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Abstract

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Trade and growth with heterogeneous firms revisited once again

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January 9, 2017

Abstract

To study the long-run growth and welfare effects of both symmetric and asymmetric trade liberalization, we extend Baldwin and Robert-Nicoud (2008) to allow for asymmetric countries. We obtain four main results. First, the dynamic effect strictly dominates the static effect on expenditure if and only if the knowledge sector is active. Second, under a generalized Coe-Helpman specification, unilateral trade liberalization can raise the balanced growth rate. Third, in the symmetric country case, we derive extended autarkiness ratio formulas for long-run growth and welfare. Fourth, growth-enhancing unilateral trade liberalization is not sufficient for higher long-run welfare for at most one country.

JEL classification: F13; F43

Keywords: Trade and growth; Heterogeneous firms; Asymmetric countries; Unilateral trade liberalization; Endogenous growth

1 Introduction

The idea of the Melitz (2003) model is that trade liberalization causes selection of heterogeneous firms. Baldwin and Robert-Nicoud (2008) (henceforth BRN) first study the implication of the liberalization-induced firm selection for long-run growth with two symmetric countries. They find that trade liberalization may either raise or lower long-run growth, depending on whether its positive effect through the increased international knowledge spillovers (which they call "the p_K-channel") is larger or smaller than its negative effect through the increased expected fixed costs due to tougher selection (which they call "the $\overline{\kappa}$-channel"). Under their Grossman-Helpman and Coe-Helpman specifications for R&D technologies, the p_K-channel is weaker than the $\overline{\kappa}$-channel, so that trade liberalization slows down long-run growth; the opposite occurs under their efficiency-linked knowledge spillovers, reverse engineering, and lab-equipment specifications.

Ourens (2016) points out that BRN's welfare calculations contain some errors. After correcting them and decomposing the total long-run welfare effect of trade liberalization into "the static effect on expenditure", "the static effect on the price index", and "the dynamic effect", he finds that the static effect on expenditure is negative if and only if the dynamic effect is positive (his implication (d) and Result 1), and that the former can outweigh the latter (his implication (e) and Result 2). Even if the static effect on the price index is always positive (his implication (a)), the total long-run welfare effect of trade liberalization depends on the sign of the sum of the static effect on expenditure and the dynamic effect.

To study the long-run growth and welfare effects of both symmetric and asymmetric trade liberalization, we extend the BRN model to allow for asymmetric countries. Considering asymmetric countries is practically important because: "(t)rade costs also vary widely across countries. On average, developing countries have significantly larger trade costs, by a factor of two or more in some important categories." (Anderson and van Wincoop, 2004, p. 747) We need to endogenize two new variables that were absent under symmetry: the relative wage and the relative number of domestic varieties. The former is determined from either country's zero balance of trade condition, whereas the latter is determined from the balanced growth condition.

The first result is a correction of Ourens' implication (e) and Result 2: the dynamic effect strictly dominates the static effect on expenditure if and only if the knowledge sector is active. As mentioned

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by BRN (based on wrong derivations though) and Sampson (2016), the equilibrium balanced growth rate is lower than the optimal one due to positive externalities from knowledge spillovers. Since a rise in the equilibrium balanced growth rate reallocates resources toward the optimal allocation, the long-run welfare rises even if the expenditure decreases. A corollary of this result is that, in the symmetric country case, bilateral trade liberalization raises the long-run welfare if it raises the balanced growth rate. This sharpens Ourens' welfare implications.

We obtain more results (see the paragraphs after the proofs of propositions for intuitions and implications). Second, under a generalized Coe-Helpman specification of the asymmetric BRN model, where the degree of international knowledge spillovers for country j is nondecreasing in country k's fraction of exporters in its domestic surviving firms with elasticity ε, unilateral trade liberalization raises the balanced growth rate if ε is sufficiently large. Third, in the symmetric country case, we derive the extended autarkiness ratio formulas of Arkolakis, Costinot, and Rodríguez-Clare (2012) (henceforth ACR) for long-run growth and welfare. Fourth, in the general case, even if unilateral trade liberalization raises the balanced growth rate, it is not sufficient for higher long-run welfare for at most one country.

This paper contributes to the growing literature on the welfare effects of trade liberalization in heterogeneous firm trade models with nonendogenous (e.g., Atkeson and Burstein, 2010; Buera and Oberfield, 2016), semiendogenous (e.g., Gustafsson and Segerstrom, 2010), or endogenous growth (e.g., Dinopoulos and Unel, 2011; Alvarez et al., 2014; Perla et al., 2015; Sampson, 2016), mostly assuming symmetric countries. In a model with two competing modes of innovation, process innovation (raising productivity) and product innovation (increasing variety), Atkeson and Burstein (2010) demonstrate that the positive welfare effects of trade liberalization through reallocations are largely offset by decreased product innovation in a steady state. Similar welfare implications arise even in an endogenous growth model of Perla et al. (2015) with both costly technology adoption and endogenous variety. Based on a continuum-good Ricardian framework, Alvarez et al. (2014) and Buera and Oberfield (2016) study the effects of trade liberalization on technology diffusion and welfare with symmetric countries or small versus large countries. Since they assume away dynamic optimization and thus use the static utility as a welfare measure, they might underestimate the long-run welfare effects of trade liberalization. Gustafsson and Segerstrom (2010) present a semiendogenous growth version of the BRN model. Focusing on the Grossman-Helpman specification for international knowledge spillovers, they find that trade liberalization raises the path of per capita real consumption if the elasticity of R&D productivity with respect to knowledge is sufficiently small. This is because the positive static effect from selection outweighs the negative dynamic effect from slower variety growth. Due to the semiendogenous growth setting, trade liberalization can affect growth only in the short run, but not in the long run. Dinopoulos and Unel (2011) and Sampson (2016) are most related to this paper: using different Melitz-based endogenous growth models, they derive a similar tradeoff between the dynamic effect and the static effect on expenditure to Ourens (2016). Sampson (2016, Proposition 3) even shows that the direct effect of growth on welfare is always stronger than its indirect effect decreasing the initial consumption, and that trade liberalization always raises welfare through faster growth as well as static reallocations. This paper is different from Sampson (2016) in that trade liberalization can either raise or lower growth and welfare in the symmetric country case of our model. More importantly, we allow for two asymmetric large countries in a Melitz-based endogenous growth model, and analytically examine the long-run growth and welfare effects of unilateral trade liberalization.

Section 2 derives a general long-run welfare formula. Section 3 examines the effects of trade liberalization. Section 4 concludes. Details of model setup and derivations are given in Appendix.

2 General long-run welfare formula of the asymmetric BRN model

Appendix shows that the long-run welfare of country $j(=1,2)$ in the asymmetric BRN model is given by:

$$\rho U_j = \ln E_j^* - \ln P_j^* + (1/\rho)\gamma_j^*/(\sigma-1); \tag{1}$$

$$E_j^* = p_j^{K*}\overline{\kappa}_j^*[\rho + w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*)] = p_j^{K*}\overline{\kappa}_j^*\rho + w_j^* L_j, \tag{2}$$

$$\gamma_j^* = (1/\sigma)w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*) - (1-1/\sigma)\rho - \delta, \tag{3}$$

where ρ is the subjective discount rate, U_j is the long-run welfare, E_j^* is the expenditure, P_j^* is the price index, γ_j^* is the growth rate of the number of domestic varieties, $\sigma(>1)$ is the elasticity of substitution between any two varieties, p_j^{K*} is: "an 'intensive form' " (BRN, 2008, p. 25) of the price of the knowledge good P_j^K, $\overline{\kappa}_j^*$ is: "the expected units of knowledge required to get a 'winner.' " (BRN, 2008, p. 25), w_j^* is the wage rate (with $w_2^* \equiv 1$), L_j is the supply of labor, δ is the death rate of manufacturing firms (e.g., Melitz, 2003; Ourens, 2016), and a superscript asterisk represents a balanced growth path (BGP). The exact form of P_j^* is omitted because it is irrelevant to our welfare analysis. Eqs. (1), (2), and (3) are the same as Eqs. (22), (19), and (20) of Ourens (2016), respectively, except that w_1^* is now endogenously determined and can be different from unity.

Totally differentiating Eq. (1) gives $\rho dU_j = \widehat{E}_j^* - \widehat{P}_j^* + (1/\rho)[1/(\sigma-1)]d\gamma_j^*$, where $\widehat{E}_j^* \equiv d\ln E_j^* \equiv dE_j^*/E_j^*$. Ourens (2016) calls the long-run welfare effects of bilateral trade liberalization coming from the first, second, and third terms of this expression "the static effect on expenditure", "the static effect on the price index", and "the dynamic effect", respectively.[1] He also points out that, from Eqs. (2), (3), and $w_j^* = 1$, the static effect on expenditure always moves in the opposite direction of the dynamic effect (his implication (d) and Result 1), and that the former can be greater than the latter (his implication (e) and Result 2). The following proposition shows that the last statement is impossible:

Proposition 1 *In the asymmetric BRN model including the symmetric country case of Ourens (2016), the dynamic effect always weakly dominates the static effect on expenditure. Moreover, the former strictly dominates the latter if and only if the knowledge sector is active.*

Proof. On a BGP: $\gamma_1^* = \gamma_2^* \equiv \gamma^*$, Eq. (3) is rewritten as $w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*) + \rho = \sigma(\rho+\delta+\gamma^*)$, or $p_j^{K*}\overline{\kappa}_j^* = w_j^* L_j/[\sigma(\rho+\delta+\gamma^*)-\rho]$, where $p_j^{K*}\overline{\kappa}_j^* > 0 \Leftrightarrow \sigma(\rho+\delta+\gamma^*)-\rho > 0 \Rightarrow \rho+\delta+\gamma^* > 0$. Using these expressions, Eq. (2) is rewritten as $E_j^* = w_j^* L_j \sigma(\rho+\delta+\gamma^*)/[\sigma(\rho+\delta+\gamma^*)-\rho]$. Using this, ρdU_j is rewritten as:

$$\rho dU_j = \widehat{w}_j^* - \widehat{P}_j^* + \Gamma^* d\gamma^* = -(1+\beta)\widehat{a}_{jj}^* + \Gamma^* d\gamma^* = -\frac{1+\beta}{\theta}\widehat{\lambda}_{jj}^* + \Gamma^* d\gamma^*; \beta \equiv \frac{\theta}{\sigma-1} > 1, \tag{4}$$

$$\Gamma^* \equiv \frac{1}{\rho+\delta+\gamma^*} - \frac{\sigma}{\sigma(\rho+\delta+\gamma^*)-\rho} + \frac{1}{\rho}\frac{1}{\sigma-1} = \frac{(\delta+\gamma^*)[(\sigma-1)\rho+\sigma(\rho+\delta+\gamma^*)]}{(\rho+\delta+\gamma^*)[\sigma(\rho+\delta+\gamma^*)-\rho]\rho(\sigma-1)},$$

where $\theta(>\sigma-1>0)$ is the shape parameter of the Pareto distribution for the unit labor requirement of firms: $G_j(a) \equiv (a/a_{j0})^\theta$, a_{jk}^* is the cutoff unit labor requirement of source country j in destination country $k(j,k=1,2)$, $\lambda_{jk}^*(\in[0,1])$ is the revenue share of varieties country j sells to country k, and $\widehat{w}_j^* - \widehat{P}_j^* = -(1+\beta)\widehat{a}_{jj}^* = -[(1+\beta)/\theta]\widehat{\lambda}_{jj}^*$ is proved in Appendix. In the definition of Γ^*, the sum of the first and second terms shows the static effect on expenditure, which is negative, whereas the third positive term represents the dynamic effect. Considering the market-clearing condition for the knowledge good $Q_j^K = \overline{\kappa}_j n_{jj}(\gamma+\delta)$, where Q_j^K is the supply of the knowledge good, and n_{jk} is the number of varieties country j sells to country k, we have $\Gamma^* \geq 0 \Leftrightarrow \delta+\gamma^* \geq 0 \Leftrightarrow Q_j^K \geq 0$, with strict inequality if and only if $Q_j^K > 0$. ■

Suppose, in contrast to our claim, that the dynamic effect were strictly dominated by the static effect on expenditure, so that $\Gamma^* < 0 \Leftrightarrow \delta+\gamma^* < 0$. The latter would mean that the number of domestic varieties (including the replacement of exiting firms) would decrease over time. This would be possible only if $Q_j^K < 0$, which would contradict with the nonnegativity of Q_j^K, a natural constraint in economics.

Eq. (4) provides a general long-run welfare formula, which is applicable to both symmetric and asymmetric country cases: country j's long-run welfare is nondecreasing in the balanced growth rate and increasing in its real wage, which is decreasing in its autarkiness ratio (i.e., domestic revenue and expenditure share) as shown by ACR. Since Ourens (2016) reveals that bilateral trade liberalization partly raises the long-run welfare through the static effect on the price index, and raises the balanced growth rate under three of the five specifications for R&D technologies by BRN, the total long-run welfare effect of bilateral trade liberalization in the symmetric BRN model is summarized as follows:

[1] BRN (2008, p. 32) wrongly derive $U_j = [E_j^*/(\rho+\gamma_j^*)](1/P_j^*)$ (assuming that $\delta = 0$), where $E_j^*/(\rho+\gamma_j^*)$ and $1/P_j^*$ capture "the dynamic welfare aspects" and "the static aspects", respectively. Their Result 2 states that: "(t)he static welfare effect of greater openness is always positive". By extending the meaning of the word "static" to include the expenditure, Ourens (2016, Result 1) states that: "(t)he static welfare effect of greater openness is not necessarily positive." The difference in their statements is simply due to the difference in the definition of the term "static".

