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JEL classification: F13; F43

Keywords: Lab-equipment model; Heterogeneous firms; Asymmetric countries; Unilateral trade liberalization; Endogenous growth

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1 Introduction

The lab-equipment model is recognized by growth economists as the simplest model of endogenous technological change. Originating in Rivera-Batiz and Romer (1991), it is one of the two, along with the knowledge-driven model, main types of expanding variety models, which, together with Schumpeterian growth models featuring creative destruction, constitute two standard endogenous technological progress models. Whereas the knowledge-driven specification assumes labor as the only private input, together with knowledge as the public input, in R&D (i.e., creation of new varieties of differentiated goods), the lab-equipment specification considers the composite final good as the only R&D input with a fixed coefficient.[1] Thanks to its simplicity (i.e., exogenous marginal cost of R&D and the absence of externalities from knowledge spillovers), the model is introduced as the first baseline model of endogenous technological change in graduate growth textbooks such as Barro and Sala-i-Martin (2004) and Acemoglu (2009), and also applied to problems such as increasing skill premium (e.g., Acemoglu, 2002). International trade is one of the earliest and best-known applications of the lab-equipment model: Rivera-Batiz and Romer (1991) show that allowing for free trade in differentiated intermediate goods (but not free flows of ideas) between two symmetric countries has no long-run growth effect under the knowledge-driven specification, whereas it raises the long-run growth rate of varieties under the lab-equipment specification.[2] The Rivera-Batiz–Romer model, including both the knowledge-driven and lab-equipment specifications, is regarded as a starting point for studying trade and growth.[3]

The Rivera-Batiz–Romer model builds on two unrealistic assumptions: homogeneous firms and symmetric countries. For extension to heterogeneous firms, Baldwin and Robert-Nicoud (2008) (BRN hereafter) blend the pathbreaking work of Melitz (2003) with Rivera-Batiz and Romer (1991) to demonstrate that the long-run growth rate is decreasing in the product of p_K and $\overline{\kappa}$, "an 'intensive form' " (BRN, 2008, p. 25) of the price of the knowledge good (normalizing the effect of variety growth), and an entrant's: "expected units of knowledge required to get a 'winner.' " (BRN, 2008, p. 25), respectively.[4] Symmetric trade liberalization, on the one hand, encourages more exports and more domestic selection (i.e., exit), which increases $\overline{\kappa}$ and hence retards growth (i.e., the $\overline{\kappa}$-channel). On the other hand, it stimulates flows of goods and/or ideas, and the resulting decrease in p_K promotes growth (i.e., the p_K-channel). The total growth effect of symmetric trade liberalization is negative under the knowledge-driven specification with international knowledge spillovers, but it is positive under the lab-equipment specification.[5] Since BRN, many papers examine the implications of liberalization-induced reallocations across heterogeneous firms for countries' long-run growth and welfare (e.g., Atkeson and Burstein, 2010; Gustafsson and Segerstrom, 2010; Dinopoulos and Unel, 2011; Perla et al., 2015; Fukuda, 2016; Sampson, 2016; Ourens, 2016; Naito, 2017), but all of them but Naito (2017) assume symmetric countries. This is understandable considering that the original motivation of the Krugman (1980) model of monopolistic competition was to explain two-way trade in differentiated products among similar countries. However, sticking to this assumption prevents us from studying the effects of policy reforms which

[1] As explained by Rivera-Batiz and Romer (1991), this is equivalent to assuming that, whatever inputs are used, the R&D production function is the same as the final good production function up to a proportionality constant.

[2] According to their intuitions, the difference arises from whether the price of a patent is variable or fixed: under the lab-equipment specification with the price of a patent fixed by the exogenous marginal cost, an increase in the total profit of a potential entrant after trade must be accompanied by an increase in the interest rate, leading to faster long-run growth.

[3] See, for example, Grossman and Helpman (1991), Acemoglu (2009), and Feenstra (2016).

[4] In BRN, the term "knowledge" refers to both the flow of a good used as fixed entry and overhead costs, and the existing number of varieties creating positive externalities in R&D under the knowledge-driven specification. To distinguish between them, we call the former the "knowledge good". This is the same as patents in Rivera-Batiz and Romer (1991).

[5] They consider two types of the knowledge-driven specification, namely the Grossman–Helpman and Coe–Helpman specifications. Whereas the coefficient of international knowledge spillovers is constant in the former, it is increasing in the fraction of exporters in domestic surviving firms with elasticity one in the latter.

are mostly asymmetric across countries. A typical example is trade liberalization: according to a survey on trade costs by Anderson and van Wincoop (2004): "(o)n average, developing countries have significantly larger trade costs, by a factor of two or more in some important categories." (Anderson and van Wincoop, 2004, p. 747). This suggests that developing countries still have a long way to catch up with developed countries in terms of liberalizing trade. To examine the effects of asymmetric trade liberalization on countries' long-run growth and welfare through intraindustry reallocations, we have to depart from the conventional assumption. To this end, we extend Rivera-Batiz and Romer (1991) to include both heterogeneous firms and asymmetric countries.

So far there is only one paper that successfully takes account of both heterogeneous firms and asymmetric countries in the Rivera-Batiz–Romer model. Naito (2017) finds that the BRN model can be extended to asymmetric countries by focusing on a balanced growth path (BGP), where all variables grow at constant (including zero) rates. Specifically, in addition to the zero cutoff profit and free entry conditions, which are two basic ingredients of the Melitz model, he uses the balanced trade and balanced growth conditions to determine the relative wage and relative number of domestic varieties simultaneously with the cutoff unit labor requirements for production of differentiated goods. Based on the extended model and focusing on the knowledge-driven specification, he shows that even unilateral trade liberalization can raise long-run growth if the liberalization-induced exports and domestic selection increase countries' coefficients of international knowledge spillovers sufficiently that BRN's p_K-channel is stronger than the $\overline{\kappa}$-channel. However, the required elasticity of the coefficients is so high that it may not be satisfied in reality. A natural question is: can unilateral trade liberalization raise countries' long-run growth and welfare under the lab-equipment specification? If yes, then this paper can provide further support for the recent empirical evidence finding the positive effect of trade liberalization on economic growth (e.g., Wacziarg and Welch, 2008; Estevadeordal and Taylor, 2013). The specific purpose of this paper is to answer this question.

One might think that, since the lab-equipment specification is simpler than the knowledge-driven specification in Rivera-Batiz and Romer (1991), this paper is technically less demanding than Naito (2017). This turns out to be wrong: each country's price of the final good depends on its wage and intermediate good price index, which in turn depends on both countries' final good prices, instead of wages as in Naito (2017). Such simultaneous input-output structure makes it more difficult to solve for a BGP, especially with asymmetric countries having different final good prices as well as wages. Even under the more complicated lab-equipment specification with heterogeneous firms and asymmetric countries, we can characterize a BGP.

In spite of the technical difficulty, we find much stronger and more useful results than Naito (2017). First, we derive extended ACR (Arkolakis–Costinot–Rodríguez-Clare) formulas for long-run growth and welfare changes even with asymmetric countries. The original ACR formula invented by Arkolakis et al. (2012) states that, in a wide class of modern trade models (including Armington, Krugman, Eaton–Kortum, and Melitz), the logarithmic change in a country's real wage in terms of the composite final good is proportional to the logarithmic change in its share of domestic expenditure, with the elasticity equal to the inverse of its "trade elasticity" (i.e., the elasticity of a country's imports relative to its domestic expenditure with respect to its import trade cost, which is negative). Our long-run growth and welfare formulas are based on the following properties: (i) each country's long-run welfare is increasing in its real wage and long-run growth rate; (ii) as in the original ACR formula, each country's real wage is decreasing in its domestic revenue share (which is equal to its domestic expenditure share from its zero balance of trade); and (iii) each country's long-run growth rate is increasing in its real wage. This is because only the final good is used for R&D: an increase in the real wage is equivalent to a decrease in the price of the final good in terms of labor, which necessarily decreases BRN's $p_K \times \overline{\kappa}$. It is true that Naito (2017) also derives properties (i) and (ii) with asymmetric countries, but he can only obtain property (iii) with symmetric countries.[6] This paper is the first to derive the extended ACR formulas with both endogenous growth and asymmetric countries. Moreover, our dynamic model produces much greater welfare gains from openness than the static Melitz model. For plausible values of parameters and the original growth rate, a 1% decrease in a country's domestic revenue share brings about more than four times as large a welfare gain (expressed in flow terms) as the static counterpart.

Second, even unilateral trade liberalization always raises both countries' long-run growth and welfare. Suppose that country 1 liberalizes its imports. This directly facilitates country 2's exports and domestic selection, and the resulting tougher competition in market 2 hinders country 1's exports and domestic selection, with the relative wage given. However, for country 1's trade deficit to be cleared, country 1's relative wage and hence its relative price of the final good decrease. This indirectly promotes country 1's exports and domestic selection whereas retards those of country 2. It turns out that the direct, partial equilibrium effect outweighs the indirect, general equilibrium effect for the partner country 2, whereas the indirect, general equilibrium effect dominates for the liberalizing country 1. Since tougher domestic selection decreases the domestic revenue share, each country's growth rate increases, with the relative number of domestic varieties given. Even if the relative number of domestic varieties is adjusted to equalize countries' growth rates, the new balanced growth rate is higher than the old one. Finally, from the extended ACR welfare formula, both countries' long-run welfare also increases. Unlike Naito (2017), where the long-run growth effect of unilateral trade liberalization is ambiguous depending on the elasticity of countries' coefficients of international knowledge spillovers, it is unambiguously positive under the lab-equipment specification. This result also implies that the positive long-run growth effect of symmetric trade liberalization in the lab-equipment models of Rivera-Batiz and Romer (1991) and BRN is robust to unilateral trade liberalization with asymmetric countries.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 characterizes a BGP, and derives long-run growth and welfare formulas. Section 4 examines the long-run effects of unilateral trade liberalization. Section 5 concludes.

2 The model

We extend the original lab-equipment model of Rivera-Batiz and Romer (1991) to allow for heterogeneous firms in line with BRN, and asymmetric countries. In country $j(= 1, 2)$, there are three sectors, namely a final good sector, an intermediate good sector, and an R&D (i.e., knowledge good) sector. The intermediate goods are differentiated and tradable, whereas the final good, knowledge good, and labor are homogeneous and nontradable.[7] The final good is produced from a variety of intermediate goods and labor under constant returns to scale and perfect competition, and is used for consumption, production of the intermediate goods, and R&D. Each intermediate good firm uses the knowledge good as the fixed input, and the final good as the variable input. The intermediate good firms are heterogeneous in a, the amount of the final good each firm requires to produce a unit of each intermediate good. The lower a is, the more productive a firm is.

[6] In a Melitz-based endogenous growth model where entrants learn from the average productivity of incumbents, Sampson (2016) finds that both the static and dynamic welfare gains are increasing in countries' common import penetration ratio (i.e., one minus countries' common domestic expenditure share). However, his model is also limited to symmetric countries.

[7] In line with Acemoglu (2002, 2009), we assume that the intermediate goods are nondurable unlike Rivera-Batiz and Romer (1991). This reduces the number of state variables without affecting the results.

Finally, the knowledge good is produced from the final good under constant returns to scale and perfect competition. Notations basically follow BRN.

