLS with Constraints (9)

	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
α_0	-1.922	8.603	-0.068	5.116	0.037	2.035	0.000	1.556
α_1	-0.299	1.314	-0.010	0.785	0.005	0.312	0.000	0.240
α_2	-0.013	0.104	-0.002	0.063	0.001	0.024	0.000	0.019
α_3	-0.165	0.774	-0.007	0.472	-0.004	0.185	-0.001	0.152
γ_0	-1.767	14.394	-1.001	6.530	-0.208	1.993	-0.156	1.566
γ_1	0.255	1.949	0.132	0.838	0.034	0.229	0.027	0.174
γ_2	0.125	1.097	0.053	0.475	0.017	0.150	0.019	0.119
γ_3	0.099	0.903	0.062	0.481	0.007	0.149	0.014	0.120
θ	-0.098	0.441	-0.060	0.421	-0.061	0.319	-0.058	0.281
Runs converged (%)		98.100		98.700		100.000		100.000
Sample size (T)		100		200		1000		1500

(c) N2SLS with Constraints (9), (10), and (11)

	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
α_0	-0.905	6.954	0.120	5.001	0.072	2.042	0.052	1.563
α_1	-0.141	1.053	0.018	0.768	0.010	0.313	0.008	0.241
α_2	-0.006	0.101	0.000	0.062	0.001	0.024	0.001	0.019
α_3	-0.088	0.620	0.007	0.475	-0.001	0.186	0.003	0.152
γ_0	-1.748	14.206	-0.938	6.428	0.015	1.995	0.163	1.570
γ_1	0.254	1.927	0.129	0.825	0.018	0.226	0.003	0.170
γ_2	0.117	1.083	0.049	0.467	0.008	0.148	0.007	0.116
γ_3	0.098	0.890	0.058	0.478	-0.001	0.148	0.003	0.118
θ	-0.100	0.441	-0.072	0.424	-0.121	0.351	-0.148	0.333
Runs converged (%)		99.600		99.900		100.000		100.000
Sample size (T)		100		200		1000		1500

Note: The error terms are drawn from a normal distribution, $N(0, \sigma)$. True values: $\alpha_0 = 20.0, \alpha_1 = 1.0, \alpha_2 = 0.1, \alpha_3 = 1.0, \gamma_0 = 5.0, \gamma_1 = 1.0, \gamma_2 = 1.0, \gamma_3 = 1.0, \theta = 0.5$ and $\sigma = 1.0$. See online appendix A.3 and Matsumura and Otani (2024) for the setting.

Table 2: Ad hoc method using (3) to compute ε_t^c and (4) with Constraints (9), (10), and (11)

	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
α_0	-0.614	5.995	-0.213	4.315	0.077	2.034	0.063	1.555
α_1	-0.085	0.902	-0.024	0.663	0.011	0.312	0.010	0.240
α_2	-0.028	0.105	-0.022	0.073	0.000	0.025	0.001	0.020
α_3	-0.070	0.549	-0.019	0.431	-0.001	0.185	0.004	0.152
γ_0	-5.106	15.922	-2.379	6.990	-0.375	1.959	-0.398	1.533
γ_1	0.386	2.047	0.141	0.839	0.045	0.229	0.044	0.175
γ_2	0.190	1.155	0.054	0.475	0.022	0.150	0.027	0.120
γ_3	0.163	1.006	0.065	0.482	0.013	0.149	0.023	0.121
θ	0.186	0.442	0.158	0.422	-0.007	0.275	0.014	0.217
Runs converged (%)		100.000		100.000		100.000		100.000
Sample size (T)		100		200		1000		1500

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