Corollary 1 *In the symmetric BRN model, bilateral trade liberalization raises the long-run welfare under efficiency-linked knowledge spillovers, reverse engineering, and lab-equipment specifications by BRN. The total long-run welfare effect is ambiguous under their Grossman-Helpman and Coe-Helpman specifications.*

3 Long-run growth and welfare effects of trade liberalization

3.1 Long-run growth effect of unilateral trade liberalization

Appendix provides a complete long-run analysis of the asymmetric BRN model under a generalized Coe-Helpman specification for R&D technologies:

$$a_j^K(n_{jj}, n_{kk}) = 1/(n_{jj} + \widetilde{\psi}_j n_{kk}); \widetilde{\psi}_j \equiv \psi_j (G_k(a_{kj})/G_k(a_{kk}))^{\varepsilon} = (a_{kj}/a_{kk})^{\varepsilon\theta}; \psi_j \in [0,1], \varepsilon \geq 0,$$

where a_j^K is the unit labor requirement of the representative R&D firm in country j, and $\widetilde{\psi}_j$ is the degree of international knowledge spillovers for country j, which is nondecreasing in $G_k(a_{kj})/G_k(a_{kk})$: "the fraction of foreign varieties that are imported" (BRN, 2008, p. 29), with elasticity ε. This encompasses BRN's Grossman-Helpman ($\varepsilon = 0$) and Coe-Helpman specifications ($\varepsilon = 1$) as special cases.

In addition to doubling the number of endogenous variables, the asymmetric BRN model has two more variables: the relative wage of country 1 to country 2 w_1, and the relative number of domestic varieties of country 1 to country 2 $\chi \equiv n_{11}/n_{22}$. They additionally affect countries' growth rates through both "the $\overline{\kappa}_j$-channel" and "the a_j^K-channel".[2] w_1^* is determined from country 1's zero balance of trade condition, whereas χ^* is determined from the balanced growth condition. The following proposition states the long-run growth effect of unilateral trade liberalization:

Proposition 2 *Under a generalized Coe-Helpman specification of the asymmetric BRN model, unilateral trade liberalization raises the balanced growth rate if ε is sufficiently large.*

Proof. Appendix shows that, after endogenizing all cutoffs and w_1^*, $d\gamma_1^*$ and $d\gamma_2^*$ are given by:

$$\begin{aligned}
d\gamma_1^* &= (1/\sigma)[L_2/(a_2^{K*}\overline{\kappa}_2^*)]\{1/[(\sigma-1)\widetilde{B}^*]\}[-\widetilde{D}_1^*\widehat{\chi}^* + \theta(\sigma-1)(I_1^*\widehat{\tau}_{21} + J_1^*\widehat{\tau}_{12})], && (5)\\
d\gamma_2^* &= (1/\sigma)[L_2/(a_2^{K*}\overline{\kappa}_2^*)]\{1/[(\sigma-1)\widetilde{B}^*]\}[\widetilde{D}_2^*\widehat{\chi}^* + \theta(\sigma-1)(I_2^*\widehat{\tau}_{12} + J_2^*\widehat{\tau}_{21})]; && (6)\\
\widetilde{B}^* &\equiv 2\beta\sigma - (1 - \lambda_{12}^* - \lambda_{21}^*) + \varepsilon\beta(\alpha_1^* + \alpha_2^*)(\sigma-1) > 0,\\
\alpha_1^* &\equiv (\widetilde{\psi}_1^*/\chi^*)/(1 + \widetilde{\psi}_1^*/\chi^*) \in [0,1], \alpha_2^* \equiv \widetilde{\psi}_2^*\chi^*/(1 + \widetilde{\psi}_2^*\chi^*) \in [0,1],\\
\widetilde{D}_j^* &\equiv (\sigma-1)\widetilde{B}^*\alpha_j^* - \theta[\sigma(\alpha_1^* + \alpha_2^*) + 1 - \alpha_1^* - \alpha_2^*](\lambda_{jk}^* + \varepsilon\alpha_j^*), k \neq j,\\
I_j^* &\equiv \lambda_{jk}^*[\beta\sigma + \lambda_{kj}^* + \varepsilon\beta\alpha_j^*(\sigma-1)] - \varepsilon\alpha_j^*[\beta\sigma - \lambda_{jj}^* + \varepsilon\beta\alpha_k^*(\sigma-1)], k \neq j,\\
J_j^* &\equiv \lambda_{jk}^*[\beta\sigma - \lambda_{kk}^* + \varepsilon\beta\alpha_j^*(\sigma-1)] - \varepsilon\alpha_j^*[\beta\sigma + \lambda_{jk}^* + \varepsilon\beta\alpha_k^*(\sigma-1)], k \neq j,
\end{aligned}$$

where $\tau_{jk}(\geq 1)$ is the iceberg trade cost factor of delivering one unit of a variety from country j to country k, and $\widetilde{D}_j^* > 0 \forall j$ is assumed to ensure local stability around a BGP. Substituting Eqs. (5) and (6) into $d\gamma_1^* = d\gamma_2^*$ to solve for $\widehat{\chi}^*$, and substituting the result back into Eq. (6), we obtain:

$$\begin{aligned}
\widehat{\chi}^* &= [\theta(\sigma-1)/\widetilde{D}^*][(I_1^* - J_2^*)\widehat{\tau}_{21} - (I_2^* - J_1^*)\widehat{\tau}_{12}]; \widetilde{D}^* \equiv \widetilde{D}_1^* + \widetilde{D}_2^* > 0, && (7)\\
d\gamma^* &= (1/\sigma)[L_2/(a_2^{K*}\overline{\kappa}_2^*)][\theta/(\widetilde{B}^*\widetilde{D}^*)][(\widetilde{D}_2^*I_1^* + \widetilde{D}_1^*J_2^*)\widehat{\tau}_{21} + (\widetilde{D}_1^*I_2^* + \widetilde{D}_2^*J_1^*)\widehat{\tau}_{12}]. && (8)
\end{aligned}$$

Eq. (8) shows that $\partial\gamma^*/\partial \ln\tau_{kj} < 0 \Leftrightarrow \widetilde{D}_k^* I_j^* + \widetilde{D}_j^* J_k^* < 0$, which is true if ε is sufficiently large. ■

[2] Using $p_1^K \equiv n_{11}P_1^K = w_1 a_1^K(1, 1/\chi)$ and $p_2^K \equiv n_{22}P_2^K = a_2^K(1,\chi)$, p_j^K/w_j in Eq. (3) is rewritten as $p_1^K/w_1 = a_1^K(1, 1/\chi)$ and $p_2^K/w_2 = a_2^K(1,\chi)$. This is why we use the term "the a_j^K-channel" instead of "the p_K-channel" (BRN, 2008, p. 27).

In Eqs. (5) and (6), I_j^* and J_j^* summarize the direct effects of changes in τ_{kj} and τ_{jk}, respectively, on γ_j^*. Suppose that country 1 unilaterally liberalizes its imports, that is, τ_{21} falls. This encourages country 2's exports (i.e., increases a_{21}^*) and induces more domestic selection (i.e., decreases a_{22}^*). Since this tends to create country 1's trade deficit, w_1^* should fall for its balance of trade to go back to zero. This causes more exports (i.e., increases a_{12}^*) and more domestic selection (i.e., decreases a_{11}^*) for the liberalizing country 1. The decrease in a_{22}^* is bad for country 2's growth through the $\overline{\kappa}_j$-channel (i.e., the first term of J_2^*), but is good for country 1's growth through the a_j^K-channel by increasing the degree of international knowledge spillovers for country 1 (i.e., the second term of I_1^*). Similarly, the decrease in a_{11}^* partly lowers country 1's growth (i.e., the first term of I_1^*), but partly raises country 2's growth (i.e., the second term of J_2^*). If ε is so large that the positive growth effects of a fall in τ_{21} through the the a_j^K-channels are dominant, the balanced growth rate rises.

3.2 Bilateral trade liberalization in the symmetric country case

In the symmetric country case, we obtain sharper results:

Proposition 3 *Under a generalized Coe-Helpman specification of the symmetric BRN model, bilateral trade liberalization raises the balanced growth rate if and only if $\varepsilon > \lambda_{21}^*/\alpha_2^*$. The long-run welfare necessarily rises in this case.*

Proof. Appendix shows that, in the symmetric country case where $\widehat{\tau}_{12} = \widehat{\tau}_{21}$, Eq. (8) reduces to:

$$d\gamma^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \{[\sigma(\rho+\delta+\gamma^*)-\rho]/\sigma\}[(\lambda_{21}^*-\varepsilon\alpha_2^*)/\lambda_{21}^*]\widehat{\lambda}_{22}^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}};\ \widehat{\lambda}_{22}^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \theta\lambda_{21}^* > 0. \quad (9)$$

Eq. (9) shows that $d\gamma^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} < 0 \Leftrightarrow \varepsilon > \lambda_{21}^*/\alpha_2^*$. Substituting Eq. (9) into Eq. (4) gives:

$$\rho \left.\frac{dU_j}{\widehat{\tau}_{21}}\right|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \left[-\frac{1+\beta}{\theta} + \Gamma^* \frac{\sigma(\rho+\delta+\gamma^*)-\rho}{\sigma}\frac{\lambda_{21}^*-\varepsilon\alpha_2^*}{\lambda_{21}^*}\right] \left.\frac{\widehat{\lambda}_{22}^*}{\widehat{\tau}_{21}}\right|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}}. \quad (10)$$

Eq. (10) implies that $\rho dU_j/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} < 0 \Leftarrow \varepsilon > \lambda_{21}^*/\alpha_2^*$. ■

Eq. (9) is the ACR formula for the balanced growth rate: a decrease in a country's autarkiness ratio due to bilateral trade liberalization is associated with a rise in the balanced growth rate if and only if $\varepsilon > \lambda_{21}^*/\alpha_2^*$. BRN demonstrate that bilateral trade liberalization lowers the balanced growth rate under both their Grossman-Helpman ($\varepsilon = 0$) and Coe-Helpman specifications ($\varepsilon = 1$). However, bilateral trade liberalization can raise the balanced growth rate under a generalized Coe-Helpman specification.

Eq. (10) is the ACR formula for the long-run welfare. Compared with the static Melitz model, where only the first term in the square brackets is present, the symmetric BRN model adds the second term, which either reinforces or counteracts the first term depending on whether $\varepsilon > \lambda_{21}^*/\alpha_2^*$ or not.

3.3 Unilateral trade liberalization in the general case

In the general asymmetric country case, we have another interesting result for the long-run welfare:

Proposition 4 *Under a generalized Coe-Helpman specification of the asymmetric BRN model, even if unilateral trade liberalization raises the balanced growth rate, it is not sufficient for higher long-run welfare for at most one country.*

Proof. Suppose that $\widetilde{D}_k^* I_j^* + \widetilde{D}_j^* J_k^* < 0 \Leftrightarrow \partial\gamma^*/\partial\ln\tau_{kj} < 0$, implying that a fall in τ_{kj} raises γ^*. Its effects on countries' real wages can be seen from $\widehat{w}_j^* - \widehat{P}_j^* = -(1+\beta)\widehat{a}_{jj}^*$, where (see Appendix for derivations):

$$\begin{aligned}\widehat{a}_{11}^* = &\{\lambda_{12}^*/[(\sigma-1)\widetilde{B}^*]\}\{[\sigma(\alpha_1^*+\alpha_2^*)+1-\alpha_1^*-\alpha_2^*]\widehat{\chi}^* \\ &+ (\sigma-1)\{[\beta\sigma+\lambda_{21}^*+\varepsilon\beta\alpha_1^*(\sigma-1)]\widehat{\tau}_{21} + [\beta\sigma-\lambda_{22}^*+\varepsilon\beta\alpha_1^*(\sigma-1)]\widehat{\tau}_{12}\}\}, \quad (11)\\ \widehat{a}_{22}^* = &\{\lambda_{21}^*/[(\sigma-1)\widetilde{B}^*]\}\{-[\sigma(\alpha_1^*+\alpha_2^*)+1-\alpha_1^*-\alpha_2^*]\widehat{\chi}^* \\ &+ (\sigma-1)\{[\beta\sigma+\lambda_{12}^*+\varepsilon\beta\alpha_2^*(\sigma-1)]\widehat{\tau}_{12} + [\beta\sigma-\lambda_{11}^*+\varepsilon\beta\alpha_2^*(\sigma-1)]\widehat{\tau}_{21}\}\}. \quad (12)\end{aligned}$$

Eqs. (11) and (12) show that a fall in τ_{kj} directly decreases both a_{11}^* and a_{22}^*. However, it indirectly increases either a_{11}^* or a_{22}^*, depending on the sign of $\partial \ln \chi^* / \partial \ln \tau_{kj}$ from Eq. (7), which is generally ambiguous. Due to the last effect, the real wage of at most one country can fall, and so can its long-run welfare from Eq. (4). ■

Consider the case where $\widetilde{D}_2^* I_1^* + \widetilde{D}_1^* J_2^* < 0 \Leftrightarrow \partial \gamma^* / \partial \ln \tau_{21} < 0$ and $I_1^* - J_2^* > 0 \Leftrightarrow \partial \ln \chi^* / \partial \ln \tau_{21} > 0$. Then a fall in τ_{21} decreases χ^*. Since this lowers w_1^* from the home market effect in wages, exporters from country 2 become relatively less competitive, causing more inefficient firms to remain in their domestic market. If this effect is so strong, country 2's real wage or even its long-run welfare might fall even if it partly gains from faster long-run growth.