2.1 Households

The representative household in country j maximizes its overall utility $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho t)dt$, subject to its budget constraint $\dot{W}_{jt} = r_{jt}W_{jt} + w_{jt}L_j - E_{jt}; \dot{W}_{jt} \equiv dW_{jt}/dt, E_{jt} \equiv p_{jt}^Y C_{jt}$, with $\{r_{jt}, w_{jt}, p_{jt}^Y\}_{t=0}^\infty$ and W_{j0} given, where $t(\in [0,\infty))$ is time (omitted whenever no confusion arises), C_j is consumption, ρ is the subjective discount rate, W_j is the asset, r_j is the interest rate, w_j is the wage rate, L_j is the supply of labor, E_j is the consumption expenditure, and p_j^Y is the price of the final good. Unless otherwise stated, only parameters without country subscripts (e.g., ρ) are assumed to be the same across countries. Dynamic optimization with respect to E_j implies the Euler equation $\dot{E}_{jt}/E_{jt} = r_{jt} - \rho$.

2.2 Final good firms

The representative final good firm in country j maximizes its profit $\pi_j^Y = p_j^Y Y_j - \int_{\Theta_j} p_j(i)x_j(i)di - w_j L_j^Y$, subject to its production function $Y_j = A_j X_j^{\alpha_j}(L_j^Y)^{1-\alpha_j}, X_j = (\int_{\Theta_j} x_j(i)^{(\sigma-1)/\sigma}di)^{\sigma/(\sigma-1)}$, with $p_j^Y, \{p_j(i)\}_{i\in\Theta_j}$, and w_j given, where Y_j is the supply of the final good, Θ_j is the set of available varieties of intermediate goods, $p_j(i)$ is the demand price of variety i, $x_j(i)$ is the demand for variety i, L_j^Y is the demand for labor, A_j is an arbitrary constant, X_j is the index of the intermediate goods, $\alpha_j(\in (0,1))$ is the Cobb-Douglas cost share of the intermediate goods, and $\sigma(>1)$ is the elasticity of substitution across varieties.

The profit maximization problem is solved in three steps. First, the minimized cost of producing X_j units of the intermediate good index is defined as $P_j X_j \equiv \min_{\{x_j(i)\}_{i\in\Theta_j}}\{\int_{\Theta_j} p_j(i)x_j(i)di : X_j = (\int_{\Theta_j} x_j(i)^{(\sigma-1)/\sigma}di)^{\sigma/(\sigma-1)}\}; P_j \equiv (\int_{\Theta_j} p_j(i)^{1-\sigma}di)^{1/(1-\sigma)}$, where P_j is the price index of the intermediate goods (i.e., minimized cost of producing a unit of the intermediate good index). Second, the minimized cost of producing Y_j units of the final good is defined as $c_j^Y(P_j, w_j)Y_j \equiv \min_{X_j, L_j^Y}\{P_j X_j + w_j L_j^Y : Y_j = A_j X_j^{\alpha_j}(L_j^Y)^{1-\alpha_j}\}; c_j^Y(P_j, w_j) \equiv P_j^{\alpha_j} w_j^{1-\alpha_j}$, where we set $A_j \equiv \alpha_j^{-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)}$ to simplify the unit cost function $c_j^Y(P_j, w_j)$. Third, the first-order condition for profit maximization is given by $p_j^Y = c_j^Y(P_j, w_j)$, which implies the free entry condition $p_j^Y Y_j = \int_{\Theta_j} p_j(i)x_j(i)di + w_j L_j^Y$.

2.3 Intermediate good firms

An intermediate good firm indexed by the unit final good requirement a in the source country j maximizes its profit in the destination country $k(=1,2)$ $\pi_{jk}(a) = p_{jk}^f(a)y_{jk}(a) - p_j^Y a y_{jk}(a)$, subject to the market-clearing condition $y_{jk}(a) = \tau_{jk}x_{jk}(a)$, the conditional demand function $x_{jk}(a) = p_{jk}(a)^{-\sigma}P_k^\sigma X_k = (\tau_{jk}p_{jk}^f(a))^{-\sigma}P_k^\sigma X_k$, with p_j^Y, P_k, and X_k given, where $p_{jk}^f(a)$ is the supply price of the firm's variety, $y_{jk}(a)$ is the supply of the firm's variety, $\tau_{jk}(\geq 1)$ is the iceberg trade cost factor of delivering one unit of a variety from country j to country k (with $\tau_{jj} = 1$), $x_{jk}(a)$ is country k's demand for the firm's variety, and $p_{jk}(a)$ is country k's demand price of the firm's variety.[8] For $k \neq j$, τ_{jk} represents country k's import trade cost, which is the only policy variable in this paper. The first-order condition for profit maximization is rewritten as $(p_{jk}^f(a) - p_j^Y a)/p_{jk}^f(a) = 1/\sigma \Leftrightarrow p_{jk}^f(a) = p_j^Y a/(1-1/\sigma)$, and the resulting revenue and gross profit are given by $e_{jk}(a) \equiv p_{jk}^f(a)y_{jk}(a) = [\tau_{jk}p_j^Y a/(1-1/\sigma)]^{1-\sigma}P_k^\sigma X_k$ and $\pi_{jk}(a) = e_{jk}(a)/\sigma = [\tau_{jk}p_j^Y a/(1-1/\sigma)]^{1-\sigma}P_k^\sigma X_k/\sigma$, respectively. The gross firm value is defined as usual:

[8]The conditional demand function is derived from Shephard's lemma: $x_k(i) = (\partial P_k/\partial p_k(i))X_k = p_k(i)^{-\sigma}P_k^\sigma X_k$.

$v_{jkt}(a) \equiv \int_t^\infty \pi_{jks}(a)\exp(-\int_t^s (r_{ju}+\delta)du)ds$, where δ is the exogenous rate of a bad shock forcing a firm to exit (e.g., Melitz, 2003).

As in Melitz (2003), there are two conditions determining the equilibrium productivity distribution: the zero cutoff profit and free entry conditions. The zero cutoff profit condition means that the gross value of the cutoff firm just covers the one-time fixed overhead cost:

$$v_{jkt}(a_{jkt}) = P_{jt}^K \kappa_{jk}, j, k = 1, 2, \tag{1}$$

where a_{jk} is the value of a for the cutoff firm, P_j^K is the price of the knowledge good, and κ_{jk} is country j's fixed overhead cost in market k in terms of the knowledge good. We assume that $\kappa_{jk} > \kappa_{jj}, k \neq j$, that is, firms incur a larger fixed overhead cost for exports than domestic sales. Using Eq. (1) and $e_{jks}(a)/e_{jks}(a_{jkt}) = (a/a_{jkt})^{1-\sigma} = \pi_{jks}(a)/\pi_{jks}(a_{jkt})$, $v_{jkt}(a)$ is rewritten as $v_{jkt}(a) = (a/a_{jkt})^{1-\sigma} P_{jt}^K \kappa_{jk}$. This implies that firms with $a > a_{jk}$ exit from, whereas those with $a \leq a_{jk}$ enter, market k. It is assumed that $a_{jk} < a_{jj} \forall j, k = 1, 2, k \neq j$, that is, only the most productive domestic surviving firms with $a \leq a_{jk}$ can profitably export.

After paying a one-time fixed entry cost, a is randomly drawn from a source-specific distribution function $G_j(a); a \in [0, a_{j0}]$, and the corresponding density function $g_j(a)$. Let $\mu_{jk}(a|a_{jk}) \equiv g_j(a)/G_j(a_{jk})$ be the density function conditional on survival in market k, with $\int_0^{a_{jk}} \mu_{jk}(a|a_{jk})da = 1$. The free entry condition requires that the sum of the expected net firm values over all markets is equal to the fixed entry cost:

$$\begin{aligned}\textstyle\sum_k \int_0^{a_{jk}} (v_{jk}(a) - P_j^K \kappa_{jk}) g_j(a) da = P_j^K \kappa_j^e \Leftrightarrow \textstyle\sum_k \kappa_{jk} H_{jk}(a_{jk}) = \kappa_j^e; \\ H_{jk}(a_{jk}) \equiv G_j(a_{jk}) h_{jk}(a_{jk}), h_{jk}(a_{jk}) \equiv (\overline{a}_{jk}(a_{jk})/a_{jk})^{1-\sigma} - 1, \\ \overline{a}_{jk}(a_{jk}) \equiv (\int_0^{a_{jk}} a^{1-\sigma} \mu_{jk}(a|a_{jk}) da)^{1/(1-\sigma)},\end{aligned} \tag{2}$$

where κ_j^e is country j's fixed entry cost in terms of the knowledge good, $H_{jk}(a_{jk})$ is country j's expected net firm value in market k relative to the fixed overhead cost $P_j^K \kappa_{jk}$, $h_{jk}(a_{jk})$ is the conditional version of $H_{jk}(a_{jk})$, and $\overline{a}_{jk}(a_{jk})$ is the aggregate unit final good requirement of surviving firms. Since $H_{jk}(a_{jk})$ is increasing in a_{jk} mainly through an increase in the probability of survival $G_j(a_{jk})$, Eq. (2) implies that, for $k \neq j$, a_{jj} and a_{jk} always move in the opposite directions.[9] In other words, whenever more firms exit, or selected out, from their domestic market (i.e., a_{jj} decreases), more firms enter their export market (i.e., a_{jk} increases), and vice versa.

2.4 R&D firms

The representative R&D firm in country j maximizes its profit $\pi_j^K = P_j^K Q_j^K - p_j^Y D_j$, subject to the production function $Q_j^K = D_j$, with P_j^K and p_j^Y given, where Q_j^K is the supply of the knowledge good, and D_j is the demand for the final good from the R&D sector. The first-order condition for profit maximization is given by $P_j^K = p_j^Y$, implying the free entry condition $P_j^K Q_j^K = p_j^Y D_j$.

2.5 Markets

The market-clearing conditions for the asset, labor, knowledge good, and final good are given by, respectively:

[9] Increasingness of $H_{jk}(a_{jk})$ can be proved in a similar manner to Melitz (2003, Appendix B).

$$
\begin{aligned}
W_j &= \textstyle\sum_k n_j^e \displaystyle\int_0^{a_{jk}} v_{jk}(a) g_j(a) da = \textstyle\sum_k n_{jk} \displaystyle\int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk}) da; n_{jk} \equiv n_j^e G_j(a_{jk}), j = 1, 2, \\
L_j &= L_j^Y, j = 1, 2, \\
Q_j^K &= \overline{\kappa}_j(\dot{n}_{jj} + \delta n_{jj}); \overline{\kappa}_j \equiv (\textstyle\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e)/G_j(a_{jj}), j = 1, 2, \\
Y_j &= C_j + D_j + F_j; F_j \equiv \textstyle\sum_k n_j^e \displaystyle\int_0^{a_{jk}} a y_{jk}(a) g_j(a) da = \textstyle\sum_k n_{jk} \displaystyle\int_0^{a_{jk}} a y_{jk}(a)\mu_{jk}(a|a_{jk}) da, j = 1, 2,
\end{aligned}
$$

where n_j^e is the number of entrants in country j, n_{jk} is the number of entrants in country j surviving in country k, or the number of varieties country j sells to country k, $\overline{\kappa}_j$ is an entrant's: "expected units of knowledge required to get a 'winner.' " (BRN, 2008, p. 25), or its expected total fixed costs in terms of the knowledge good conditional on domestic survival, and F_j is the demand for the final good from the intermediate good sector.

To close the model, we impose:[10]

$$
\begin{aligned}
\textstyle\sum_k n_{jk} \displaystyle\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk}) da &= \textstyle\sum_k n_{kj} \displaystyle\int_0^{a_{kj}} e_{kj}(a)\mu_{kj}(a|a_{kj}) da = P_j X_j, \\
n_{jk} \int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk}) da &= n_{kj} \int_0^{a_{kj}} e_{kj}(a)\mu_{kj}(a|a_{kj}) da, k \neq j.
\end{aligned}
$$

The first line shows country j's national budget constraint, requiring that its total revenue of selling the intermediate goods to all destinations is equal to its total expenditure for buying the intermediate goods from all sources. Subtracting country j's domestic revenue and expenditure from the first line, we obtain the second line, which represents country j's zero balance of trade. Defining the revenue share of varieties country j sells to country k as $\lambda_{jk} \equiv n_{jk} \int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk}) da / \sum_l n_{jl} \int_0^{a_{jl}} e_{jl}(a)\mu_{jl}(a|a_{jl}) da; \sum_k \lambda_{jk} = 1$, country j's zero balance of trade is simplified to $\lambda_{jk} P_j X_j = \lambda_{kj} P_k X_k, k \neq j$.

Having specified our model, we characterize a BGP in the next section.

3 Balanced growth path

3.1 Characterization

Let labor in country 2 be the numeraire: $w_2 \equiv 1$. From now on, we focus on a BGP, where all variables grow at constant (including zero) rates. Suppose that the world economy is on a BGP for $t \geq 0$. One of the main variables of interest is country j's growth rate of domestic varieties $\gamma_j^* \equiv (\dot{n}_{jj}/n_{jj})^*$ (see Appendix A for derivation):

$$
\gamma_j^* = [\alpha_j/(1-\alpha_j)](1/\sigma) w_j^* L_j / (p_j^{K*} \overline{\kappa}_j^*) - \rho - \delta,
$$

where $p_j^K \equiv n_{jj} P_j^K$ is: "an 'intensive form' " (BRN, 2008, p. 25) of P_j^K normalizing the negative effect of variety growth on $P_j^K = p_j^Y = P_j^{\alpha_j} w_j^{1-\alpha_j}$, and a superscript asterisk represents a BGP. The above expression reveals that country j's growth rate depends on p_j^{K*}/w_j^* and $\overline{\kappa}_j^*$, which correspond to: "the p_K-channel and

[10] In fact, country j's national budget constraint is derived from its Walras' law, which in turn is derived by time differentiating the asset market-clearing condition, and using the no-arbitrage condition $\dot{v}_{jk}(a) = (r_j + \delta) v_{jk}(a) - \pi_{jk}(a)$, household budget constraint, and free entry conditions for all sectors.

the $\overline{\kappa}$-channel" (BRN, 2008, p. 27), respectively. Noting that $\overline{\kappa}_j^*$ depends only on $\{a_{jk}^*\}$, which are constant from Eq. (2), constancy of γ_j^* implies that p_j^{K*}/w_j^* is constant.

Ensuring constancy of p_j^{K*}/w_j^* requires a careful investigation of prices. Country j's intermediate good price index is rewritten as:

$$\begin{aligned} P_j &= \{\textstyle\sum_k n_{kj}[\tau_{kj}p_k^Y\overline{a}_{kj}(a_{kj})/(1-1/\sigma)]^{1-\sigma}\}^{1/(1-\sigma)} = n_{jj}^{1/(1-\sigma)}p_j^Y\overline{m}_j/(1-1/\sigma); \\ \overline{m}_j &\equiv \{\textstyle\sum_k (n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}^{1/(1-\sigma)}, \end{aligned} \tag{3}$$

where $p_j^Y\overline{m}_j$ is: "a weighted average of firms' marginal selling costs in a particular market" (BRN, 2008, p. 24), which is market j in the present case. Substituting Eq. (3) into $p_j^Y = P_j^{\alpha_j}w_j^{1-\alpha_j}$, and solving the resulting equation for p_j^Y with $n_{jj}, \overline{m}_j$, and w_j given, we obtain $p_j^Y = [n_{jj}^{1/(1-\sigma)}\overline{m}_j/(1-1/\sigma)]^{\alpha_j/(1-\alpha_j)}w_j$.[11] Since constant growth of n_{jj} continues to decrease p_j^Y/w_j with the elasticity of $[1/(\sigma-1)]\alpha_j/(1-\alpha_j)$, for $p_j^K/w_j = n_{jj}P_j^K/w_j = n_{jj}p_j^Y/w_j$ to be constant over time, the elasticity must be unity. Solving $[1/(\sigma-1)]\alpha_j/(1-\alpha_j) = 1$ for α_j gives:[12]

$$\alpha_j = 1 - 1/\sigma = (\sigma-1)/\sigma.$$

Under this restriction on α_j for the existence of a BGP, p_j^K is expressed as:

$$\begin{aligned} p_j^K &= [\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j; \\ \overline{m}_1 &= \{\overline{a}_{11}(a_{11})^{1-\sigma} + (1/\chi)(G_2(a_{21})/G_2(a_{22}))[(\tau_{21}p_2^Y/p_1^Y)\overline{a}_{21}(a_{21})]^{1-\sigma}\}^{1/(1-\sigma)}, \\ \overline{m}_2 &= \{\overline{a}_{22}(a_{22})^{1-\sigma} + \chi(G_1(a_{12})/G_1(a_{11}))[(\tau_{12}p_1^Y/p_2^Y)\overline{a}_{12}(a_{12})]^{1-\sigma}\}^{1/(1-\sigma)}, \\ \chi &\equiv n_{11}/n_{22}, \end{aligned}$$

where χ is the relative number of domestic varieties of country 1 to country 2. Then the growth equation is finally rewritten as:

$$\gamma_j^* = (1-1/\sigma)w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*) - \rho - \delta = (1-1/\sigma)L_j/\{[\overline{m}_j^*/(1-1/\sigma)]^{\sigma-1}\overline{\kappa}_j^*\} - \rho - \delta. \tag{4}$$

It is worthwhile at this point to compare Eq. (4) with Naito's (2017) growth equation (3): $\gamma_j^* = (1/\sigma)L_j/(A_j^{K*}\overline{\kappa}_j^*) - (1-1/\sigma)\rho - \delta; A_j^{K*} \equiv 1/(1+\widetilde{\psi}_j^*(n_{kk}/n_{jj})^*), \widetilde{\psi}_j^* \equiv \psi_j(G_k(a_{kj}^*)/G_k(a_{kk}^*))^\varepsilon, \psi_j \in [0,1], \varepsilon \geq 0$, where A_j^K is the unit labor requirement for R&D (normalizing the effect of variety growth), and $\widetilde{\psi}_j$ is country j's coefficient of international knowledge spillovers, which is nondecreasing in country $k(\neq j)$'s fraction of exporters $G_k(a_{kj})/G_k(a_{kk})$. The only qualitative difference between Naito (2017) and the present model comes from BRN's p_K-channel: in Naito (2017), $p_j^K/w_j = A_j^K$ depends only on a_{kk}, a_{kj}, and n_{kk}/n_{jj}. In the present model, $p_j^K/w_j = [\overline{m}_j/(1-1/\sigma)]^{\sigma-1}$ depends not only on a_{kk}, a_{kj}, and n_{kk}/n_{jj}, but also on

[11] It is important to note that this expression is not the explicit solution for p_j^Y because $\overline{m}_j$ still contains p_k^Y/p_j^Y. We will soon show that $\overline{m}_j^*$ and $(p_1^Y/p_2^Y)^*$ are simultaneously determined.

[12] This is different from Eq. (23) of BRN, where $\alpha_j = \sigma - 1$. The difference comes from the fact that labor instead of the final good is used as the variable input in the differentiated good sector of BRN. Then p_j^Y in Eq. (3) of the present model is replaced by w_j, and $p_j^Y = [n_{jj}^{1/(1-\sigma)}w_j\overline{m}_j/(1-1/\sigma)]^{\alpha_j}w_j^{1-\alpha_j} = [n_{jj}^{1/(1-\sigma)}\overline{m}_j/(1-1/\sigma)]^{\alpha_j}w_j$. This implies that $[1/(\sigma-1)]\alpha_j = 1 \Leftrightarrow \alpha_j = \sigma - 1$ for a BGP to exist. One advantage of our specification over BRN is that the Cobb-Douglas restriction $\alpha_j \in (0,1)$ imposes no upper bound for σ in our model, whereas it implies that $\sigma < 2$ in BRN, which might be restrictive on empirical grounds.

$a_{jj}, p_k^Y/p_j^Y$, and τ_{kj}. In contrast to the conventional wisdom that the lab-equipment specification is simpler than the knowledge-driven specification, our lab-equipment model is more complicated than Naito's (2017) knowledge-driven model.

Eqs. (3) and (4) imply that, with the cutoffs $\{a_{jk}^*\}$ given, country j's growth rate of domestic varieties γ_j^* is decreasing in its import trade cost $\tau_{kj}, k \neq j$, decreasing in the relative price of the final good of country k to country j p_k^Y/p_j^Y, and decreasing in the relative number of domestic varieties of country j to country k n_{jj}/n_{kk}. The last result means that a country's faster past growth slows down its present growth, suggesting growth convergence. Indeed, on a BGP, χ^* is determined to equalize countries' growth rates:

$$\gamma_1^* = \gamma_2^* \equiv \gamma^* \Leftrightarrow L_1/(\overline{m}_1^{*\sigma-1}\overline{\kappa}_1^*) = L_2/(\overline{m}_2^{*\sigma-1}\overline{\kappa}_2^*), \tag{5}$$

where we call a common growth rate on a BGP γ^* the "balanced growth rate".

Since $\overline{m}_j^*$ is constant from Eq. (4), and χ^* is constant from Eq. (5), Eq. (3) implies that the relative price of the final goods $(p_1^Y/p_2^Y)^*$ is constant. Using $p_j^K = n_{jj}p_j^Y = [\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j$, this is given by:

$$(p_1^Y/p_2^Y)^* = (w_1^*/\chi^*)(\overline{m}_1^*/\overline{m}_2^*)^{\sigma-1}. \tag{6}$$

From Eqs. (3) and (6), $\overline{m}_j^*$ and $(p_1^Y/p_2^Y)^*$ are solved as functions of $w_1^*, \chi^*, \{a_{jk}^*\}$, and $\{\tau_{jk}\}$. Eq. (6) also implies that w_1^* is constant, and so is p_j^{K*}.

As mentioned in section 2.3, the cutoffs are determined from Eqs. (1) and (2). Specifically, dividing Eq. (1) by itself with $j = k$ gives $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = P_{j0}^K\kappa_{jk}/(P_{k0}^K\kappa_{kk}), j \neq k$, which is rewritten as (see Appendix A for derivations):

$$a_{12}^*/a_{22}^* = v^{*-1}\tau_{12}^{-1}(\kappa_{12}/\kappa_{22})^{-1/(\sigma-1)}, \tag{7}$$
$$a_{21}^*/a_{11}^* = v^*\tau_{21}^{-1}(\kappa_{21}/\kappa_{11})^{-1/(\sigma-1)}; v^* \equiv (p_1^Y/p_2^Y)^{*\sigma/(\sigma-1)}. \tag{8}$$

An increase in a_{jk}^*/a_{kk}^* means that country $j(\neq k)$ becomes relatively more competitive in country k in that relatively more firms from the former enter the latter. Taking Eq. (7) for example, country 1 becomes relatively more competitive in country 2 if the latter liberalizes its imports (i.e., τ_{12} decreases) and/or the former's final good becomes relatively cheaper (i.e., $(p_1^Y/p_2^Y)^*$ decreases). Considering Eqs. (3) and (6), the free entry condition (2) and the relative competitiveness conditions (7) and (8) are solved for the cutoffs as:

$$a_{jk}^* = a_{jk}^*(w_1^*, \chi^*, \tau_{21}, \tau_{12}), j, k = 1, 2. \tag{9}$$

The last piece to characterize a BGP is country 1's (or 2's) zero balance of trade, which is rewritten as (see Appendix A for derivation):

$$\lambda_{12}^*w_1^*L_1 = \lambda_{21}^*L_2; \lambda_{jk}^* \equiv (H_{jk}(a_{jk}^*) + G_j(a_{jk}^*))\kappa_{jk}/\textstyle\sum_l(H_{jl}(a_{jl}^*) + G_j(a_{jl}^*))\kappa_{jl}. \tag{10}$$

As in Krugman (1980), the relative wage w_1^* is determined to satisfy Eq. (10). To sum up, the balanced growth condition (5), the cutoff functions (9), and the balanced trade condition (10), determine a BGP: $(\chi^*, \{a_{jk}^*\}, w_1^*)$.