4 Concluding remarks

We first provide an analytically well-behaved model dealing with heterogeneous firms, R&D-based endogenous growth, and asymmetric countries at the same time. The model can be applied to a variety of policy issues requiring asymmetric countries such as the optimal tariff problem. The model could also be extended for more than two countries to study the long-run growth and welfare effects of regional trade agreements.

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Abstract

We extend Baldwin and Robert-Nicoud (2008) to allow for asymmetric countries. After formulating the model and characterizing a balanced growth path, we examine the long-run growth effects of unilateral trade liberalization under the Grossman-Helpman and generalized Coe-Helpman specifications. Finally, we study the long-run welfare effects of unilateral trade liberalization in the general case and bilateral trade liberalization in the symmetric country case. In the symmetric country case, we derive extended ACR formulas for the balanced growth rate and long-run welfare. In the general case, growth-enhancing unilateral trade liberalization is not sufficient for higher long-run welfare for at most one country.

1 The model

Our model is the same as Baldwin and Robert-Nicoud (2008) (henceforth BRN), except that there are two possibly asymmetric countries: $j = 1, 2$.

1.1 Households

The representative household in country j maximizes its overall utility $U_j = \int_0^\infty \ln D_{jt} \exp(-\rho t)dt, D_{jt} = (\int_{\Theta_{jt}} d_{jt}(i)^{(\sigma-1)/\sigma} di)^{\sigma/(\sigma-1)}$, subject to its budget constraint:

$$\dot{W}_{jt} = r_{jt}W_{jt} + w_{jt}L_j - E_{jt}; \dot{W}_{jt} \equiv dW_{jt}/dt, E_{jt} = \int_{\Theta_{jt}} p_{jt}(i)d_{jt}(i)di, \tag{1}$$

with $\{r_{jt}, w_{jt}, \{p_{jt}(i)\}_{i\in\Theta_{jt}}\}_{t=0}^\infty$ and W_{j0} given, where $t(\in [0, \infty))$ is time, D_j is the consumption index, ρ is the subjective discount rate, Θ_j is the set of available varieties of manufacturing goods, $d_j(i)$ is the demand for variety i, $\sigma(> 1)$ is the elasticity of substitution between any two varieties, W_j is the asset, r_j is the interest rate, w_j is the wage rate, L_j is the supply of labor, and E_j is the expenditure. The time subscript is omitted whenever no confusion arises. Minimizing $\int_{\Theta_j} p_j(i)d_j(i)di$ subject to the consumption index with $\{p_j(i)\}_{i\in\Theta_j}$ and D_j given implies the conditional demand function for variety i:

$$d_j(i) = p_j(i)^{-\sigma} P_j^\sigma D_j; P_j \equiv (\int_{\Theta_j} p_j(i)^{1-\sigma} di)^{1/(1-\sigma)}, \tag{2}$$

where P_j is the price index defined as the minimized expenditure to obtain a unit of the consumption index: $\int_{\Theta_j} p_j(i)d_j(i)di = P_jD_j = E_j$. Substituting $D_j = E_j/P_j$ into the overall utility function, the latter is rewritten as $U_j = \int_0^\infty (\ln E_{jt} - \ln P_{jt}) \exp(-\rho t)dt$. Maximizing this subject to Eq. (1) with $\{P_{jt}, r_{jt}, w_{jt}\}_{t=0}^\infty$ and W_{j0} given implies the Euler equation:

$$\dot{E}_{jt}/E_{jt} = r_{jt} - \rho. \tag{3}$$

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1.2 Manufacturing firms

A manufacturing firm with the unit labor requirement a in source country j maximizes its gross profit in destination country k: $\pi_{jk}(a) = p^f_{jk}(a)y_{jk}(a) - w_j l_{jk}(a)$, subject to its variable cost function in terms of labor $l_{jk}(a) = ay_{jk}(a)$, the market-clearing condition for its variety $y_{jk}(a) = \tau_{jk}d_{jk}(a)$, and the demand function for its variety $d_{jk}(a) = p_{jk}(a)^{-\sigma}P_k^{\sigma-1}E_k = (\tau_{jk}p^f_{jk}(a))^{-\sigma}P_k^{\sigma-1}E_k$ from Eq. (2), with w_j, P_k, and E_k given, where $p^f_{jk}(a)$ is the FOB supply price of the firm's variety, $y_{jk}(a)$ is the supply of the firm's variety, $l_{jk}(a)$ is the firm's demand for labor as its variable cost, $d_{jk}(a)$ is country k's demand for the firm's variety, $p_{jk}(a)$ is country k's demand price of the firm's variety, and $\tau_{jk}(\geq 1)$ is the iceberg trade cost factor of delivering one unit of a variety from country j to country k, with $\tau_{jj} = 1$. For $k \neq j$, a fall in τ_{jk} is interpreted as country k's import liberalization. Profit maximization gives:

$$(p^f_{jk}(a) - w_j a)/p^f_{jk}(a) = 1/\sigma \Leftrightarrow p^f_{jk}(a) = w_j a/(1 - 1/\sigma). \tag{4}$$

The resulting revenue, gross profit, and firm value are given by, respectively:

$$e_{jk}(a) \equiv p^f_{jk}(a)y_{jk}(a) = (\tau_{jk}w_j a)^{1-\sigma}[(1 - 1/\sigma)P_k]^{\sigma-1}E_k, \tag{5}$$

$$\pi_{jk}(a) = (p^f_{ij}(a) - w_j a)y_{jk}(a) = e_{jk}(a)/\sigma = (\tau_{jk}w_j a)^{1-\sigma}[(1 - 1/\sigma)P_k]^{\sigma-1}E_k/\sigma, \tag{6}$$

$$v_{jkt}(a) \equiv \int_t^\infty \pi_{jks}(a)\exp(-\int_t^s (r_{ju} + \delta)du)ds, \tag{7}$$

where δ is the death rate of manufacturing firms (e.g., Melitz, 2003; Ourens, 2016). Differentiating Eq. (7) with respect to t gives the no-arbitrage condition:

$$\dot{v}_{jkt}(a) = (r_{jt} + \delta)v_{jkt}(a) - \pi_{jkt}(a). \tag{8}$$

In the BRN model, both the fixed entry and overhead costs are paid only once at the time of entry. In period t, a manufacturing firm with a in country j survives (i.e., makes a nonnegative firm value net of the fixed overhead cost) in country k if and only if $a \leq a_{jkt}$, where the cutoff unit labor requirement a_{jkt} is determined by the zero cutoff profit condition:

$$v_{jkt}(a_{jkt}) = P^K_{jt}\kappa_{jk}, j, k = 1, 2, \tag{9}$$

where P^K_j is the price of the knowledge good (i.e., blueprint), and κ_{jk} is the amount of the knowledge good required for selling each variety from country j to country k. In line with the literature, the variable and fixed trade costs are assumed to be so high that not all domestic firms export:

$$a_{jkt} < a_{jjt} \forall j, k = 1, 2, k \neq j.$$

Using $e_{jks}(a)/e_{jks}(a_{jkt}) = (a/a_{jkt})^{1-\sigma} = \pi_{jks}(a)/\pi_{jks}(a_{jkt})$ from Eqs. (5) and (6), and the zero cutoff profit condition (9), Eq. (7) for $a \leq a_{jkt}$ is rewritten as $v_{jkt}(a) = (a/a_{jkt})^{1-\sigma}P^K_{jt}\kappa_{jk}(\geq P^K_{jt}\kappa_{jk})$.

After paying the fixed entry cost, an entrant in country j draws a from a country-specific distribution $G_j(a)$ and the corresponding density $g_j(a)$. The firm also pays the fixed overhead cost to sell its variety to country k if and only if $a \leq a_{jk}$. The free entry condition requires that the total expected firm value should be equal to the total expected fixed entry and overhead costs. The expected firm value and overhead cost of a manufacturing firm in country j selling its variety to country k are given by, respectively:

$$\int_0^{a_{jk}} v_{jk}(a)g_j(a)da = G_j(a_{jk})\int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da = P_j^K\kappa_{jk}(H_{jk}(a_{jk}) + G_j(a_{jk})),$$
$$\int_0^{a_{jk}} P_j^K\kappa_{jk}g_j(a)da = G_j(a_{jk})\int_0^{a_{jk}} P_j^K\kappa_{jk}\mu_{jk}(a|a_{jk})da = P_j^K\kappa_{jk}G_j(a_{jk});$$
$$\mu_{jk}(a|a_{jk}) \equiv g_j(a)/G_j(a_{jk}), H_{jk}(a_{jk}) \equiv G_j(a_{jk})h_{jk}(a_{jk}), h_{jk}(a_{jk}) \equiv (\overline{a}_{jk}(a_{jk})/a_{jk})^{1-\sigma} - 1,$$
$$\overline{a}_{jk}(a_{jk}) \equiv (\int_0^{a_{jk}} a^{1-\sigma}\mu_{jk}(a|a_{jk})da)^{1/(1-\sigma)},$$

where $\mu_{jk}(a|a_{jk})$ is the density of a conditional on survival, with $\int_0^{a_{jk}} \mu_{jk}(a|a_{jk})da = 1$, $\overline{a}_{jk}(a_{jk})$ is interpreted as the aggregate unit labor requirement of surviving firms, and $P_j^K\kappa_{jk}H_{jk}(a_{jk}) = \int_0^{a_{jk}} v_{jk}(a)g_j(a)da - \int_0^{a_{jk}} P_j^K\kappa_{jk}g_j(a)da$ represents the expected firm value of a manufacturing firm in country j selling its variety to country k, net of its expected overhead cost. Using these expressions, the free entry condition is compactly written as:

$$\sum\nolimits_k \int_0^{a_{jk}} v_{jk}(a)g_j(a)da = \sum\nolimits_k \int_0^{a_{jk}} P_j^K\kappa_{jk}g_j(a)da + P_j^K\kappa_j^e \Leftrightarrow \sum\nolimits_k\kappa_{jk}H_{jk}(a_{jk}) = \kappa_j^e, \tag{10}$$

where κ_j^e is the amount of the knowledge good required for entry. It can be easily verified that:

$$(d/da_{jk})\overline{a}_{jk}(a_{jk})^{1-\sigma} = -a_{jk}^{1-\sigma}g_{jk}h_{jk}/G_{jk}; G_{jk} \equiv G_j(a_{jk}), g_{jk} \equiv g_j(a_{jk}),$$
$$h'_{jk} = -g_{jk}h_{jk}/G_{jk} + (h_{jk}+1)(\sigma-1)/a_{jk},$$
$$H'_{jk} = G_{jk}(h_{jk}+1)(\sigma-1)/a_{jk} > 0,$$
$$H'_{jk}a_{jk}/H_{jk} = [(h_{jk}+1)/h_{jk}](\sigma-1) > \sigma - 1 > 0.$$

The free entry condition (10), together with the increasingness of $H_{jk}(a_{jk})$, implies that country j' s export cutoff a_{jk} is always negatively related to its domestic cutoff a_{jj}: when domestic selection becomes tougher (i.e., a_{jj} decreases), the expected net firm value from domestic sales decreases. For the free entry condition to be restored, the expected net firm value from exports should increase, which means that more firms enter their export market (i.e., a_{jk} increases).

1.3 R&D firms

The representative R&D firm in country j maximizes its profit $\pi_j^K = P_j^KQ_j^K - w_jL_j^K$, subject to its cost function in terms of labor $L_j^K = a_j^KQ_j^K$, with P_j^K, w_j, and a_j^K given, where Q_j^K is the supply of the knowledge good, L_j^K is the firm's demand for labor, and a_j^K is the firm's unit labor requirement given by $a_j^K = a_j^K(n_{jj}, n_{kk}; \{a_{lm}\}); n_{jk} \equiv n_j^eG_j(a_{jk})$, where n_j^e is the number of entrants in country j, and n_{jk} is the number of entrants in country j successfully selling varieties to country k. The a_j^K function is assumed to be decreasing and homogeneous of degree minus one in (n_{jj}, n_{kk}), representing domestic and international knowledge spillovers, respectively. Also, the cutoffs in the manufacturing sector $\{a_{lm}\}$ could affect a_j^K by changing the degrees of knowledge spillovers. Profit maximization under constant returns to scale and perfect competition implies the zero profit condition:

$$P_j^K = w_ja_j^K(n_{jj}, n_{kk}) \Leftrightarrow P_j^KQ_j^K = w_jL_j^K. \tag{11}$$

1.4 Markets

The market-clearing conditions for the asset, labor, knowledge good, and manufacturing goods are given by:

$$W_j = \sum_k n_j^e \int_0^{a_{jk}} v_{jk}(a) g_j(a) da = \sum_k n_{jk} \int_0^{a_{jk}} v_{jk}(a) \mu_{jk}(a|a_{jk}) da, j = 1, 2, \tag{12}$$

$$L_j = \sum_k n_j^e \int_0^{a_{jk}} l_{jk}(a) g_j(a) da + L_j^K = \sum_k n_{jk} \int_0^{a_{jk}} l_{jk}(a) \mu_{jk}(a|a_{jk}) da + L_j^K, j = 1, 2, \tag{13}$$

$$Q_j^K = \overline{\kappa}_j(\dot{n}_{jj} + \delta n_{jj}); \overline{\kappa}_j(\{a_{jk}\}) \equiv (\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e)/G_j(a_{jj}), j = 1, 2, \tag{14}$$

$$y_{jk}(a) = \tau_{jk} d_{jk}(a), j, k = 1, 2. \tag{15}$$

In Eq. (14), $\overline{\kappa}_j$ represents an entrant's: "expected units of knowledge required to get a 'winner.' " (BRN, 2008, p. 25) The demand for the knowledge good is $\overline{\kappa}_j$ times $\dot{n}_{jj} + \delta n_{jj}$, the number of new successful entrants including the replacement of exiting firms due to bad shocks. We check Walras' law to endorse these market-clearing conditions. We start from rewriting Eq. (12) using Eq. (10) as:

$$\begin{aligned} W_j &= n_{jj} \sum_k (G_j(a_{jk})/G_j(a_{jj})) \int_0^{a_{jk}} v_{jk}(a) \mu_{jk}(a|a_{jk}) da \\ &= (n_{jj}/G_j(a_{jj})) P_j^K (\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e) = p_j^K \overline{\kappa}_j; p_j^K \equiv n_{jj} P_j^K, \end{aligned} \tag{16}$$

where p_j^K is "an 'intensive form' of P_K" (BRN, 2008, p. 25): n_{jj} compensates for a decrease in P_j^K through knowledge spillovers. Eq. (16) means that country j's asset is simply equal to $p_j^K \overline{\kappa}_j$. We next differentiate the first equality of Eq. (16) with respect to time and use Eqs. (8) and (16) to obtain:

$$\begin{aligned} \dot{W}_j &= \dot{n}_{jj} \sum_k (G_j(a_{jk})/G_j(a_{jj})) \int_0^{a_{jk}} v_{jk}(a) \mu_{jk}(a|a_{jk}) da \\ &+ n_{jj} \sum_k (G_j(a_{jk})/G_j(a_{jj})) \int_0^{a_{jk}} \dot{v}_{jk}(a) \mu_{jk}(a|a_{jk}) da \\ &= (\dot{n}_{jj}/G_j(a_{jj})) P_j^K (\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e) + (r_j + \delta) W_j - \sum_k n_{jk} \int_0^{a_{jk}} \pi_{jk}(a) \mu_{jk}(a|a_{jk}) da. \end{aligned} \tag{17}$$

Combining Eqs. (1), (11), and (17), and using Eq. (16) and $E_j = \sum_k n_{kj} \int_0^{a_{kj}} \tau_{kj} p_{kj}^f(a) d_{kj}(a) \mu_{kj}(a|a_{kj}) da$, we obtain Walras' law for country j:

$$\begin{aligned} 0 &= w_j (\sum_k n_{jk} \int_0^{a_{jk}} l_{jk}(a) \mu_{jk}(a|a_{jk}) da + L_j^K - L_j) + P_j^K [\overline{\kappa}_j (\dot{n}_{jj} + \delta n_{jj}) - Q_j^K] \\ &+ \sum_k n_{kj} \int_0^{a_{kj}} p_{kj}^f(a) \tau_{kj} d_{kj}(a) \mu_{kj}(a|a_{kj}) da - \sum_k n_{jk} \int_0^{a_{jk}} p_{jk}^f(a) y_{jk}(a) \mu_{jk}(a|a_{jk}) da. \end{aligned} \tag{18}$$

Eq. (18) implies two things. First, summing it up for all j gives Walras' law for the world:

$$\begin{aligned} 0 &= \sum_j w_j (\sum_k n_{jk} \int_0^{a_{jk}} l_{jk}(a) \mu_{jk}(a|a_{jk}) da + L_j^K - L_j) + \sum_j P_j^K [\overline{\kappa}_j (\dot{n}_{jj} + \delta n_{jj}) - Q_j^K] \\ &+ \sum_j \sum_k n_{jk} \int_0^{a_{jk}} p_{jk}^f(a) (\tau_{jk} d_{jk}(a) - y_{jk}(a)) \mu_{jk}(a|a_{jk}) da. \end{aligned}$$

This is totally consistent with the three market clearing conditions (13) to (15). Second, substituting Eqs. (13) to (15) into Eq. (18), we obtain:

$$\textstyle\sum_k n_{jk} \int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da = \sum_k n_{kj} \int_0^{a_{kj}} e_{kj}(a)\mu_{kj}(a|a_{kj})da = E_j, \tag{19}$$

$$n_{jk} \int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da = n_{kj} \int_0^{a_{kj}} e_{kj}(a)\mu_{kj}(a|a_{kj})da, k \neq j. \tag{20}$$

Eq. (19) says that country j's total revenue from selling to all destinations is equal to its total expenditure for buying from all sources. Eq. (20) means that country j's balance of trade is zero: country j's value of exports (i.e., the left-hand side) is equal to its value of imports (i.e., the right-hand side). Now let:

$$\lambda_{jk} \equiv n_{jk} \int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da / \textstyle\sum_l n_{jl} \int_0^{a_{jl}} e_{jl}(a)\mu_{jl}(a|a_{jl})da; \sum_k \lambda_{jk} = 1, \tag{21}$$

be the revenue share of varieties country j sells to country k. Then Eqs. (19) and (20) imply that λ_{jk} is also equal to the expenditure share of varieties country j buys from country k, and that the zero balance of trade condition (20) is simply rewritten as:

$$\lambda_{jk} E_j = \lambda_{kj} E_k. \tag{22}$$

Finally, using Eq. (4) and the definition of $\overline{a}_{kj}(a_{kj})$, country j's price index in Eq. (2) is simplified to:

$$\begin{aligned} P_j &= \{\textstyle\sum_k n_{kj}[\tau_{kj} w_k \overline{a}_{kj}(a_{kj})/(1 - 1/\sigma)]^{1-\sigma}\}^{1/(1-\sigma)} = n_{jj}^{1/(1-\sigma)} \overline{m}_j/(1 - 1/\sigma); \\ \overline{m}_j &\equiv [\textstyle\sum_k (n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))(\tau_{kj} w_k \overline{a}_{kj}(a_{kj}))^{1-\sigma}]^{1/(1-\sigma)}, \end{aligned} \tag{23}$$

where $\overline{m}_j$ is interpreted as: "a weighted average of firms' marginal selling costs in a particular market" (BRN, 2008, p. 24), which is market j in the present case. Compared with BRN, allowing for asymmetric countries adds two variables to $\overline{m}_j$: n_{kk}/n_{jj} and w_k, which are to be determined in general equilibrium.

2 Balanced growth path

2.1 Characterization

Let labor in country 2 be the numeraire: $w_2 \equiv 1$. Since $a_j^K(n_{jj}, n_{kk})$ is homogeneous of degree minus one in (n_{jj}, n_{kk}), we have $a_j^K((1/n_{jj})n_{jj}, (1/n_{jj})n_{kk}) = (1/n_{jj})^{-1} a_j^K(n_{jj}, n_{kk})$, or $a_j^K(n_{jj}, n_{kk}) = (1/n_{jj}) a_j^K((1/n_{jj})n_{jj}, (1/n_{jj})n_{kk}) = (1/n_{jj}) a_j^K(1, n_{kk}/n_{jj})$. Combining this with Eq. (11), the intensive-form prices of the knowledge good for the two countries are given by:

$$p_1^K = n_{11} P_1^K = w_1 a_1^K(1, 1/\chi), p_2^K = n_{22} P_2^K = a_2^K(1, \chi); \chi \equiv n_{11}/n_{22}, \tag{24}$$

where χ represents the number of domestic varieties in country 1 relative to country 2, and w_1 is now the relative wage of country 1 to country 2. Eq. (24) highlights the fundamental difficulty of the present model: whereas p_j^K is fixed at $a_j^K(1, 1)$ in BRN and Ourens (2016), it now depends on two endogenous variables w_1 and χ.

We derive some differential equations. Rewriting Eq. (17) using Eqs. (6), (16), and (19) gives:

$$\dot{W}_j/W_j = \gamma_j + r_j + \delta - Z_j/\sigma; \gamma_j \equiv \dot{n}_{jj}/n_{jj}, Z_j \equiv E_j/W_j,$$

where γ_j is the growth rate of the number of domestic varieties in country j, one of the main variables of interest. Using Eq. (3), the growth rate of a transformed variable Z_j is calculated as:

$$\dot{Z}_j/Z_j = \dot{E}_j/E_j - \dot{W}_j/W_j = Z_j/\sigma - \rho - \delta - \gamma_j. \tag{25}$$

Using Eqs. (6), (11), (14), (16), and (19), Eq. (13) is rewritten as:

$$\gamma_j = w_j L_j/(p_j^K \overline{\kappa}_j) - (1-1/\sigma)Z_j - \delta. \tag{26}$$

From now on, we focus on a balanced growth path (BGP), a path along which all variables grow at constant (including zero) rates. From Eq. (25), both γ_j and Z_j must be constant at a BGP, which implies that $\dot{Z}_j/Z_j = 0$. And from Eq. (26), $w_j/(p_j^K \overline{\kappa}_j)$ must be constant at a BGP. Using a superscript asterisk to represent a BGP, Z_j^* and γ_j^* are solved from Eqs. (25), (26), and $\dot{Z}_j/Z_j = 0$ as:

$$Z_j^* = \rho + w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*), \tag{27}$$
$$\gamma_j^* = (1/\sigma)w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*) - (1-1/\sigma)\rho - \delta, \tag{28}$$

with w_j^*, p_j^{K*}, and $\overline{\kappa}_j^*$ given at this point.

Using Eq. (28), the growth rate of χ^* is simply given by:

$$\dot{\chi}^*/\chi^* = \gamma_1^* - \gamma_2^* = (1/\sigma)L_1/(a_1^K(1,1/\chi^*)\overline{\kappa}_1^*) - (1-1/\sigma)\rho - \delta - [(1/\sigma)L_2/(a_2^K(1,\chi^*)\overline{\kappa}_2^*) - (1-1/\sigma)\rho - \delta], \tag{29}$$

where Eq. (24) is used to eliminate w_1^*. Since constancy of $\dot{\chi}^*/\chi^*$ requires constancy of χ^* from Eq. (29), we have $\dot{\chi}^*/\chi^* = 0$, implying that both countries grow at the same rate at a BGP:

$$\gamma_1^* = \gamma_2^* \equiv \gamma^* \Leftrightarrow L_1/(a_1^K(1,1/\chi^*;\{a_{lm}^*\})\overline{\kappa}_1^*(\{a_{1k}^*\})) = L_2/(a_2^K(1,\chi^*;\{a_{lm}^*\})\overline{\kappa}_2^*(\{a_{2k}^*\})). \tag{30}$$

With the cutoffs $\{a_{lm}^*\}$ given, the balanced growth condition (30) determines χ^*. we call γ^* the balanced growth rate.