Before looking at full general equilibrium effects of unilateral trade liberalization, we derive formulas for long-run growth and welfare changes, which will elegantly simplify the following analysis.

3.2 Long-run growth and welfare formulas

First of all, the logarithmically differentiated form of Eq. (2) is generally calculated as:[13]

$$0 = \sum_k \lambda_{jk}^* \widehat{a}_{jk}^*; \widehat{a}_{jk}^* \equiv d \ln a_{jk}^* \equiv da_{jk}^*/a_{jk}^*. \tag{11}$$

From now on, we specify $G_j(a)$ as Pareto, which is widely used in applications of the Melitz model:

$$G_j(a) \equiv (a/a_{j0})^{\theta} = a_{j0}^{-\theta} a^{\theta}; \theta > \sigma - 1,$$

where a_{j0} is a country-specific scale parameter, and θ is a common shape parameter. Then we obtain:

$$\begin{aligned}
\overline{a}_{jk}(a_{jk})^{1-\sigma} &= [\beta/(\beta-1)]a_{jk}^{1-\sigma}; \beta \equiv \theta/(\sigma-1) > 1, \\
h_{jk}(a_{jk}) &= 1/(\beta-1), H_{jk}(a_{jk}) = G_j(a_{jk})/(\beta-1), \\
H_{jk}(a_{jk}) + G_j(a_{jk}) &= G_j(a_{jk})\beta/(\beta-1) = \beta H_{jk}(a_{jk}), \\
(H'_{jk} + g_{jk})a_{jk}/(H_{jk} + G_{jk}) &= g_{jk}a_{jk}/G_{jk} = H'_{jk}a_{jk}/H_{jk} = \theta \forall j,k; g_{jk} \equiv g_j(a_{jk}), G_{jk} \equiv G_j(a_{jk}).
\end{aligned}$$

Using Eq. (11) and the above results, the logarithmically differentiated forms of λ_{jk}^* and $\overline{\kappa}_j^*$ are given by:

$$\widehat{\lambda}_{jk}^* = \theta \widehat{a}_{jk}^*, \tag{12}$$

$$\widehat{\overline{\kappa}}_j^* = -\theta \widehat{a}_{jj}^*. \tag{13}$$

Eq. (12) implies that an increase a_{jk}^* increases country j's probability of survival in market k, which increases the corresponding revenue share. Eq. (13) means that a decrease in a_{jj}^* makes it less likely for country j's potential entrant to survive, which increases $\overline{\kappa}_j^*$.

The rate of change in country j's real wage in terms of the final good w_j^*/p_j^{Y*}, where $p_j^{Y*} \equiv p_{j0}^Y$ is evaluated at the initial period of a BGP, is simply expressed as (see Appendix B for derivation):

$$\widehat{w}_j^* - \widehat{p}_j^{Y*} = -(\sigma - 1)\widehat{a}_{jj}^* = -(1/\beta)\widehat{\lambda}_{jj}^*. \tag{14}$$

Eq. (14) implies that country j's real wage increases if and only if domestic selection becomes tougher (i.e., its cutoff unit final good requirement for domestic sales a_{jj}^* decreases), or the country becomes more open (i.e., its domestic revenue share λ_{jj}^* decreases). This is qualitatively the same as, but quantitatively slightly different from, the ACR formula of Arkolakis et al. (2012). The latter states that $\widehat{w}_j^* - \widehat{p}_j^{Y*} = (1/\varepsilon_j^{kk})\widehat{\lambda}_{jj}^*$, where $\varepsilon_j^{kk}, k \neq j$, is the "trade elasticity", that is, the elasticity of country j's imports relative to its domestic expenditure with respect to its import trade cost. In the present Melitz-based model with a common Pareto shape parameter θ, it is verified that $\varepsilon_j^{kk} = -\theta \forall j,k,k \neq j$ (see Appendix B for derivation). Then Eq. (14) is rewritten as $\widehat{w}_j^* - \widehat{p}_j^{Y*} = [(\sigma-1)/\varepsilon_j^{kk}]\widehat{\lambda}_{jj}^*$. This means that the rate of change in w_j^*/p_j^{Y*} in the present model is $\sigma - 1$ times the original ACR formula. This difference comes from the difference in technologies: the composite final good is produced not only from the intermediate goods but also from labor, and the final good instead of labor is used as the variable input in the intermediate good sector.

Although Eq. (14) is not enough to discuss country j's long-run welfare in our dynamic model, it is

[13] We use $H'_{jk}a_{jk}/H_{jk} = [(h_{jk}+1)/h_{jk}](\sigma-1) = [(H_{jk}+G_{jk})/H_{jk}](\sigma-1)$, where $G_{jk} \equiv G_j(a_{jk})$, and the definition of λ_{jk}^*.

closely related to the balanced growth rate. To see this, we logarithmically differentiate $p_j^{K*} = n_{jj0}p_{j0}^Y = n_{j0}^e G_j(a_{jj}^*)p_j^{Y*}$, with n_{j0}^e predetermined, to obtain $\widehat{p}_j^{K*} = \theta\widehat{a}_{jj}^* + \widehat{p}_j^{Y*}$. Using this and Eq. (13), the rate of change in $w_j^*/(p_j^{K*}\overline{\kappa}_j^*)$ in Eq. (4) is rewritten as:

$$\widehat{w}_j^* - \widehat{p}_j^{K*} - \widehat{\overline{\kappa}}_j^* = \widehat{w}_j^* - \theta\widehat{a}_{jj}^* - \widehat{p}_j^{Y*} + \theta\widehat{a}_{jj}^* = \widehat{w}_j^* - \widehat{p}_j^{Y*}.$$

This means that γ_j^* increases if and only if w_j^*/p_j^{Y*} increases. Indeed, substituting the above expression and Eq. (14) into the differentiated form of Eq. (4): $d\gamma_j^* = (1-1/\sigma)[w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*)](\widehat{w}_j^* - \widehat{p}_j^{K*} - \widehat{\overline{\kappa}}_j^*) = (\rho+\delta+\gamma^*)(\widehat{w}_j^* - \widehat{p}_j^{K*} - \widehat{\overline{\kappa}}_j^*)$, we obtain:

$$d\gamma_j^* = -(\sigma-1)(\rho+\delta+\gamma^*)\widehat{a}_{jj}^* = -[(\rho+\delta+\gamma^*)/\beta]\widehat{\lambda}_{jj}^*. \tag{15}$$

It is true that country j's tougher domestic selection (i.e., a decrease in a_{jj}^*) partly decreases its growth through an increase in $\overline{\kappa}_j^*$. However, the negative growth effect is more than offset by the positive growth effect through a decrease in p_j^{K*}/w_j^*. Overall, country j's tougher domestic selection, or equivalently its increased openness (i.e., a decrease in λ_{jj}^*), is good for its growth.

Finally, country j's long-run welfare (expressed in flow terms) is given by (see Appendix B for derivation):[14]

$$\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \ln w_j^* + \ln L_j + \ln\{[(1-1/\sigma)\rho+\rho+\delta+\gamma^*]/(\rho+\delta+\gamma^*)\} - \ln p_j^{Y*} + (1/\rho)\gamma^*. \tag{16}$$

Eq. (16) implies that country j's long-run welfare depends only on its real wage w_j^*/p_j^{Y*} and growth rate γ^*. In particular, an increase in γ^* directly increases ρU_j through a faster fall in p_j^Y over time, whereas it indirectly decreases ρU_j through a decrease in the asset and hence E_j^*. Simple calculation shows that:

$$\begin{aligned}
\rho dU_j &= \widehat{w}_j^* - \widehat{p}_j^{Y*} + \Gamma^* d\gamma^*;\\
\Gamma^* &\equiv 1/[(1-1/\sigma)\rho+\rho+\delta+\gamma^*] - 1/(\rho+\delta+\gamma^*) + 1/\rho\\
&= [(1-1/\sigma)\rho(\delta+\gamma^*) + (\rho+\delta+\gamma^*)^2]/\{[(1-1/\sigma)\rho+\rho+\delta+\gamma^*](\rho+\delta+\gamma^*)\rho\} > 0.
\end{aligned}$$

Since $Q_j^K = \overline{\kappa}_j^* n_{jj}(\gamma^*+\delta) \geq 0$, we have $\Gamma^* > 0$, meaning that the positive direct effect of an increase in γ^* on ρU_j always outweighs its negative indirect effect. Substituting Eqs. (14) and (15) into the above expression, we finally obtain:

$$\rho dU_j = -(1/\beta)(1+\Omega^*)\widehat{\lambda}_{jj}^*; \Omega^* \equiv \Gamma^*(\rho+\delta+\gamma^*). \tag{17}$$

Eq. (17) is an extended ACR formula for long-run welfare. Without endogenous growth, a 1% decrease in λ_{jj}^* would increase ρU_j by the amount $(1/\beta)\times 0.01$ through an increase in w_j^*/p_j^{Y*}. With endogenous growth, however, the same 1% decrease in λ_{jj}^* additionally increases ρU_j by the amount $(1/\beta)\times\Omega^*\times 0.01$ through an increase in γ^*. The positive static welfare effect of greater openness is always reinforced by its positive dynamic welfare effect.

Eqs. (14), (15), and (17), together with Eq. (5), imply the following proposition:

[14] $U_j = (1/\rho)[\ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*] = [\ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*]\int_0^\infty \exp(-\rho t)dt$ can be interpreted as the representative household having a constant utility flow $\ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \rho U_j$ discounted by a factor $\exp(-\rho t)$ over an infinite horizon.

Proposition 1 *In the long run, each country's welfare increases through increases in both its real wage and growth rate if and only if its domestic revenue share decreases. Moreover, both countries' welfare changes by the same amount.*

Since countries' growth rates are always equalized in the long run from Eq. (5), and ρ, δ, σ, and θ are the same across countries, their domestic revenue shares change by the same rate from Eq. (15), and hence their long-run welfare changes by the same amount from Eq. (17). It should be emphasized that our long-run growth and welfare formulas hold even if countries are asymmetric in terms of parameters such as $L_j, \{\tau_{jk}\}, \{\kappa_{jk}\}, \kappa_j^e$, and a_{j0}. Under a knowledge-driven specification, where both the intermediate good and R&D firms use labor as the only variable input, Naito (2017) derives similar growth and welfare formulas only with symmetric countries, but they are not available with asymmetric countries. This is because p_j^{K*}/w_j^* in country j's growth equation is not monotonically related to a_{jj}^* in general.[15] Only under the lab-equipment specification, our growth and welfare formulas are even applicable to asymmetric countries.