Multiplying Eq. (27) by $W_j^* = p_j^{K*}\overline{\kappa}_j^*$ from Eq. (16), E_j^* is obtained as:

$$E_j^* = p_j^{K*}\overline{\kappa}_j^*[\rho + w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*)] = p_j^{K*}\overline{\kappa}_j^*\rho + w_j^* L_j. \tag{31}$$

By definition, $\overline{\kappa}_j^*$ is constant as long as the cutoffs are constant. And from Eq. (24), p_j^{K*} is constant as long as w_j^* is constant. So suppose that w_1^* is constant at a BGP, which will be verified later. Since E_j^* is constant from Eq. (31), Eq. (3) implies that:

$$r_j^* = \rho \forall j. \tag{32}$$

We next see how the cutoffs are determined. Suppose that the world economy is on a BGP for all $s \geq t$. Dividing the zero cutoff profit condition (9) by itself for $j = k$ gives:

$$\frac{v_{jkt}(a_{jkt})}{v_{kkt}(a_{kkt})} = \frac{P_{jt}^K \kappa_{jk}}{P_{kt}^K \kappa_{kk}}, j \neq k. \tag{33}$$

Eq. (7) is rewritten as:

$$\begin{aligned} v_{jkt}(a) &= \int_t^\infty \pi_{jkt}(a)\exp(\int_t^s (\dot{\pi}_{jku}(a)/\pi_{jku}(a))du)\exp(-\int_t^s (r_{ju}+\delta)du)ds \\ &= \pi_{jkt}(a)\Delta_{jkt}(a); \Delta_{jkt}(a) \equiv \int_t^\infty \exp(-\int_t^s (r_{ju} + \delta - \dot{\pi}_{jku}(a)/\pi_{jku}(a))du)ds. \end{aligned}$$

Noting that n_{kjs} grows at the rate of γ^* for all k and j on a BGP, Eq. (23) implies that:

$$P_{js} = P_{jt}e^{-[\gamma^*/(\sigma-1)](s-t)}. \tag{34}$$

From Eqs. (6) and (34), we have $\pi_{jks}(a) = \pi_{jkt}(a)e^{-\gamma^*(s-t)}$, which implies that $\dot{\pi}_{jks}(a)/\pi_{jks}(a) = -\gamma^*$. Using this and Eq. (32), $\Delta_{jkt}(a)$ is calculated as $\Delta_{jkt}(a) = 1/(\rho + \delta + \gamma^*)$. Eq. (7) is thus simplified to:

$$v_{jkt}(a) = \pi_{jkt}(a)/(\rho + \delta + \gamma^*). \tag{35}$$

Using Eqs. (6) and (35), the left-hand side of Eq. (33) is rewritten as:

$$\frac{v_{jkt}(a_{jk}^*)}{v_{kkt}(a_{kk}^*)} = \frac{(\tau_{jk}w_j^* a_{jk}^*)^{1-\sigma}[(1-1/\sigma)P_{ks}]^{\sigma-1}(E_k^*/\sigma)/(\rho+\delta+\gamma^*)}{(w_k^* a_{kk}^*)^{1-\sigma}[(1-1/\sigma)P_{ks}]^{\sigma-1}(E_k^*/\sigma)/(\rho+\delta+\gamma^*)} = \left(\frac{\tau_{jk}w_j^* a_{jk}^*}{w_k^* a_{kk}^*}\right)^{1-\sigma}.$$

In the right-hand side of Eq. (33), P_{jt}^K/P_{kt}^K is rewritten using Eq. (24) as:

$$\frac{P_{jt}^K}{P_{kt}^K} = \frac{w_j^*(1/n_{jj})a_j^K(1, n_{kk}/n_{jj})}{w_k^*(1/n_{kk})a_k^K(1, n_{jj}/n_{kk})} = \frac{w_j^*}{w_k^*}\frac{n_{kk}}{n_{jj}}\frac{a_j^K(1, n_{kk}/n_{jj})}{a_k^K(1, n_{jj}/n_{kk})}.$$

From these results, Eq. (33) is simplified to:

$$a_{12}^*/a_{22}^* = v^{*-1}\tau_{12}^{-1}(\kappa_{12}/\kappa_{22})^{-1/(\sigma-1)}, \tag{36}$$

$$a_{21}^*/a_{11}^* = v^*\tau_{21}^{-1}(\kappa_{21}/\kappa_{11})^{-1/(\sigma-1)}; \tag{37}$$

$$v^*(w_1^*, \chi^*; \{a_{lm}^*\}) \equiv (w_1^{*\sigma}A(\chi^*; \{a_{lm}^*\}))^{1/(\sigma-1)}, A(\chi^*; \{a_{lm}^*\}) \equiv (1/\chi^*)a_1^K(1, 1/\chi^*; \{a_{lm}^*\})/a_2^K(1, \chi^*; \{a_{lm}^*\}).$$

Eqs. (36) and (37) show how the relative competitiveness of foreign versus domestic firms are affected by the relative wage, relative number of domestic varieties, and iceberg trade costs (to focus on τ_{jk} as the only policy variable, we take κ_{jk} as given throughout). In Eq. (36), for example, the lower w_1^* is and/or the lower τ_{12} is, the larger a_{12}^*/a_{22}^* is, meaning that exporters from country 1 becomes relatively more competitive in market 2 (the discussion of χ^* is left to the following sections, where a_j^K will be specified). From Eqs. (10), (36), and (37), we solve for the four cutoffs as functions of w_1^*, χ^*, and iceberg trade costs:

$$a_{jk}^* = a_{jk}^*(w_1^*, \chi^*, \tau_{21}, \tau_{12}), j, k = 1, 2. \tag{38}$$

In line with the typical monopolistic competition models with asymmetric countries since Krugman (1980), the relative wage w_1^* is determined from the zero balance of trade condition. The revenue share (21) is rewritten as:

$$\lambda_{jk}^* = \frac{n_{jjt}(G_j(a_{jk}^*)/G_j(a_{jj}^*))\int_0^{a_{jk}^*}e_{jk}(a)\mu_{jk}(a|a_{jk}^*)da}{\sum_l n_{jjt}(G_j(a_{jl}^*)/G_j(a_{jj}^*))\int_0^{a_{jl}^*}e_{jl}(a)\mu_{jl}(a|a_{jl}^*)da} = \frac{G_j(a_{jk}^*)\int_0^{a_{jk}^*}e_{jk}(a)\mu_{jk}(a|a_{jk}^*)da}{\sum_l G_j(a_{jl}^*)\int_0^{a_{jl}^*}e_{jl}(a)\mu_{jl}(a|a_{jl}^*)da}.$$

Since $\int_0^{a_{jk}^*}e_{jk}(a)\mu_{jk}(a|a_{jk}^*)da = (h_{jk}(a_{jk}^*)+1)e_{jk}(a_{jk}^*) = (h_{jk}(a_{jk}^*)+1)\sigma(\rho+\delta+\gamma^*)P_j^K\kappa_{jk}$ from Eqs. (5), (6), (9), and (35), λ_{jk}^* is further simplified to:

$$\lambda_{jk}^* = \frac{(H_{jk}(a_{jk}^*)+G_j(a_{jk}^*))\kappa_{jk}}{\sum_l(H_{jl}(a_{jl}^*)+G_j(a_{jl}^*))\kappa_{jl}} \equiv \lambda_{jk}^*(\{a_{jl}^*\}). \tag{39}$$

Using Eqs. (24), (30), (31), and (39), the zero balance of trade condition (22) is rewritten as:

$$\lambda_{12}^*(\{a_{1k}^*\})w_1^* a_1^K(1, 1/\chi^*; \{a_{lm}^*\})\overline{\kappa}_1^*(\{a_{1k}^*\}) = \lambda_{21}^*(\{a_{2k}^*\})a_2^K(1, \chi^*; \{a_{lm}^*\})\overline{\kappa}_2^*(\{a_{2k}^*\}). \tag{40}$$

The balanced growth condition (30), the cutoff functions (38), and the zero balance of trade condition (40), characterize a BGP: $(\chi^*, \{a_{jk}^*\}, w_1^*)$. Suppose that $\{a_{jk}^*\}$ is constant. Then χ^* is constant from Eq. (30), and hence w_1^* is constant from Eq. (40). Since χ^* and w_1^* are constant, Eq. (38) ensures that $\{a_{jk}^*\}$ is indeed constant. From now on, we assume that there exists a unique BGP. All other variables can be determined by substituting $(\chi^*, \{a_{jk}^*\}, w_1^*)$ back into the appropriate equations.

2.2 Preliminary results

We derive some preliminary results, which will simplify the following analysis. First, logarithmically differentiating the free entry condition (10), and using Eq. (39) and $H_{jk}' a_{jk}/H_{jk} = [(h_{jk}+1)/h_{jk}](\sigma-1) = [(H_{jk}+G_{jk})/H_{jk}](\sigma-1)$, we obtain:

$$0 = \textstyle\sum_k \lambda_{jk}^* \widehat{a}_{jk}^*; \widehat{a}_{jk}^* \equiv d\ln a_{jk}^* \equiv da_{jk}^*/a_{jk}^*. \tag{41}$$

From now on, we follow BRN and Ourens (2016) in assuming that a is Pareto distributed with a country-specific scale parameter a_{j0} and a common shape parameter θ:

$$G_j(a) \equiv (a/a_{j0})^{\theta} = a_{j0}^{-\theta} a^{\theta}; a \in [0, a_{j0}], \theta > \sigma - 1 > 0.$$

This implies that $g_j(a) = \theta a_{j0}^{-\theta} a^{\theta-1}, G_j(a_{jk}) = a_{j0}^{-\theta} a_{jk}^{\theta}, \mu_{jk}(a|a_{jk}) = \theta a_{jk}^{-\theta} a^{\theta-1}, \overline{a}_{jk}(a_{jk})^{1-\sigma} = [\beta/(\beta-1)] a_{jk}^{1-\sigma}; \beta \equiv \theta/(\sigma-1) > 1, h_{jk}(a_{jk}) = 1/(\beta-1), H_{jk}(a_{jk}) = a_{j0}^{-\theta} a_{jk}^{\theta}/(\beta-1)$.

Second, logarithmically differentiating Eq. (39), and using Eqs. (39), (41), and $(H_{jk}^{*\prime} + g_{jk}^{*}) a_{jk}^{*}/(H_{jk}^{*} + G_{jk}^{*}) = \theta$, $\widehat{\lambda}_{jk}^{*}$ is given by:

$$\widehat{\lambda}_{jk}^{*} = \theta \widehat{a}_{jk}^{*}. \tag{42}$$

Eq. (42) simply says that country j's revenue share in market k increases if and only if more firms in country j enter market k.

Third, logarithmically differentiating $\overline{\kappa}_j^{*} = (\sum_k \kappa_{jk} G_j(a_{jk}^{*}) + \kappa_j^{e})/G_j(a_{jj}^{*}) = [\sum_k \kappa_{jk}(H_{jk}(a_{jk}^{*}) + G_j(a_{jk}^{*}))]/G_j(a_{jj}^{*})$ from Eqs. (10) and (14), and using Eqs. (39), (41), $(H_{jk}^{*\prime} + g_{jk}^{*}) a_{jk}^{*}/(H_{jk}^{*} + G_{jk}^{*}) = \theta$, and $g_{jk}^{*} a_{jk}^{*}/G_{jk}^{*} = \theta$, $\widehat{\overline{\kappa}}_j^{*}$ is expressed as:

$$\widehat{\overline{\kappa}}_j^{*} = -\theta \widehat{a}_{jj}^{*}. \tag{43}$$

An increase in country j's domestic cutoff partly decreases country j's expected units of the knowledge good required to get a winner by decreasing: "the expected number of 'tries' it takes to get a winner" (BRN, 2008, p. 25). This is actually the total effect because $\sum_k \kappa_{jk} G_j(a_{jk}^{*}) + \kappa_j^{e}$, the numerator of $\overline{\kappa}_j^{*}$, is constant from the free entry condition.

Fourth, using Eqs. (6), (24), and (35), the zero cutoff profit condition (9) for domestic sales is rewritten as $(w_j^{*} a_{jj}^{*})^{1-\sigma}[(1-1/\sigma)P_j^{*}]^{\sigma-1}(E_j^{*}/\sigma)/(\rho+\delta+\gamma^{*}) = (p_j^{K*}/n_{jjt})\kappa_{jj}$. And the growth equation (28) is rewritten using Eqs. (16) and (27) as $\rho + \delta + \gamma^{*} = (1/\sigma)E_j^{*}/(p_j^{K*}\overline{\kappa}_j^{*})$. Putting them together gives:

$$\{w_j^{*} a_{jj}^{*}/[(1-1/\sigma)P_j^{*}]\}^{1-\sigma} \overline{\kappa}_j^{*} = \kappa_{jj}/n_{jjt}.$$

Logarithmically differentiating this, noting that n_{jjt} is predetermined, and using Eq. (43), we obtain:

$$\widehat{w}_j^{*} - \widehat{P}_j^{*} = -(1+\beta)\widehat{a}_{jj}^{*}. \tag{44}$$

Eq. (44) means that country j's real wage in terms of the consumption index rises if and only if its domestic firms are more selected.

Finally, combining Eqs. (42) and (44) immediately gives the ACR formula of Arkolakis, Costinot, and Rodríguez-Clare (2012) stating the negative relationship between country j's autarkiness ratio (i.e., domestic revenue and expenditure share) and its real wage:

$$\widehat{w}_j^{*} - \widehat{P}_j^{*} = -[(1+\beta)/\theta]\widehat{\lambda}_{jj}^{*}. \tag{45}$$

Although Eq. (45) provides sufficient information about the welfare effects of policy changes in a static setting, it is not enough in the present endogenous growth model. In the next section, we examine the long-run growth effects of trade liberalization under the benchmark Grossman-Helpman specification. In the following two sections, we omit asterisks just for notational simplicity.

3 Grossman-Helpman specification

In line with BRN, we start from the simplest Grossman-Helpman specification for a_j^K as a benchmark:

$$a_j^K(n_{jj}, n_{kk}) = 1/(n_{jj} + \psi_j n_{kk}),$$

where $\psi_j (\in [0,1])$ represents the degree of international knowledge spillovers, which is exogenous. Logarithmically differentiating $a_1^K(1, 1/\chi) = 1/(1+\psi_1/\chi)$ and $a_2^K(1,\chi) = 1/(1+\psi_2\chi)$ gives:

$$\widehat{a}_1^K(1, 1/\chi) = \alpha_1\widehat{\chi}; \alpha_1 \equiv (\psi_1/\chi)/(1+\psi_1/\chi) \in [0,1], \tag{46}$$

$$\widehat{a}_2^K(1, \chi) = -\alpha_2\widehat{\chi}; \alpha_2 \equiv \psi_2\chi/(1+\psi_2\chi) \in [0,1]. \tag{47}$$

Logarithmically differentiating $A(\chi) = (1/\chi)a_1^K(1, 1/\chi)/a_2^K(1, \chi)$, and using Eqs. (46) and (47), $\widehat{A}$ is obtained as:

$$\widehat{A} = -(1-\alpha_1-\alpha_2)\widehat{\chi}; 1-\alpha_1-\alpha_2 = (1-\psi_1\psi_2)/[(1+\psi_1/\chi)(1+\psi_2\chi)] \geq 0. \tag{48}$$

An increase in χ, the number of domestic varieties in country 1 relative to country 2, partly increases $A(\chi)$ by making R&D more difficult in country 1 but easier in country 2. However, under the assumption that international knowledge spillovers are not stronger than domestic ones (i.e., $\psi_j \leq 1$), these effects are not large enough to outweigh the inverse term $1/\chi$, so that $A(\chi)$ is nonincreasing in χ in total.

We solve for the rates and amounts of changes in endogenous variables caused by changes in iceberg trade costs in four steps: (i) we solve for the logarithmically differentiated form of the cutoff function $\widehat{a}_{jk}$ in terms of $\widehat{v}, \widehat{\tau}_{21}$, and $\widehat{\tau}_{12}$; (ii) we use the zero balance of trade condition (40), together with the result in step (i) and the definition of v, to solve for $\widehat{w}_1$ in terms of $\widehat{\chi}, \widehat{\tau}_{21}$, and $\widehat{\tau}_{12}$; (iii) using the results in steps (i) and (ii), we express $d\gamma_j$ in terms of $\widehat{\chi}, \widehat{\tau}_{21}$, and $\widehat{\tau}_{12}$; (iv) we substitute the result in step (iii) into the balanced growth condition (30), or $d\gamma_1 = d\gamma_2$, to solve for $\widehat{\chi}$ in terms of $\widehat{\tau}_{21}$ and $\widehat{\tau}_{12}$. And substituting $\widehat{\chi}$ back into $d\gamma_2$, we solve for the amount of change in the balanced growth rate in terms of $\widehat{\tau}_{21}$ and $\widehat{\tau}_{12}$.

In step (i), logarithmically differentiating Eqs. (36) and (37) gives:

$$\widehat{a}_{12} - \widehat{a}_{22} = -\widehat{v} - \widehat{\tau}_{12}, \tag{49}$$

$$\widehat{a}_{21} - \widehat{a}_{11} = \widehat{v} - \widehat{\tau}_{21}. \tag{50}$$

Substituting $\widehat{a}_{12}$ and $\widehat{a}_{21}$ from Eqs. (49) and (50), respectively, into Eq. (41), we have two equations for $\widehat{a}_{11}$ and $\widehat{a}_{22}$, which are solved as:

$$\widehat{a}_{11} = (\lambda_{12}/|\lambda|)(\widehat{v} + \lambda_{22}\widehat{\tau}_{12} - \lambda_{21}\widehat{\tau}_{21}), \tag{51}$$

$$\widehat{a}_{22} = (\lambda_{21}/|\lambda|)(-\widehat{v} + \lambda_{11}\widehat{\tau}_{21} - \lambda_{12}\widehat{\tau}_{12}); |\lambda| \equiv \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} = 1 - \lambda_{12} - \lambda_{21}. \tag{52}$$

Although it will turn out that the sign of $|\lambda|$ does not affect our main results, it is positive if $\lambda_{jk} < 1/2 \forall j,k,k \neq j$, which is likely if a_{jk} is sufficiently smaller than a_{jj} due to high trade costs. Eq. (51) is interpreted as follows, assuming for now that $|\lambda| > 0$. A fall in v (due to a fall in w_1 or a rise in χ) or a fall in τ_{12} encourages country 1's exports (i.e., increases a_{12}) from the relative competitiveness condition (49), which forces its least productive domestic firms to exit (i.e., decreases a_{11}) from the free entry condition (41). A fall in τ_{21} encourages country 2's exports from Eq. (50), which forces its least productive domestic firms to exit from Eq. (41). Due to tougher competition in market 2, country 1's least productive exporters exits from their export market (i.e., a_{12} decreases) from Eq. (49). Again from the free entry condition (41), more inefficient firms survive in their domestic market (i.e., a_{11} increases). The last result means that a country's import liberalization directly causes less exports and less domestic selection. However, the total effect of a country's import liberalization on its exports and domestic selection cannot be determined until the general equilibrium effects through w_1 and χ are considered.

In step (ii), substituting Eqs. (41), (42), (43), (46), and (47) into the logarithmically differentiated form of Eq. (40) gives:

$$-(\theta/\lambda_{12})\widehat{a}_{11} + \widehat{w}_1 + (\alpha_1 + \alpha_2)\widehat{\chi} = -(\theta/\lambda_{21})\widehat{a}_{22}.$$

The terms $-(\theta/\lambda_{12})\widehat{a}_{11}$ and $-(\theta/\lambda_{21})\widehat{a}_{22}$ correspond to the rates of changes in $\lambda_{12}\overline{\kappa}_1$ and $\lambda_{21}\overline{\kappa}_2$, respectively. The term $\widehat{w}_1$ represents the income effect (i.e., w_1^* in Eq. (40)). The term $(\alpha_1 + \alpha_2)\widehat{\chi}$ comes from a_1^K/a_2^K. Using Eqs. (48), (51), (52), and $\widehat{v} = (\sigma\widehat{w}_1 + \widehat{A})/(\sigma - 1)$, the above expression is solved for $\widehat{w}_1$ as:

$$
\begin{aligned}
&0 = -B\widehat{w}_1 + C\widehat{\chi} + \theta[(\lambda_{11}+\lambda_{21})\widehat{\tau}_{21} - (\lambda_{22}+\lambda_{12})\widehat{\tau}_{12}] \\
&\quad \Leftrightarrow \widehat{w}_1 = (1/B)\{C\widehat{\chi} + \theta[(\lambda_{11}+\lambda_{21})\widehat{\tau}_{21} - (\lambda_{22}+\lambda_{12})\widehat{\tau}_{12}]\}; \qquad (53)\\
&B \equiv 2\beta\sigma - |\lambda| = 2\beta\sigma - (1-\lambda_{12}-\lambda_{21}) > 0, C \equiv 2\beta(1-\alpha_1-\alpha_2) + |\lambda|(\alpha_1+\alpha_2).
\end{aligned}
$$

To interpret Eq. (53), suppose for now that $|\lambda| > 0$ (our main results will not depend on it). The fact that $B > 0$ means that a rise in w_1 creates country 1's trade deficit because it decreases $\lambda_{12}\overline{\kappa}_1$ but increases $\lambda_{21}\overline{\kappa}_2$, which outweighs the income effect. An increase in χ creates country 1's trade surplus both by increasing $\lambda_{12}\overline{\kappa}_1$ but decreasing $\lambda_{21}\overline{\kappa}_2$, and by increasing a_1^K/a_2^K. For country 1's trade surplus to be cleared, w_1 should go up. The result that a country's relative wage is increasing in its relative size is known as the home market effect in wages since Krugman (1980). A fall in τ_{21} creates country 1's trade deficit by encouraging country 2's exports but discouraging its own exports. Then w_1 should fall for its zero balance of trade to be restored. Similarly, a fall in τ_{12} raises w_1.

In step (iii), substituting Eq. (53) back into $\widehat{v} = [\sigma\widehat{w}_1 - (1-\alpha_1-\alpha_2)\widehat{\chi}]/(\sigma-1)$, $\widehat{v}$ is obtained as:

$$
\widehat{v} = [1/(\sigma-1)](1/B)\{|\lambda|[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\widehat{\chi} + \sigma\theta[(\lambda_{11}+\lambda_{21})\widehat{\tau}_{21} - (\lambda_{22}+\lambda_{12})\widehat{\tau}_{12}]\}. \qquad (54)
$$

In Eq. (54), the coefficient on $\widehat{\chi}$ turns from negative to positive because of the aforementioned home market effect in wages. Substituting Eq. (54) back into Eqs. (51) and (52), $\widehat{a}_{11}$ and $\widehat{a}_{22}$ are expressed as:

$$
\widehat{a}_{11} = \{\lambda_{12}/[(\sigma-1)B]\}\{[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\widehat{\chi} + (\sigma-1)[(\beta\sigma+\lambda_{21})\widehat{\tau}_{21} + (\beta\sigma-\lambda_{22})\widehat{\tau}_{12}]\}, \qquad (55)
$$

$$
\widehat{a}_{22} = \{\lambda_{21}/[(\sigma-1)B]\}\{-[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\widehat{\chi} + (\sigma-1)[(\beta\sigma+\lambda_{12})\widehat{\tau}_{12} + (\beta\sigma-\lambda_{11})\widehat{\tau}_{21}]\}. \qquad (56)
$$

It is worthwhile to compare Eqs. (55) and (56) with Eqs. (51) and (52), respectively. First, since $|\lambda|$ is eliminated, its sign does not matter after endogenizing w_1. Second, in Eq. (55), the coefficient on $\widehat{\tau}_{21}$ turns from negative to positive. This means that, with χ given, country 1's import liberalization results in more exports and more domestic selection. This is because the direct export-reducing effect of a fall in τ_{21} is outweighed by its indirect export-enhancing effect through a fall in w_1.

Totally differentiating Eq. (28), and using Eqs. (24) and (30), the amount of change in country j's growth rate is expressed as $d\gamma_j = (1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)](-\widehat{a}_j^K - \widehat{\overline{\kappa}}_j)$, where $-\widehat{a}_j^K$ and $-\widehat{\overline{\kappa}}_j$ correspond to "the p_K-channel" and "the $\overline{\kappa}$-channel", respectively, according to BRN. In the present model, however, we have to distinguish a_j^K from p_j^K due to the endogeneity of w_1. Now let us call the two channels "the a_j^K-channel" and "the $\overline{\kappa}_j$-channel", respectively. Substituting Eqs. (55) and (56) into $-\widehat{a}_1^K - \widehat{\overline{\kappa}}_1 = -\alpha_1\widehat{\chi} + \theta\widehat{a}_{11}$ and $-\widehat{a}_2^K - \widehat{\overline{\kappa}}_2 = \alpha_2\widehat{\chi} + \theta\widehat{a}_{22}$ from Eqs. (43), (46), (47), the totally differentiated forms of the growth equations are obtained as:

$$
d\gamma_1 = (1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)]\{1/[(\sigma-1)B]\}\{-D_1\widehat{\chi} + \theta(\sigma-1)\lambda_{12}[(\beta\sigma+\lambda_{21})\widehat{\tau}_{21} + (\beta\sigma-\lambda_{22})\widehat{\tau}_{12}]\}, \qquad (57)
$$

$$
\begin{aligned}
&d\gamma_2 = (1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)]\{1/[(\sigma-1)B]\}\{D_2\widehat{\chi} + \theta(\sigma-1)\lambda_{21}[(\beta\sigma+\lambda_{12})\widehat{\tau}_{12} + (\beta\sigma-\lambda_{11})\widehat{\tau}_{21}]\}; \qquad (58)\\
&D_j \equiv (\sigma-1)B\alpha_j - \theta[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\lambda_{jk}, k \neq j.
\end{aligned}
$$

In the definition of D_j, the first term shows the direct effect of an increase in χ on γ_j through the a_j^K-channel, whereas the second term represents its indirect effect through the $\overline{\kappa}_j$-channel. If the direct effect dominates so that $D_j > 0$, then a change in χ works as a stabilizing force: when country 1 has relatively more domestic varieties, R&D becomes more difficult in that country, which pulls down its growth rate (the opposite occurs in country 2). An implication of this case becomes apparent by substituting Eqs. (57) and (58) with $\widehat{\tau}_{21} = \widehat{\tau}_{12} = 0$ into the totally differentiated form of Eq. (29):

$$
d(d\ln\chi/dt)|_{d\ln\chi/dt=0} = d\gamma_1 - d\gamma_2 = -(1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)]\{1/[(\sigma-1)B]\}D\widehat{\chi}; D \equiv D_1 + D_2.
$$

The model is locally stable around a BGP (i.e., $d(d\ln\chi/dt)/d\ln\chi|_{d\ln\chi/dt=0} < 0$) if $D_j > 0\forall j$. For the model to be well-behaved, we assume this in the rest of this paper.

In step (iv), we substitute Eqs. (57) and (58) into Eq. (30): $d\gamma_1 = d\gamma_2$, to solve for $\widehat{\chi}$ as:

$$\widehat{\chi} = [\theta(\sigma-1)/D]\{[\lambda_{12}(\beta\sigma+\lambda_{21}) - \lambda_{21}(\beta\sigma-\lambda_{11})]\widehat{\tau}_{21} - [\lambda_{21}(\beta\sigma+\lambda_{12}) - \lambda_{12}(\beta\sigma-\lambda_{22})]\widehat{\tau}_{12}\}. \tag{59}$$

Eq. (59) implies that the sign of $\partial \ln \chi/\partial \ln \tau_{kj}$ is generally ambiguous. For example, a fall in τ_{21} decreases χ if and only if $\lambda_{12}(\beta\sigma+\lambda_{21}) > \lambda_{21}(\beta\sigma-\lambda_{11})$. In this case, γ_1 directly falls by more than γ_2 from Eqs. (57) and (58), so χ should decrease to clear the growth gap.