How large is Ω^*, the dynamic welfare effect? Eqs. (5), (9) (derived from Eqs. (2), (7), and (8), together with Eqs. (3) and (6)), and (10) imply that $\chi^*, \{a_{jk}^*\}$, and w_1^* depend on σ and θ, but not on ρ and δ. Then from Eq. (4), $\rho + \delta + \gamma^* = (1 - 1/\sigma)L_j/\{[\overline{m}_j^*/(1 - 1/\sigma)]^{\sigma-1}\overline{\kappa}_j^*\}$ is independent of ρ and δ: whenever ρ or δ increases, γ^* decreases by the same amount so that both sides of this equation are unchanged. Taking $\rho + \delta + \gamma^*$ as constant, $\Omega^* = \{1/[(1 - 1/\sigma)\rho + \rho + \delta + \gamma^*] - 1/(\rho + \delta + \gamma^*) + 1/\rho\}(\rho + \delta + \gamma^*)$ is decreasing and convex in ρ but independent of δ. This implies that Ω^* can be arbitrarily large as ρ decreases toward zero. Indeed, Ω^* approaches infinity as ρ approaches zero.

For example, we borrow plausible values of parameters and the original growth rate from other studies: $\sigma_0 = 3.8, \delta_0 = 0.025$ from Balistreri et al. (2011); and $\rho_0 = 0.02, \gamma_0^* = 0.02$ from Acemoglu (2009). The resulting value of Ω^* is $\Omega_0^* = 3.06518$, meaning that the total long-run welfare effect of greater openness (expressed in flow terms) is about four times as large as the static counterpart. Moreover, regarding Ω^* as a function of ρ, with the value of $\rho + \delta + \gamma^*$ fixed at $\rho_0 + \delta_0 + \gamma_0^* = 0.065$, it is easily calculated that $\Omega^*(0.05) = 0.938243, \Omega^*(0.04) = 1.31302, \Omega^*(0.03) = 1.91289$, and $\Omega^*(0.01) = 6.39818$. Therefore, for a wide domain of ρ, adding the dynamic welfare effect more than doubles the total long-run welfare effect of greater openness (expressed in flow terms).

Armed with the long-run growth and welfare formulas, we examine the long-run effects of unilateral trade liberalization in the next section.

4 Long-run effects of unilateral trade liberalization

From now on, we omit asterisks just for notational simplicity. The rates of changes in endogenous variables, in particular $(\widehat{\chi}, \{\widehat{a}_{jk}\}, \widehat{w}_1)$, can be solved as follows: (i) we use Eqs. (3) and (6) to solve for $\widehat{p_1^Y/p_2^Y} = \widehat{p_1^Y/p_2^Y}(\widehat{w}_1, \widehat{\chi}, \{\widehat{a}_{jk}\}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (ii) substituting the result from step (i) into the logarithmically differentiated forms of Eqs. (7) and (8), and combining them with Eq. (11), we solve for $\widehat{a}_{jk} = \widehat{a}_{jk}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iii) substituting the result from step (ii) into Eq. (12), and substituting it into the logarithmically differentiated form of Eq. (10), we solve for $\widehat{w}_1 = \widehat{w}_1(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iv) substituting the result from step (iii) back into $\widehat{a}_{jj} = \widehat{a}_{jj}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, and substituting it into Eq. (15), we solve for $d\gamma_j = d\gamma_j(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; and (v)

[15] As explained after Eq. (4), Naito (2017) has $p_j^{K*}/w_j^* = A_j^{K*} = 1/[1 + \psi_j(G_k(a_{kj}^*)/G_k(a_{kk}^*))^{\varepsilon}(n_{kk}/n_{jj})^*]$, which depends on a_{kk}^* and $(n_{kk}/n_{jj})^*$ (even after eliminating a_{kj}^* from the free entry condition). Only in the symmetric country case, we have $a_{kk}^* = a_{jj}^*$ and $(n_{kk}/n_{jj})^* = 1$, implying that p_j^{K*}/w_j^* depends only on a_{jj}^*.

substituting the result from step (iv) into the differentiated form of Eq. (5), we solve for $\widehat{\chi} = \widehat{\chi}(\widehat{\tau}_{21}, \widehat{\tau}_{12})$. The rates of changes in all other endogenous variables can be solved by substituting backwards.

In step (i), the rate of change in p_1^Y/p_2^Y is solved as (see Appendix C for derivation):

$$
\begin{aligned}
&\widehat{p}_1^Y - \widehat{p}_2^Y \\
&= (1/\Delta)[\widehat{w}_1 - (1-\lambda_{12}-\lambda_{21})\widehat{\chi}] + [(\sigma-1)/\Delta](\lambda_{12}\widehat{\tau}_{21} - \lambda_{21}\widehat{\tau}_{12}) \\
&+ [(\sigma-1)/\Delta]\{(1/\lambda_{12})[(1-\lambda_{12})(\lambda_{12}+\lambda_{21}) - \beta\lambda_{21}]\widehat{a}_{11} - (1/\lambda_{21})[(1-\lambda_{21})(\lambda_{21}+\lambda_{12}) - \beta\lambda_{12}]\widehat{a}_{22}\}; \quad (18) \\
&\Delta \equiv 1 + (\sigma-1)(\lambda_{12}+\lambda_{21}) > 1.
\end{aligned}
$$

As Eq. (6) shows, p_1^Y/p_2^Y is directly increasing in w_1 but decreasing in χ. An increase in χ also decreases the relative number of imported to domestic varieties in country 1, whereas it increases that in country 2. The former increases $\overline{m}_1$, whereas the latter decreases $\overline{m}_2$. Consequently, both of them indirectly increases p_1^Y/p_2^Y. However, the direct effect of χ dominates under the assumption that:

$$\lambda_{jk} < 1/2 \forall j,k,k \neq j,$$

that is, the export revenue share is smaller than the domestic revenue share for all countries.[16] This is quite reasonable considering the fixed and variable trade costs. An increase in τ_{21} increases p_1^Y/p_2^Y by increasing $\overline{m}_1$. Similarly, an increase in τ_{12} decreases p_1^Y/p_2^Y. An increase in a_{11} increases $\overline{m}_1$ on the one hand, but on the other hand it (together with a decrease in a_{12} from the free entry condition) increases $\overline{m}_2$ by decreasing the relative number of imported to domestic varieties in country 2. Its total effect on p_1^Y/p_2^Y is thus ambiguous. Similarly, an increase in a_{22} has an ambiguous total effect on p_1^Y/p_2^Y.

In step (ii), we obtain (see Appendix C for derivations):

$$
\begin{aligned}
\widehat{a}_{11} &= (\lambda_{12}/|\widetilde{\lambda}|)\{\widehat{V} + [\widetilde{\lambda}_{22} - (\sigma/\Delta)\lambda_{21}]\widehat{\tau}_{12} - [1 - (\sigma/\Delta)\lambda_{12} - \widetilde{\lambda}_{22}]\widehat{\tau}_{21}\}, \quad (19) \\
\widehat{a}_{22} &= (\lambda_{21}/|\widetilde{\lambda}|)\{-\widehat{V} + [\widetilde{\lambda}_{11} - (\sigma/\Delta)\lambda_{12}]\widehat{\tau}_{21} - [1 - (\sigma/\Delta)\lambda_{21} - \widetilde{\lambda}_{11}]\widehat{\tau}_{12}\}; \quad (20) \\
\widehat{V} &\equiv [\sigma/(\sigma-1)](1/\Delta)[\widehat{w}_1 - (1-\lambda_{12}-\lambda_{21})\widehat{\chi}], \\
\widetilde{\lambda}_{jj} &\equiv 1 - \lambda_{jk} - (\sigma/\Delta)[(1-\lambda_{jk})(\lambda_{jk}+\lambda_{kj}) - \beta\lambda_{kj}], k \neq j, \\
\widetilde{\lambda}_{jk} &\equiv (\lambda_{jk}/\lambda_{kj})\{\lambda_{kj} + (\sigma/\Delta)[(1-\lambda_{kj})(\lambda_{kj}+\lambda_{jk}) - \beta\lambda_{jk}]\}, k \neq j, \\
|\widetilde{\lambda}| &\equiv \widetilde{\lambda}_{11}\widetilde{\lambda}_{22} - \widetilde{\lambda}_{12}\widetilde{\lambda}_{21} = (1/\Delta)[(1-\lambda_{12}-\lambda_{21})^2 + \sigma(\beta-1)(\lambda_{12}+\lambda_{21})] > 0.
\end{aligned}
$$

Eq. (19) can be interpreted as follows. An increase in w_1 and/or a decrease in χ increase p_1^Y/p_2^Y (see Eq. (18)). This makes country 1 relatively less competitive in country 2 (i.e., decreases a_{12}; see Eq. (7)). The resulting decrease in country 1's expected net firm value from exports should be compensated by an increase in its expected net firm value from domestic sales, thereby causing more firms to enter their domestic market (i.e., increasing a_{11}; see Eq. (11)). An increase in τ_{12} directly has a similar effect to an increase in w_1 and/or a decrease in χ (see Eqs. (7) and (11)), but it indirectly has a counteracting effect through a decrease in p_1^Y/p_2^Y (see Eqs. (7) and (18)).[17] An increase in τ_{21} makes country 2 relatively less competitive in country

[16] See Appendix C for proof that this is true at the symmetric BGP.

[17] To disentangle the counteracting indirect effects of changes in import trade costs from Eqs. (19) and (20), we just need to eliminate the terms including (σ/Δ). For example, the coefficient of $\widehat{\tau}_{12}$ in Eq. (19) becomes $\widetilde{\lambda}_{22} - (\sigma/\Delta)\lambda_{21} = 1 - \lambda_{21} > 0$, which ensures that the direct effect of an increase in τ_{12} on a_{11} is positive.

1, which causes less exports and less domestic selection for country 2 (i.e., decreases a_{21} and increases a_{22}; see Eqs. (8) and (11)). Since this makes it easier for exporters from country 1 to survive in country 2 (i.e., increases a_{12}; see Eq. (7)), more domestic firms are selected out (i.e., a_{11} decreases; see Eq. (11)). However, an increase in τ_{21} also has a counteracting indirect effect through an increase in p_1^Y/p_2^Y (see Eqs. (8) and (18)). Eq. (20) can be interpreted similarly. At this point, with w_1 and χ given, the effects of changes in import trade costs on domestic cutoffs are ambiguous.