Substituting Eq. (59) back into Eq. (58), we finally obtain:

$$d\gamma = \frac{1}{\sigma}\frac{L_2}{a_2^K\overline{\kappa}_2}\frac{\theta}{BD}\{[D_2\lambda_{12}(\beta\sigma+\lambda_{21})+D_1\lambda_{21}(\beta\sigma-\lambda_{11})]\widehat{\tau}_{21}+[D_1\lambda_{21}(\beta\sigma+\lambda_{12})+D_2\lambda_{12}(\beta\sigma-\lambda_{22})]\widehat{\tau}_{12}\}. \tag{60}$$

Eq. (60) shows that $\partial\gamma/\partial \ln \tau_{kj} > 0, k \neq j$, that is, a fall in any import trade cost lowers the balanced growth rate. A fall in either τ_{21} or τ_{12} directly lowers both γ_1 and γ_2 through the $\overline{\kappa}_j$-channel, whereas it has no direct effect on γ_1 or γ_2 through the a_j^K-channel. Even if χ is adjusted to eliminate the growth gap, the new balanced growth rate must be lower than the old one. Therefore, the main logic of growth-reducing bilateral trade liberalization under the Grossman-Helpman specification of the symmetric country BRN model remains valid even for unilateral trade liberalization in the asymmetric country case, except that the a_j^K-channel works to equalize countries' growth rates.

4 Generalized Coe-Helpman specification

In addition to the benchmark Grossman-Helpman specification, BRN considers four other specifications for R&D technologies: Coe-Helpman, efficiency-linked knowledge spillovers, reverse engineering, and lab-equipment. The third one is a modification of the first one, whereas the fourth one turns out to be similar to the second one. The second one uses $\overline{m}_j$ and $\overline{m}_k$ as the degrees of knowledge spillovers, but as mentioned at the end of section 1, they are very complicated under asymmetric countries. In this section, we deal with the Coe-Helpman specification as the simplest of the four, with a simple generalization:

$$a_j^K(n_{jj}, n_{kk}) = 1/(n_{jj} + \widetilde{\psi}_j n_{kk}); \widetilde{\psi}_j \equiv \psi_j(G_k(a_{kj})/G_k(a_{kk}))^{\varepsilon} = (a_{kj}/a_{kk})^{\varepsilon\theta}; \psi_j \in [0,1], \varepsilon \geq 0.$$

Now $\widetilde{\psi}_j$, the degree of international knowledge spillovers for country j, is nondecreasing in $G_k(a_{kj})/G_k(a_{kk})$: "the fraction of foreign varieties that are imported" (BRN, 2008, p. 29). The elasticity of $\widetilde{\psi}_j$ with respect to $G_k(a_{kj})/G_k(a_{kk})$ is given by $\varepsilon(\geq 0)$. BRN only considers the case where $\varepsilon = 1$, but we allow ε to be different from unity. The specification reduces to the Grossman-Helpman one as a special case where $\varepsilon = 0$.

With endogeneity of $\widetilde{\psi}_j$ in mind, the analysis goes similarly. Logarithmically differentiating $a_1^K(1, 1/\chi) = 1/(1+\widetilde{\psi}_1/\chi)$ and $a_2^K(1,\chi) = 1/(1+\widetilde{\psi}_2\chi)$, and using Eq. (41), we obtain:

$$\widehat{a}_1^K(1, 1/\chi) = \alpha_1[\widehat{\chi} + (\varepsilon\theta/\lambda_{21})\widehat{a}_{22}]; \alpha_1 \equiv (\widetilde{\psi}_1/\chi)/(1+\widetilde{\psi}_1/\chi) \in [0,1], \tag{61}$$

$$\widehat{a}_2^K(1,\chi) = -\alpha_2[\widehat{\chi} - (\varepsilon\theta/\lambda_{12})\widehat{a}_{11}]; \alpha_2 \equiv \widetilde{\psi}_2\chi/(1+\widetilde{\psi}_2\chi) \in [0,1], \tag{62}$$

where α_1 and α_2 are the same as those defined in Eqs. (46) and (47), respectively, with ψ_j replaced by $\widetilde{\psi}_j$. What is new in this section is the presence of the term $(\varepsilon\theta/\lambda_{kj})\widehat{a}_{kk}$ in the square brackets: in Eq. (61), for example, a decrease in a_{22} is always followed by an increase in a_{21} from the free entry condition. Both of them increase country 2's fraction of exporters, thereby enhancing international knowledge spillovers for country 1. If country 1's import liberalization encourages country 2's exports as expected, it could directly raise country 1's growth through such a_j^K-channel.

Using Eqs. (61) and (62), $\widehat{A}$ is calculated as:

$$\widehat{A} = -(1-\alpha_1-\alpha_2)\widehat{\chi} + \varepsilon\theta[(\alpha_1/\lambda_{21})\widehat{a}_{22} - (\alpha_2/\lambda_{12})\widehat{a}_{11}]. \tag{63}$$

In step (i), from Eqs. (41), (49), and (50), we obtain the same two equations to solve for $\widehat{a}_{11}$ and $\widehat{a}_{22}$. However, v now depends on a_{11} and a_{22} through A. Using Eq. (63) and $\widehat{v} = (\sigma\widehat{w}_1 + \widehat{A})/(\sigma-1)$, and rearranging terms, we have:

$$\begin{aligned}
&\widetilde{\lambda}_{11}\widehat{a}_{11} + \widetilde{\lambda}_{12}\widehat{a}_{22} = \lambda_{12}(\widehat{V} + \widehat{\tau}_{12}),\\
&\widetilde{\lambda}_{21}\widehat{a}_{11} + \widetilde{\lambda}_{22}\widehat{a}_{22} = -\lambda_{21}(\widehat{V} - \widehat{\tau}_{21});\\
&\qquad\widetilde{\lambda}_{jj} \equiv \lambda_{jj} + \varepsilon\beta\alpha_k, \widetilde{\lambda}_{jk} \equiv \lambda_{jk}(1-\varepsilon\beta\alpha_j/\lambda_{kj}), k \neq j, \widehat{V} \equiv [\sigma\widehat{w}_1 - (1-\alpha_1-\alpha_2)\widehat{\chi}]/(\sigma-1),
\end{aligned}$$

where each new coefficient $\widetilde{\lambda}_{jk}$ consists of the original λ_{jk} and a feedback effect through A, whereas $\widehat{V}$ is the same as $\widehat{v}$ in the previous section without the feedback effects. The above equations are solved as:

$$\widehat{a}_{11} = (\lambda_{12}/|\widetilde{\lambda}|)[\widehat{V} + (\lambda_{22}+\varepsilon\beta\alpha_1)\widehat{\tau}_{12} - (\lambda_{21}-\varepsilon\beta\alpha_1)\widehat{\tau}_{21}], \tag{64}$$

$$\widehat{a}_{22} = (\lambda_{21}/|\widetilde{\lambda}|)[-\widehat{V} + (\lambda_{11}+\varepsilon\beta\alpha_2)\widehat{\tau}_{21} - (\lambda_{12}-\varepsilon\beta\alpha_2)\widehat{\tau}_{12}]; |\widetilde{\lambda}| \equiv \widetilde{\lambda}_{11}\widetilde{\lambda}_{22} - \widetilde{\lambda}_{12}\widetilde{\lambda}_{21} = |\lambda| + \varepsilon\beta(\alpha_1+\alpha_2). \tag{65}$$

In step (ii), using Eqs. (41), (42), (43), (61), and (62), the logarithmically differentiated form of Eq. (40) is rewritten as:

$$-(1+\varepsilon\alpha_2)(\theta/\lambda_{12})\widehat{a}_{11} + \widehat{w}_1 + (\alpha_1+\alpha_2)\widehat{\chi} = -(1+\varepsilon\alpha_1)(\theta/\lambda_{21})\widehat{a}_{22}.$$

The first term in each side represents the rate of change in $\lambda_{jk}\overline{\kappa}_j$ minus the rate of change in a_k^K caused by a change in a_{jj}. Substituting Eqs. (64), (65), and $\widehat{V} = [\sigma\widehat{w}_1 - (1-\alpha_1-\alpha_2)\widehat{\chi}]/(\sigma-1)$ into the above expression, $\widehat{w}_1$ is solved as:

$$\begin{aligned}
0 &= -\widetilde{B}\widehat{w}_1 + \widetilde{C}\widehat{\chi} + \theta(F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}) \Leftrightarrow \widehat{w}_1 = (1/\widetilde{B})[\widetilde{C}\widehat{\chi} + \theta(F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12})]; &(66)\\
\widetilde{B} &\equiv \beta[2+\varepsilon(\alpha_1+\alpha_2)]\sigma - |\widetilde{\lambda}| = B + \varepsilon\beta(\alpha_1+\alpha_2)(\sigma-1) > 0,\\
\widetilde{C} &\equiv \beta[2+\varepsilon(\alpha_1+\alpha_2)](1-\alpha_1-\alpha_2) + |\widetilde{\lambda}|(\alpha_1+\alpha_2),\\
F_{jk} &\equiv (1+\varepsilon\alpha_k)(\lambda_{kk}+\varepsilon\beta\alpha_j) + (1+\varepsilon\alpha_j)(\lambda_{jk}-\varepsilon\beta\alpha_k), k \neq j.
\end{aligned}$$

Although Eq. (66) looks more complicated than Eq. (53), the former can be interpreted in the same way as the latter.

In step (iii), substituting Eq. (66) back into $\widehat{V} = [\sigma\widehat{w}_1 - (1-\alpha_1-\alpha_2)\widehat{\chi}]/(\sigma-1)$, and substituting the result back into Eqs. (64), (65), we obtain:

$$\widehat{V} = [1/(\sigma-1)](1/\widetilde{B})\{|\widetilde{\lambda}|[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\widehat{\chi} + \sigma\theta(F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12})\}, \tag{67}$$

$$\begin{aligned}
\widehat{a}_{11} &= \{\lambda_{12}/[(\sigma-1)\widetilde{B}]\}\{[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\widehat{\chi}\\
&\quad + (\sigma-1)\{[\beta\sigma + \lambda_{21} + \varepsilon\beta\alpha_1(\sigma-1)]\widehat{\tau}_{21} + [\beta\sigma - \lambda_{22} + \varepsilon\beta\alpha_1(\sigma-1)]\widehat{\tau}_{12}\}\}, &(68)\\
\widehat{a}_{22} &= \{\lambda_{21}/[(\sigma-1)\widetilde{B}]\}\{-[\sigma(\alpha_1+\alpha_2)+1-\alpha_1-\alpha_2]\widehat{\chi}\\
&\quad + (\sigma-1)\{[\beta\sigma + \lambda_{12} + \varepsilon\beta\alpha_2(\sigma-1)]\widehat{\tau}_{12} + [\beta\sigma - \lambda_{11} + \varepsilon\beta\alpha_2(\sigma-1)]\widehat{\tau}_{21}\}\}. &(69)
\end{aligned}$$

Eqs. (67), (68), and (69) are qualitatively similar to Eqs. (54), (55), and (56), respectively. In particular, with χ given, a fall in any import trade cost still induces more exports and more domestic selection in both countries even under the Coe-Helpman specification.