In step (iii), using Eqs. (11), (12), (19), and (20) to rewrite the logarithmically differentiated form of Eq. (10), we obtain:

$$\begin{aligned}
0 &= -\widetilde{B}\widehat{w}_1 + \widetilde{C}\widehat{\chi} + \theta(F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}) \Leftrightarrow \widehat{w}_1 = (1/\widetilde{B})[\widetilde{C}\widehat{\chi} + \theta(F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12})]; \qquad (21)\\
\widetilde{B} &\equiv \beta(\sigma/\Delta)(2 - \lambda_{12} - \lambda_{21}) - |\widetilde{\lambda}|\\
&= (1/\Delta)\{\sigma + (1 - \lambda_{12} - \lambda_{21})[\sigma(2\beta - 1) - (1 - \lambda_{12} - \lambda_{21})]\} > \sigma/\Delta,\\
\widetilde{C} &\equiv \beta(\sigma/\Delta)(1 - \lambda_{12} - \lambda_{21})(2 - \lambda_{12} - \lambda_{21}) > 0,\\
F_{jk} &\equiv (1 - \lambda_{jk})[\widetilde{\lambda}_{kk} - (\sigma/\Delta)\lambda_{kj}] + (1 - \lambda_{kj})[1 - (\sigma/\Delta)\lambda_{kj} - \widetilde{\lambda}_{jj}], k \neq j.
\end{aligned}$$

As we saw in step (ii), an increase in w_1 discourages country 1's exports but encourages country 2's exports. Since the sum of these effects are stronger than its positive income effect on country 1's value of exports, country 1's balance of trade decreases. An increase in χ encourages country 1's exports but discourages country 2's exports, thereby creating country 1's trade surplus. For the surplus to be cleared, w_1 increases. An increase in τ_{21} discourages country 2's exports but encourages country 1's exports (as long as its counteracting indirect effect through an increase in p_1^Y/p_2^Y is minor), which causes w_1 to increase. Similarly, an increase in τ_{12} creates country 1's trade deficit, which is cleared through a decrease in w_1.

In step (iv), substituting Eq. (21) back into Eqs. (19) and (20), noting that $\widetilde{\lambda}_{11} + \widetilde{\lambda}_{22} - 1 = |\widetilde{\lambda}|$ and $\Delta - \sigma(\lambda_{21} + \lambda_{12}) = 1 - \lambda_{12} - \lambda_{21}$, and substituting the results back into Eq. (15), $d\gamma_1$ and $d\gamma_2$ are expressed as, respectively:

$$\begin{aligned}
d\gamma_1 &= -(\rho + \delta + \gamma)(1/\widetilde{B})(\lambda_{12}/\Delta)[\sigma(1 - \lambda_{12} - \lambda_{21})\widehat{\chi} + (\sigma - 1)(J_1\widehat{\tau}_{12} + I_1\widehat{\tau}_{21})], \qquad (22)\\
d\gamma_2 &= -(\rho + \delta + \gamma)(1/\widetilde{B})(\lambda_{21}/\Delta)[-\sigma(1 - \lambda_{12} - \lambda_{21})\widehat{\chi} + (\sigma - 1)(J_2\widehat{\tau}_{21} + I_2\widehat{\tau}_{12})]; \qquad (23)\\
J_j &\equiv (1 - \lambda_{jk} - \lambda_{kj})[\beta\sigma - (1 - \lambda_{kj})] + \sigma\lambda_{kj} > 0, k \neq j,\\
I_j &\equiv (1 - \lambda_{jk} - \lambda_{kj})[\beta\sigma - (1 - \lambda_{kj})] + \Delta - \sigma\lambda_{jk} > J_j, k \neq j.
\end{aligned}$$

In line with Eq. (4), an increase in χ decreases γ_1 but increases γ_2. This might sound contradictory with Eqs. (19) and (20), where an increase in χ decreases a_{11} but increases a_{22}. In fact, an increase in χ increases w_1 so much that its partial equilibrium effects, with w_1 given, are overturned. Eqs. (22) and (23) also show that a decrease in any import trade cost increases any country's growth rate, with χ given.[18] Eq. (19) implies that a decrease in τ_{12} directly encourages country 1's exports and domestic selection (i.e., decreases a_{11}), but its encouraged exports causes w_1 to increase from Eq. (21), which indirectly discourages country 1's exports and domestic selection (i.e., increases a_{11}). Similarly, a decrease in τ_{21} directly increases a_{11} but

[18] Just to deliver the following intuitions, we assume for the moment that the counteracting indirect effects of changes in import trade costs through changes in p_1^Y/p_2^Y are minor. However, Eqs. (22) and (23) are obtained regardless of whether this assumption is true.

indirectly decreases a_{11} through a decrease in w_1. It turns out that the direct, partial equilibrium effect is stronger than the indirect, general equilibrium effect for τ_{12}, the import trade cost of country 1's trading partner, whereas the indirect, general equilibrium effect dominates for τ_{21}, country 1's own import trade cost. Therefore, a decrease in any import trade cost increases country 1's growth rate, with χ given. The same is true for country 2's growth rate.

In step (v), substituting Eqs. (22) and (23) into the differentiated form of Eq. (5), we obtain:

$$\widehat{\chi} = [(\sigma - 1)/\sigma]\{1/[(1 - \lambda_{12} - \lambda_{21})(\lambda_{12} + \lambda_{21})]\}[(\lambda_{21}I_2 - \lambda_{12}J_1)\widehat{\tau}_{12} - (\lambda_{12}I_1 - \lambda_{21}J_2)\widehat{\tau}_{21}]. \tag{24}$$

This simply says that χ is adjusted to eliminate the gap between countries' growth rates created by trade cost changes. For example, when $\lambda_{12}I_1 > \lambda_{21}J_2$ at the old BGP, a decrease in τ_{21} increases γ_1 by more than γ_2. Then χ increases so that γ_1 decreases but γ_2 increases until they are equalized.

Finally, substituting Eq. (24) back into Eq. (23), and noting that $I_j + J_k = \Delta\widetilde{B}\forall j, k, k \neq j$, we obtain:

$$d\gamma = -(\sigma - 1)(\rho + \delta + \gamma)[\lambda_{12}\lambda_{21}/(\lambda_{12} + \lambda_{21})](\widehat{\tau}_{21} + \widehat{\tau}_{12}). \tag{25}$$

Eq. (25) immediately implies that:

$$\partial\gamma/\partial \ln \tau_{jk} = \partial\gamma/\partial \ln \tau_{kj} = -(\sigma - 1)(\rho + \delta + \gamma)\lambda_{12}\lambda_{21}/(\lambda_{12} + \lambda_{21}) < 0\forall j, k, k \neq j.$$

Combining this with Proposition 1, we obtain the following proposition:

Proposition 2 *A decrease in the import trade cost of either country by the same rate increases the balanced growth rate and both countries' long-run welfare by the same amounts.*

It should be remarked that Proposition 2 does not depend on $\lambda_{jk} < 1/2\forall j, k, k \neq j$ assumed after Eq. (18). It is true that this assumption is used to sign $1 - \lambda_{12} - \lambda_{21}, \widetilde{B}, \widetilde{C}, J_j$, and I_j, and thus to interpret what is going on in each step. However, since their signs are irrelevant to Eq. (25), the assumption is unnecessary as far as the final result is concerned.

Proposition 2 has rich implications. First, even unilateral trade liberalization raises long-run growth. BRN briefly state (without working out a full model) that, in a lab-equipment model with heterogeneous firms and symmetric countries, symmetric trade liberalization always raises the balanced growth rate. By considering the general equilibrium effects through the relative wage and number of domestic varieties, we show that even unilateral trade liberalization is enough for faster long-run growth. Second, unilateral trade liberalization always raises long-run growth. Under BRN's Coe-Helpman specification, where each country's coefficient of international knowledge spillovers is nondecreasing in the fraction of exporters of each other country, Naito (2017) demonstrates that unilateral trade liberalization can raise the balanced growth rate only if the elasticity of the spillover coefficients is positive and large enough. Our result implies that the positive long-run growth effect of unilateral trade liberalization generally holds under the original lab-equipment model of Rivera-Batiz and Romer (1991) extended to heterogeneous firms and asymmetric countries. Third, even with asymmetric countries, τ_{21} and τ_{12} have quantitatively the same long-run growth effect. Differences in country characteristics such as $L_j, \{\tau_{jk}\}, \{\kappa_{jk}\}, \kappa^e_j$, and a_{j0} are fully translated into w_1, χ, and $\{a_{jk}\}$ so that a one percent change in the import trade cost of either country 1 or 2 makes no quantitative difference in the amount of change in the balanced growth rate. Fourth, all of the three implications for long-run growth also apply to long-run welfare. Even unilateral trade liberalization always raises long-run welfare of both countries by the same amount, and it does not matter whether a larger or a smaller country liberalizes its imports by one percent.

5 Concluding remarks

This paper contributes to the literature on both trade and growth theories. For the trade side, we augment the ACR formula of Arkolakis et al. (2012) to allow for both endogenous growth and asymmetric countries. The additional dynamic welfare gains (expressed in flow terms) from a 1% decrease in a country's domestic revenue share is about three times as large as the static welfare gains for realistic parameter values. By taking growth seriously, countries gain much more from openness than without it. Also, we allow countries to be asymmetric in terms of endowments, technologies, and trade costs. Our extended ACR welfare formula is not limited to trade between the same countries as in Sampson (2016) and Naito (2017), but applicable to trade between realistically different countries. Our model also naturally adds endogenous technological change to the asymmetric Melitz models such as Felbermayr et al. (2013) and Demidova and Rodríguez-Clare (2013) with sufficient tractability.

For the growth side, we extend the lab-equipment model of Rivera-Batiz and Romer (1991) to include both heterogeneous firms and asymmetric countries. Although the expanding variety models of endogenous growth have been extended to heterogeneous firms since BRN, the literature has been almost monopolized by symmetric country models, preventing us from studying the long-run growth and welfare effects of asymmetric policy reforms such as unilateral trade liberalization. In spite of the increased richness and difficulty coming from our extension, our main result is surprisingly clean: even unilateral trade liberalization always raises both countries' long-run growth and welfare. This provides a reason to choose our lab-equipment specification over the knowledge-driven specification of Naito (2017) as the main theoretical framework to consider endogenous growth, heterogeneous firms, and asymmetric countries.

Appendix A. Derivations of key equations in section 3.1

Derivation of $\gamma_j^* = [\alpha_j/(1-\alpha_j)](1/\sigma)w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*) - \rho - \delta$

Applying Shephard's lemma to $c_j^Y(P_j, w_j)Y_j$, and using $p_j^Y = c_j^Y(P_j, w_j)$, the minimized expenditures for the intermediate goods and labor are expressed as, respectively:

$$P_j X_j = \alpha_j p_j^Y Y_j, \tag{A.1}$$

$$w_j L_j^Y = (1-\alpha_j) p_j^Y Y_j. \tag{A.2}$$

The asset market-clearing condition $W_j = \sum_k n_{jk} \int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da$ is rewritten using Eq. (2) as:

$$\begin{aligned} W_j &= n_{jj} \textstyle\sum_k (G_j(a_{jk})/G_j(a_{jj})) \displaystyle\int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da \\ &= (n_{jj}/G_j(a_{jj}))P_j^K(\textstyle\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e) = p_j^K \overline{\kappa}_j; p_j^K \equiv n_{jj} P_j^K. \end{aligned} \tag{A.3}$$

Time differentiating Eq. (A.3), and using Eq. (A.3) and the no-arbitrage condition $\dot{v}_{jk}(a) = (r_j + \delta)v_{jk}(a) - \pi_{jk}(a)$, we obtain:

$$\dot{W}_j = (\dot{n}_{jj}/G_j(a_{jj}))P_j^K(\textstyle\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e) + (r_j+\delta)W_j - \sum_k n_{jk} \displaystyle\int_0^{a_{jk}} \pi_{jk}(a)\mu_{jk}(a|a_{jk})da.$$

Further rewriting this using Eqs. (A.1), (A.3), $\pi_{jk}(a) = e_{jk}(a)/\sigma$, and country j's national budget constraint gives:

$$\dot{W}_j = W_j(\gamma_j + r_j + \delta) - (1/\sigma)\alpha_j p_j^Y Y_j; \gamma_j \equiv \dot{n}_{jj}/n_{jj}.$$

Multiplying $Y_j = C_j + D_j + F_j$ by p_j^Y, and using Eqs. (A.1), (A.3), $\pi_{jk}(a) = e_{jk}(a)/\sigma, P_j^K Q_j^K = p_j^Y D_j, Q_j^K = \overline{\kappa}_j(\dot{n}_{jj} + \delta n_{jj})$, and country j's national budget constraint, $p_j^Y Y_j$ is expressed as:

$$p_j^Y Y_j = [1/(1-\alpha_j + \alpha_j/\sigma)][E_j + p_j^K \overline{\kappa}_j(\gamma_j + \delta)]. \tag{A.4}$$

Substituting Eq. (A.4) into the last expression for $\dot{W}_j$, and using Eq. (A.3), we obtain:

$$\dot{W}_j/W_j = r_j + [(1-\alpha_j)/(1-\alpha_j+\alpha_j/\sigma)](\gamma_j + \delta) - [(\alpha_j/\sigma)/(1-\alpha_j+\alpha_j/\sigma)]Z_j; Z_j \equiv E_j/W_j.$$

Substituting the above expression and the Euler equation $\dot{E}_j/E_j = r_j - \rho$ into $\dot{Z}_j/Z_j = \dot{E}_j/E_j - \dot{W}_j/W_j$, the growth rate of a transformed variable Z_j is expressed in terms of Z_j and γ_j:

$$\dot{Z}_j/Z_j = [(\alpha_j/\sigma)/(1-\alpha_j+\alpha_j/\sigma)]Z_j - \rho - [(1-\alpha_j)/(1-\alpha_j+\alpha_j/\sigma)](\gamma_j + \delta). \tag{A.5}$$

We next rewrite the labor market-clearing condition $L_j = L_j^Y$ using Eqs. (A.2), (A.3), and (A.4) to express γ_j in terms of Z_j:

$$\gamma_j = [(1 - \alpha_j + \alpha_j/\sigma)/(1 - \alpha_j)] w_j L_j / (p_j^K \overline{\kappa}_j) - Z_j - \delta. \tag{A.6}$$

By definition of a BGP, both $\dot{Z}_j/Z_j$ and γ_j are constant. Then Eq. (A.5) implies that Z_j is constant. From Eqs. (A.5), (A.6), and $\dot{Z}_j/Z_j = 0$, Z_j and γ_j are solved as:

$$Z_j^* = \rho + w_j^* L_j / (p_j^{K*} \overline{\kappa}_j^*), \tag{A.7}$$

$$\gamma_j^* = [\alpha_j/(1 - \alpha_j)](1/\sigma) w_j^* L_j / (p_j^{K*} \overline{\kappa}_j^*) - \rho - \delta. \tag{A.8}$$

Eq. (A.8) is the desired equation.

Derivations of Eqs. (7) and (8)

The right-hand side of $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = P_{j0}^K \kappa_{jk} / (P_{k0}^K \kappa_{kk}), j \neq k$, is simply rewritten as $(p_j^Y/p_k^Y)^* \kappa_{jk}/\kappa_{kk}$. In the left-hand side, $v_{jk0}(a)$ is given by:

$$v_{jk0}(a) = \pi_{jk0}(a) \Delta_{jk0}(a); \Delta_{jk0}(a) \equiv \int_0^\infty \exp(-\int_0^t (r_{js} + \delta - \dot{\pi}_{jks}(a)/\pi_{jks}(a)) ds) dt.$$

To evaluate $\Delta_{jk0}(a)$, we have to calculate r_{js} and $\dot{\pi}_{jks}(a)/\pi_{jks}(a)$ on a BGP. For r_{js}, multiplying Eq. (A.7) by $W_j^* = p_j^{K*} \overline{\kappa}_j^*$ from Eq. (A.3) gives:

$$E_j^* = p_j^{K*} \overline{\kappa}_j^* [\rho + w_j^* L_j / (p_j^{K*} \overline{\kappa}_j^*)] = p_j^{K*} \overline{\kappa}_j^* \rho + w_j^* L_j. \tag{A.9}$$

Since $\overline{\kappa}_j^*, w_j^*$, and p_j^{K*} are constant, E_j^* is constant from Eq. (A.9). This and the Euler equation imply that $r_j^* = \rho$.

For $\dot{\pi}_{jks}(a)/\pi_{jks}(a)$, noting that $P_j X_j = (\sigma - 1) w_j L_j$ from Eqs. (A.1), (A.2), $L_j = L_j^Y$, and $\alpha_j/(1 - \alpha_j) = \sigma - 1$, $\pi_{jk}(a) = [\tau_{jk} p_j^Y a/(1 - 1/\sigma)]^{1-\sigma} P_k^\sigma X_k / \sigma$ is rewritten as:

$$\pi_{jkt}(a) = [\tau_{jk} a/(1 - 1/\sigma)]^{1-\sigma} (P_{kt}/p_{jt}^Y)^{\sigma-1} (1 - 1/\sigma) w_k^* L_k. \tag{A.10}$$

Dividing P_{kt} from Eq. (3) by p_{jt}^Y gives $P_{kt}/p_{jt}^Y = n_{kkt}^{1/(1-\sigma)} (p_k^Y/p_j^Y)^* \overline{m}_k^* / (1 - 1/\sigma)$. Since n_{kkt} grows at the rate γ^*, P_{kt}/p_{jt}^Y grows at the rate $-\gamma^*/(\sigma - 1)$. Then Eq. (A.10) implies that $\pi_{jkt}(a)$ grows at the rate $-\gamma^*$: $\dot{\pi}_{jkt}(a)/\pi_{jkt}(a) = -\gamma^*$.

Substituting the results into the definition of $\Delta_{jk0}(a)$, we obtain $\Delta_{jk0}(a) = 1/(\rho + \delta + \gamma^*)$, and hence:

$$v_{jk0}(a) = \pi_{jk0}(a)/(\rho + \delta + \gamma^*). \tag{A.11}$$

Dividing Eq. (A.11) by itself with $j = k$, and using Eq. (A.10), we obtain $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = \pi_{jk0}(a_{jk}^*)/\pi_{kk0}(a_{kk}^*) = [\tau_{jk} (p_j^Y/p_k^Y)^* a_{jk}^*/a_{kk}^*]^{1-\sigma}$. Therefore, the original equation is rewritten as:

$$[\tau_{jk} (p_j^Y/p_k^Y)^* a_{jk}^*/a_{kk}^*]^{1-\sigma} = (p_j^Y/p_k^Y)^* \kappa_{jk}/\kappa_{kk}.$$

Solving this for a_{12}^*/a_{22}^* and a_{21}^*/a_{11}^*, we obtain Eqs. (7) and (8), respectively.

Derivation of Eq. (10)

Using Eqs. (1), (A.11), $\pi_{jk}(a) = e_{jk}(a)/\sigma$, and $e_{jk}(a)/e_{jk}(a^*_{jk}) = (a/a^*_{jk})^{1-\sigma}$, $\int_0^{a^*_{jk}} e_{jk}(a)\mu_{jk}(a|a^*_{jk})da$ in $\lambda^*_{jk} = n_{jk0}\int_0^{a^*_{jk}} e_{jk}(a)\mu_{jk}(a|a^*_{jk})da/\sum_l n_{jl0}\int_0^{a^*_{jl}} e_{jl}(a)\mu_{jl}(a|a^*_{jl})da$ is rewritten as $\int_0^{a^*_{jk}} e_{jk}(a)\mu_{jk}(a|a^*_{jk})da = (h_{jk}(a^*_{jk})+1)\sigma(\rho+\delta+\gamma^*)P^K_{j0}\kappa_{jk}$. Substituting this and $n_{jk0} = n_{jj0}G_j(a^*_{jk})/G_j(a^*_{jj})$ into the expression for λ^*_{jk}, we obtain $\lambda^*_{jk} = (H_{jk}(a^*_{jk}) + G_j(a^*_{jk}))\kappa_{jk}/\sum_l(H_{jl}(a^*_{jl}) + G_j(a^*_{jl}))\kappa_{jl}$. Finally, substituting $P^*_j X^*_j = (\sigma-1)w^*_j L_j$ into $\lambda^*_{jk}P^*_j X^*_j = \lambda^*_{kj}P^*_k X^*_k$, we obtain Eq. (10).

Appendix B. Derivations of key equations in section 3.2

Derivation of Eq. (14)

Rewriting $v_{jj0}(a^*_{jj}) = P^K_{j0}\kappa_{jj}$ from Eq. (1) for $k = j$ using Eqs. (A.10), (A.11), and $p^{K*}_j = n_{jj0}P^K_{j0} = n^e_{j0}G_j(a^*_{jj})P^K_{j0}$ gives:

$$[a^*_{jj}/(1-1/\sigma)]^{1-\sigma}(P^*_j/p^{Y*}_j)^{\sigma-1}(1-1/\sigma)w^*_j L_j/(\rho+\delta+\gamma^*) = [p^{K*}_j/(n^e_{j0}G_j(a^*_{jj}))]\kappa_{jj}; P^*_j \equiv P_{j0}, p^{Y*}_j \equiv p^Y_{j0}.$$

Using Eq. (5), Eq. (4) is rewritten as:

$$\rho+\delta+\gamma^* = (1-1/\sigma)w^*_j L_j/(p^{K*}_j\overline{\kappa}^*_j) \Leftrightarrow (1-1/\sigma)w^*_j L_j/(\rho+\delta+\gamma^*) = p^{K*}_j\overline{\kappa}^*_j. \tag{B.1}$$

Using Eq. (B.1), the previous expression is simplified to:

$$[a^*_{jj}/(1-1/\sigma)]^{1-\sigma}(P^*_j/p^{Y*}_j)^{\sigma-1}\overline{\kappa}^*_j = \kappa_{jj}/(n^e_{j0}G_j(a^*_{jj})).$$

Logarithmically differentiating this expression, using Eq. (13), and noting that n^e_{j0} is a predetermined state variable, we obtain:

$$\widehat{p}^{Y*}_j - \widehat{P}^*_j = -\widehat{a}^*_{jj}.$$

Logarithmically differentiating $p^{Y*}_j = P^{*\alpha_j}_j w^{*1-\alpha_j}_j$ gives $\widehat{p}^{Y*}_j = \alpha_j\widehat{P}^*_j + (1-\alpha_j)\widehat{w}^*_j$, which is rewritten as $\widehat{w}^*_j - \widehat{p}^{Y*}_j = \alpha_j(\widehat{w}^*_j - \widehat{P}^*_j) = \alpha_j(\widehat{w}^*_j - \widehat{p}^{Y*}_j + \widehat{p}^{Y*}_j - \widehat{P}^*_j)$, or $\widehat{w}^*_j - \widehat{p}^{Y*}_j = [\alpha_j/(1-\alpha_j)](\widehat{p}^{Y*}_j - \widehat{P}^*_j) = (\sigma-1)(\widehat{p}^{Y*}_j - \widehat{P}^*_j)$. Substituting $\widehat{p}^{Y*}_j - \widehat{P}^*_j = -\widehat{a}^*_{jj}$ into this expression, and using Eq. (12), we obtain Eq. (14).