Country j's growth rate is still given by $d\gamma_j = (1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)](-\widehat{a}_j^K - \widehat{\overline{\kappa}}_j)$, where $-\widehat{a}_j^K - \widehat{\overline{\kappa}}_j$ is rewritten using Eqs. (43), (61), and (62) as:

$$-\widehat{a}_1^K - \widehat{\overline{\kappa}}_1 = -\alpha_1[\widehat{\chi} + (\varepsilon\theta/\lambda_{21})\widehat{a}_{22}] + \theta\widehat{a}_{11} = -\alpha_1\widehat{\chi} + \theta[\widehat{a}_{11} - (\varepsilon\alpha_1/\lambda_{21})\widehat{a}_{22}],$$
$$-\widehat{a}_2^K - \widehat{\overline{\kappa}}_2 = \alpha_2[\widehat{\chi} - (\varepsilon\theta/\lambda_{12})\widehat{a}_{11}] + \theta\widehat{a}_{22} = \alpha_2\widehat{\chi} + \theta[\widehat{a}_{22} - (\varepsilon\alpha_2/\lambda_{12})\widehat{a}_{11}].$$

As explained in the paragraph right after Eqs. (61) and (62), the presence of the term $(\varepsilon\theta/\lambda_{kj})\widehat{a}_{kk}$ in the first square brackets opens up the possibility that a country's import liberalization directly raises its growth through the a_j^K-channel. Using Eqs. (68) and (69), the totally differentiated forms of the growth equations are obtained as:

$$d\gamma_1 = (1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)]\{1/[(\sigma-1)\widetilde{B}]\}[-\widetilde{D}_1\widehat{\chi} + \theta(\sigma-1)(I_1\widehat{\tau}_{21} + J_1\widehat{\tau}_{12})], \tag{70}$$
$$d\gamma_2 = (1/\sigma)[L_2/(a_2^K\overline{\kappa}_2)]\{1/[(\sigma-1)\widetilde{B}]\}[\widetilde{D}_2\widehat{\chi} + \theta(\sigma-1)(I_2\widehat{\tau}_{12} + J_2\widehat{\tau}_{21})]; \tag{71}$$
$$\widetilde{D}_j \equiv (\sigma-1)\widetilde{B}\alpha_j - \theta[\sigma(\alpha_1+\alpha_2) + 1 - \alpha_1 - \alpha_2](\lambda_{jk} + \varepsilon\alpha_j), k \neq j,$$
$$I_j \equiv \lambda_{jk}[\beta\sigma + \lambda_{kj} + \varepsilon\beta\alpha_j(\sigma-1)] - \varepsilon\alpha_j[\beta\sigma - \lambda_{jj} + \varepsilon\beta\alpha_k(\sigma-1)], k \neq j,$$
$$J_j \equiv \lambda_{jk}[\beta\sigma - \lambda_{kk} + \varepsilon\beta\alpha_j(\sigma-1)] - \varepsilon\alpha_j[\beta\sigma + \lambda_{jk} + \varepsilon\beta\alpha_k(\sigma-1)], k \neq j.$$

$\widetilde{D}_j$ is similar to D_j in the previous section, and we assume that $\widetilde{D}_j > 0 \forall j$ to ensure local stability around a BGP. I_j and J_j summarize the direct effects of changes in τ_{kj} and τ_{jk}, respectively, on γ_j. In the definition of I_j, the first term shows the direct growth effect through the $\overline{\kappa}_j$-channel, which was also present under the Grossman-Helpman specification. The second term represents the direct growth effect through the a_j^K-channel, which is new to the generalized Coe-Helpman specification. Since I_j is quadratic in ε with a negative leading coefficient, it is negative if ε is sufficiently large. In this case, we have $\partial\gamma_j/\partial\ln\tau_{kj} < 0$, meaning that country j's import liberalization directly raises its growth. Similarly, we have $\partial\gamma_j/\partial\ln\tau_{jk} < 0$ if ε is sufficiently large that $J_j < 0$.

In step (iv), $\widehat{\chi}$ is solved from Eqs. (30), (70), (71) as:

$$\widehat{\chi} = [\theta(\sigma-1)/\widetilde{D}][(I_1 - J_2)\widehat{\tau}_{21} - (I_2 - J_1)\widehat{\tau}_{12}]; \widetilde{D} \equiv \widetilde{D}_1 + \widetilde{D}_2 > 0. \tag{72}$$

Substituting Eq. (72) back into Eq. (71), we finally obtain:

$$d\gamma = \frac{1}{\sigma}\frac{L_2}{a_2^K\overline{\kappa}_2}\frac{\theta}{\widetilde{B}\widetilde{D}}[(\widetilde{D}_2I_1 + \widetilde{D}_1J_2)\widehat{\tau}_{21} + (\widetilde{D}_1I_2 + \widetilde{D}_2J_1)\widehat{\tau}_{12}]. \tag{73}$$

Eq. (73) implies that $\partial\gamma/\partial\ln\tau_{kj} < 0$ if and only if $\widetilde{D}_kI_j + \widetilde{D}_jJ_k < 0, k \neq j$, which is true if ε is sufficiently large. In this case, even unilateral trade liberalization by country j raises the balanced growth rate. This was impossible under the Grossman-Helpman specification, where the a_j^K-channel does not work to raise growth directly.

5 Long-run welfare effects of trade liberalization

5.1 Unilateral trade liberalization in the general case

Suppose that the world economy is on a BGP for all $t \geq 0$. Noting that $E_{jt} = E_j^*$ and $P_{jt} = P_j^* e^{-[\gamma^*/(\sigma-1)]t}$ from Eq. (34), country j's overall utility is rewritten as:

$$\rho U_j = \ln E_j^* - \ln P_j^* + (1/\rho)\gamma^*/(\sigma-1).$$

This long-run welfare measure is the same as Eq. (22) of Ourens (2016). He calls the long-run welfare effects working through the first, second, and third terms of the above expression "the static effect on expenditure", "the static effect on the price index", and "the dynamic effect", respectively. He also uses Eq. (31) of this paper directly, with $w_j^* = 1$ by symmetry, as the level of expenditure. Instead of doing the same, we further express E_j^* in terms of γ^*. To do this, Eq. (28) is rewritten as $w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*) + \rho = \sigma(\rho + \delta + \gamma^*)$, or $p_j^{K*}\overline{\kappa}_j^* = w_j^* L_j/[\sigma(\rho+\delta+\gamma^*)-\rho]$. For $p_j^{K*}\overline{\kappa}_j^*$ to be positive, we assume that $\sigma(\rho+\delta+\gamma^*)-\rho > 0$, which implies that $\rho+\delta+\gamma^* > 0$. Using these expressions, Eq. (31) is rewritten as:

$$E_j^* = p_j^{K*}\overline{\kappa}_j^*[\rho + w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*)] = w_j^* L_j \sigma(\rho+\delta+\gamma^*)/[\sigma(\rho+\delta+\gamma^*)-\rho].$$

This means that country j's expenditure E_j^* is its total wage $w_j^* L_j$ times the multiplier $\sigma(\rho+\delta+\gamma^*)/[\sigma(\rho+\delta+\gamma^*)-\rho](>1)$, which is decreasing in γ^*. Using this, ρU_j is rewritten as:

$$\rho U_j = \ln w_j^* + \ln L_j + \ln \frac{\sigma(\rho+\delta+\gamma^*)}{\sigma(\rho+\delta+\gamma^*)-\rho} - \ln P_j^* + \frac{1}{\rho}\frac{\gamma^*}{\sigma-1}. \tag{74}$$

The natural log of "the level of expenditure" of Ourens (2016) consists of the second (constant) and third terms in Eq. (74), where the third term is actually an additional "dynamic effect" counteracting the original "dynamic effect". The first term $\ln w_j^*$ was $\ln 1 = 0$ in the symmetric country case of BRN and Ourens (2016), but cannot be ignored in the asymmetric country case. Totally differentiating Eq. (74), and using the ACR formula for the real wage (45), our general welfare formula is given by:

$$\rho dU_j = \widehat{w}_j^* - \widehat{P}_j^* + \Gamma^* d\gamma^* = -\frac{1+\beta}{\theta}\widehat{\lambda}_{jj}^* + \Gamma^* d\gamma^*; \tag{75}$$

$$\Gamma^* \equiv \frac{1}{\rho+\delta+\gamma^*} - \frac{\sigma}{\sigma(\rho+\delta+\gamma^*)-\rho} + \frac{1}{\rho}\frac{1}{\sigma-1} = \frac{(\delta+\gamma^*)[(\sigma-1)\rho+\sigma(\rho+\delta+\gamma^*)]}{(\rho+\delta+\gamma^*)[\sigma(\rho+\delta+\gamma^*)-\rho]\rho(\sigma-1)}.$$

What is the sign of Γ^*, the total dynamic effect? we already know that $\sigma(\rho+\delta+\gamma^*)-\rho > 0$ and $\rho+\delta+\gamma^* > 0$. Moreover, from the market-clearing condition for the knowledge good (14), $\delta+\gamma^* \geq 0$ if and only if $Q_j^K \geq 0$. Therefore, we always have $\Gamma^* \geq 0$, with equality if and only if $Q_j^K = 0$. Ourens (2016, Result 2) leaves the possibility that: "the static effect on expenditure prevails over the dynamic effect", or $\Gamma^* < 0$, but this case is true if and only if $Q_j^K < 0$, which is economically impossible. Rather, the dynamic effect is stronger than the static effect on expenditure as long as the R&D sector is active.

We can use Eq. (75) to evaluate the long-run welfare effects of unilateral trade liberalization by country 1: $d\tau_{21} < 0$, in the general case. Suppose that $\widetilde{D}_2^* I_1^* + \widetilde{D}_1^* J_2^* < 0$. From Eq. (73), we have $\partial\gamma^*/\partial\ln\tau_{21} < 0$, meaning that the balanced growth rate goes up, contributing to higher welfare for both countries 1 and 2. For real wages, from Eq. (44), w_j^*/P_j^* goes up if and only if a_{jj}^* decreases. In view of Eqs. (68) and (69), a fall in τ_{21} directly decreases both a_{11}^* and a_{22}^*. However, since Eq. (72) implies that $\partial\ln\chi^*/\partial\ln\tau_{21} = [\theta(\sigma-1)/\widetilde{D}^*](I_1^* - J_2^*)$, whose sign is generally ambiguous, a fall in τ_{21} indirectly increases either a_{11}^* or a_{22}^* through the change in χ^*, depending on whether $\partial\ln\chi^*/\partial\ln\tau_{21}$ is negative or positive, respectively. If the last effect is dominant, unilateral trade liberalization by country 1 might lower the long-run welfare of at most one country in spite of faster long-run growth.

5.2 Bilateral trade liberalization in the symmetric country case

We next consider a special case where the two countries are always symmetric. Letting $\widetilde{D}_1^* = \widetilde{D}_2^*, I_1^* = I_2^*, J_1^* = J_2^*$, and $\widehat{\tau}_{12} = \widehat{\tau}_{21}$ in Eq. (73), using $L_2/(p_2^{K*}\overline{\kappa}_2^*) = \sigma(\rho+\delta+\gamma^*)-\rho$ from Eq. (28), and noting that $I_2^* + J_2^* = \widetilde{B}^*(\lambda_{21}^* - \varepsilon\alpha_2^*)$, we obtain:

$$d\gamma^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \{[\sigma(\rho+\delta+\gamma^*)-\rho]/\sigma\}\theta(\lambda_{21}^* - \varepsilon\alpha_2^*). \tag{76}$$

Eq. (76) corresponds to the long-run growth effect of bilateral trade liberalization in BRN and Ourens (2016). BRN shows that $d\gamma^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} > 0$ under both the Grossman-Helpman (i.e., $\varepsilon = 0$) and Coe-Helpman (i.e., $\varepsilon = 1$) specifications. In contrast, by allowing ε to take any nonnegative value, we make it possible that $d\gamma^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} < 0$ if and only if $\varepsilon > \lambda_{21}^*/\alpha_2^*$.

Turning to Eq. (72), we obtain:

$$\widehat{\chi}^*/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = [\theta(\sigma-1)/(2\widetilde{D}_2^*)][(I_2^* - J_2^*) - (I_2^* - J_2^*)] = 0. \tag{77}$$

This is consistent with the fact that $\chi^* = 1$ under symmetry. Using Eq. (77), Eq. (69) is simplified to:

$$\widehat{a}_{22}^{*}/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \{\lambda_{21}^{*}/[(\sigma-1)\widetilde{B}^{*}]\}(\sigma-1)[\beta\sigma+\lambda_{12}^{*}+\varepsilon\beta\alpha_{2}^{*}(\sigma-1)+\beta\sigma-\lambda_{11}^{*}+\varepsilon\beta\alpha_{2}^{*}(\sigma-1)] = \lambda_{21}^{*} > 0. \quad (78)$$

This means that bilateral trade liberalization decreases a_{22}^{*}. From Eqs. (44) and (78), country 2's (and, by symmetry, country 1's) real wage increases.

Using Eqs. (42) and (78), Eq. (76) is rewritten as:

$$d\gamma^{*}/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \{[\sigma(\rho+\delta+\gamma^{*})-\rho]/\sigma\}[(\lambda_{21}^{*}-\varepsilon\alpha_{2}^{*})/\lambda_{21}^{*}]\widehat{\lambda}_{22}^{*}/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}};\, \widehat{\lambda}_{22}^{*}/\widehat{\tau}_{21}|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \theta\lambda_{21}^{*} > 0. \quad (79)$$

Eq. (79) is considered as the ACR formula for the balanced growth rate: there is a monotonic relationship between a country's autarkiness ratio and its long-run growth rate, with the elasticity of the relationship depending on the value of ε.

Finally, substituting Eq. (79) into Eq. (75), we obtain the ACR formula for the long-run welfare:

$$\rho\,\frac{dU_j}{\widehat{\tau}_{21}}\bigg|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}} = \left[-\frac{1+\beta}{\theta}+\Gamma^{*}\frac{\sigma(\rho+\delta+\gamma^{*})-\rho}{\sigma}\frac{\lambda_{21}^{*}-\varepsilon\alpha_{2}^{*}}{\lambda_{21}^{*}}\right]\frac{\widehat{\lambda}_{22}^{*}}{\widehat{\tau}_{21}}\bigg|_{\widehat{\tau}_{12}=\widehat{\tau}_{21}}. \quad (80)$$

In the square brackets of Eq. (80), the first negative term indicates the long-run welfare effect through the real wage, which was also present in the static Melitz model. What is new here is the second term, the long-run welfare effect through the balanced growth rate. It is nonpositive if and only if $\varepsilon \geq \lambda_{21}^{*}/\alpha_{2}^{*}$. Bilateral trade liberalization raises country j's (and, by symmetry, country k's) long-run welfare if $\varepsilon \geq \lambda_{21}^{*}/\alpha_{2}^{*}$. Even if $\varepsilon < \lambda_{21}^{*}/\alpha_{2}^{*}$, so that bilateral trade liberalization lowers the balanced growth rate, the long-run welfare can rise if the real wage effect dominates.

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