Derivation of the trade elasticity

Let $E_{jk} \equiv n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da$ be country j's revenue of selling the intermediate goods to country k, or country k's expenditure for buying the intermediate goods from country j. Using $e_{jk}(a) = [\tau_{jk}p^Y_j a/(1-1/\sigma)]^{1-\sigma}P^\sigma_k X_k$ and $P_j X_j = (\sigma-1)w_j L_j$, this is rewritten as:

$$E_{jk} = (\sigma-1)n^e_j[\tau_{jk}p^Y_j/(1-1/\sigma)]^{1-\sigma}G_j(a_{jk})\overline{a}_{jk}(a_{jk})^{1-\sigma}(w_k L_k/P^{1-\sigma}_k).$$

Substituting this into country j's national budget constraint: $\sum_l E_{jl} = \sum_l E_{lj} = P_j X_j$, and again using $P_j X_j = (\sigma-1)w_j L_j$, we obtain:

$$w_j L_j = n^e_j\Pi^{1-\sigma}_j; \Pi^{1-\sigma}_j \equiv \textstyle\sum_l[\tau_{jl}p^Y_j/(1-1/\sigma)]^{1-\sigma}G_j(a_{jl})\overline{a}_{jl}(a_{jl})^{1-\sigma}(w_l L_l/P^{1-\sigma}_l).$$

Substituting n_j^e from the last expression back into E_{jk}, the gravity equation is derived as:

$$E_{jk} = (\sigma - 1)[\tau_{jk} p_j^Y/(1 - 1/\sigma)]^{1-\sigma} G_j(a_{jk})\overline{a}_{jk}(a_{jk})^{1-\sigma}(w_j L_j/\Pi_j^{1-\sigma})(w_k L_k/P_k^{1-\sigma}).$$

Taking p_j^Y and $(w_j L_j/\Pi_j^{1-\sigma})(w_k L_k/P_k^{1-\sigma})$ as given, logarithmic differentiation of the gravity equation with respect to τ_{jk} implies that $\partial \ln E_{jk}/\partial \ln \tau_{jk} = -(\sigma - 1) + [\theta - (\sigma - 1)]\partial \ln a_{jk}/\partial \ln \tau_{jk}$ and $\partial \ln E_{kk}/\partial \ln \tau_{jk} = [\theta - (\sigma - 1)]\partial \ln a_{kk}/\partial \ln \tau_{jk}$. Using these and Eqs. (7) and (8), the trade elasticity is calculated as:

$$\begin{aligned}
\varepsilon_k^{jj} &\equiv \partial \ln(E_{jk}/E_{kk})/\partial \ln \tau_{jk} \\
&= -(\sigma - 1) + [\theta - (\sigma - 1)]\partial \ln(a_{jk}/a_{kk})/\partial \ln \tau_{jk} \\
&= -\theta = \varepsilon_j^{kk} \forall j, k, k \neq j.
\end{aligned}$$

Derivation of Eq. (16)

Noting that $p_{jt}^Y = p_j^{K*}/n_{jjt} = p_j^{Y*}\exp(-\gamma^* t)$, we obtain $\ln C_{jt} = \ln E_j^* - \ln p_j^{Y*} + \gamma^* t$. Substituting this into $U_j = \int_0^\infty \ln C_{jt}\exp(-\rho t)dt$, and applying integration by parts, we obtain:

$$\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*.$$

Substituting Eq. (B.1) into Eq. (A.9), E_j^* is rewritten as:

$$E_j^* = w_j^* L_j[(1 - 1/\sigma)\rho + \rho + \delta + \gamma^*]/(\rho + \delta + \gamma^*).$$

Substituting this into the expression for ρU_j gives Eq. (16).

Appendix C. Derivations of key equations in section 4

Derivation of Eq. (18)

Logarithmically differentiating $\overline{m}_j$ in Eq. (3) gives:

$$\begin{aligned}
\widehat{\overline{m}}_j &= [1/(1-\sigma)]\textstyle\sum_k \zeta_{kj}\, d\ln\{(n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}; \\
\zeta_{kj} &\equiv \frac{(n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}}{\sum_l (n_{ll}/n_{jj})(G_l(a_{lj})/G_l(a_{ll}))[(\tau_{lj}p_l^Y/p_j^Y)\overline{a}_{lj}(a_{lj})]^{1-\sigma}} = \frac{n_{kj}\int_0^{a_{kj}} e_{kj}(a)\mu_{kj}(a|a_{kj})da}{\sum_l n_{lj}\int_0^{a_{lj}} e_{lj}(a)\mu_{lj}(a|a_{lj})da},
\end{aligned}$$

where ζ_{kj} is rewritten using $e_{kj}(a) = p_{kj}(a)^{1-\sigma}P_j^\sigma X_j$ as the expenditure share of varieties country j buys from country k. From country j's national budget constraint and its zero balance of trade, we obtain $\zeta_{kj} = \lambda_{jk}\forall j, k$. Using this and Eq. (11), $\widehat{\overline{m}}_j$ is expressed as:

$$\widehat{\overline{m}}_j = (1 - \lambda_{jk})\widehat{a}_{jj} + \lambda_{jk}\{[1/(1-\sigma)]d\ln(n_{kk}/n_{jj}) + \{[\beta - (1 - \lambda_{kj})]/\lambda_{kj}\}\widehat{a}_{kk} + \widehat{\tau}_{kj} + \widehat{p}_k^Y - \widehat{p}_j^Y\}, k \neq j.$$

Then the difference between $\widehat{\overline{m}}_1$ and $\widehat{\overline{m}}_2$ is given by:

$$
\begin{aligned}
&\widehat{\widehat{m}}_1 - \widehat{\widehat{m}}_2 \\
&= (1/\lambda_{12})[(1-\lambda_{12})(\lambda_{12}+\lambda_{21}) - \beta\lambda_{21}]\widehat{a}_{11} - (1/\lambda_{21})[(1-\lambda_{21})(\lambda_{21}+\lambda_{12}) - \beta\lambda_{12}]\widehat{a}_{22} \\
&- (\lambda_{12}+\lambda_{21})[\widehat{p}_1^Y - \widehat{p}_2^Y - \widehat{\chi}/(\sigma-1)] + \lambda_{12}\widehat{\tau}_{21} - \lambda_{21}\widehat{\tau}_{12}.
\end{aligned}
$$

Substituting this into the logarithmically differentiated form of Eq. (6), $\widehat{p}_1^Y - \widehat{p}_2^Y$ is solved as Eq. (18).

Proof that $\lambda_{jk} < 1/2 \forall j, k, k \neq j$ at the symmetric BGP

Suppose that all parameters are the same across countries: $L_j = L, \kappa_j^e = \kappa^e, a_{j0} = a_0$, $\tau_{jk} = 1$ for $k = j$; $\tau_{jk} = \tau(\geq 1)$ for $k \neq j$, and $\kappa_{jk} = \kappa_d$ for $k = j$; $\kappa_{jk} = \kappa_x(> \kappa_d)$ for $k \neq j$. At the symmetric BGP, let $w_1 = 1, \chi = 1$, and $a_{jk} = a_d$ for $k = j$; $a_{jk} = a_x$ for $k \neq j$. Then $\lambda_{jk} = (H_{jk}(a_{jk}) + G_j(a_{jk}))\kappa_{jk}/\sum_l (H_{jl}(a_{jl}) + G_j(a_{jl}))\kappa_{jl}$ for $k \neq j$ is simplified to $\lambda_{jk} = a_x^\theta \kappa_x/(a_d^\theta \kappa_d + a_x^\theta \kappa_x)$. Since $a_x/a_d = \tau^{-1}(\kappa_x/\kappa_d)^{-1/(\sigma-1)} < 1$ from Eqs. (7), (8), $\tau \geq 1$, and $\kappa_x > \kappa_d$, we have $(a_x/a_d)^\theta \kappa_x/\kappa_d = \tau^{-\theta}(\kappa_x/\kappa_d)^{-(\beta-1)} < 1$, or $a_x^\theta \kappa_x < a_d^\theta \kappa_d$. Therefore:

$$
\lambda_{jk} < a_x^\theta \kappa_x/(a_x^\theta \kappa_x + a_x^\theta \kappa_x) = 1/2, k \neq j.
$$

Derivations of Eqs. (19) and (20)

Logarithmically differentiating Eqs. (7) and (8) gives:

$$
\begin{aligned}
\widehat{a}_{12} - \widehat{a}_{22} &= -\widehat{v} - \widehat{\tau}_{12}, \\
\widehat{a}_{21} - \widehat{a}_{11} &= \widehat{v} - \widehat{\tau}_{21}.
\end{aligned}
$$

Using them to eliminate $\widehat{a}_{12}$ and $\widehat{a}_{21}$ from Eq. (11), we obtain:

$$
\begin{aligned}
(1-\lambda_{12})\widehat{a}_{11} + \lambda_{12}\widehat{a}_{22} &= \lambda_{12}(\widehat{v} + \widehat{\tau}_{12}), \\
\lambda_{21}\widehat{a}_{11} + (1-\lambda_{21})\widehat{a}_{22} &= -\lambda_{21}(\widehat{v} - \widehat{\tau}_{21}).
\end{aligned}
$$

Substituting Eq. (18) into $\widehat{v} = [\sigma/(\sigma-1)](\widehat{p}_1^Y - \widehat{p}_2^Y)$, and substituting it into the above expressions, they are rewritten as:

$$
\begin{aligned}
\widetilde{\lambda}_{11}\widehat{a}_{11} + \widetilde{\lambda}_{12}\widehat{a}_{22} &= \lambda_{12}\{\widehat{V} + [1 - (\sigma/\Delta)\lambda_{21}]\widehat{\tau}_{12} + (\sigma/\Delta)\lambda_{12}\widehat{\tau}_{21}\}, \\
\widetilde{\lambda}_{21}\widehat{a}_{11} + \widetilde{\lambda}_{22}\widehat{a}_{22} &= \lambda_{21}\{-\widehat{V} + [1 - (\sigma/\Delta)\lambda_{12}]\widehat{\tau}_{21} + (\sigma/\Delta)\lambda_{21}\widehat{\tau}_{12}\}; \\
\widehat{V} &\equiv [\sigma/(\sigma-1)](1/\Delta)[\widehat{w}_1 - (1-\lambda_{12}-\lambda_{21})\widehat{\chi}], \\
\widetilde{\lambda}_{jj} &\equiv 1 - \lambda_{jk} - (\sigma/\Delta)[(1-\lambda_{jk})(\lambda_{jk}+\lambda_{kj}) - \beta\lambda_{kj}], k \neq j, \\
\widetilde{\lambda}_{jk} &\equiv (\lambda_{jk}/\lambda_{kj})\{\lambda_{kj} + (\sigma/\Delta)[(1-\lambda_{kj})(\lambda_{kj}+\lambda_{jk}) - \beta\lambda_{jk}]\}, k \neq j.
\end{aligned}
$$

Solving them for $\widehat{a}_{11}$ and $\widehat{a}_{22}$, we obtain Eqs. (19) and (20).

References

[1] Acemoglu, D., 2002. Directed technical change. Review of Economic Studies 69, 781–809.

[2] Acemoglu, D., 2009. Introduction to Modern Economic Growth. Princeton University Press, Princeton.

[3] Anderson, J. E., Wincoop, E. v., 2004. Trade costs. Journal of Economic Literature 42, 691–751.

[4] Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102, 94–130.

[5] Atkeson, A., Burstein, A. T., 2010. Innovation, firm dynamics, and international trade. Journal of Political Economy 118, 433–484.

[6] Baldwin, R. E., Robert-Nicoud, F., 2008. Trade and growth with heterogeneous firms. Journal of International Economics 74, 21–34.

[7] Balistreri, E. J., Hillberry, R. H., Rutherford, T. F., 2011. Structural estimation and solution of international trade models with heterogeneous firms. Journal of International Economics 83, 95–108.

[8] Barro, R. J., Sala-i-Martin, X., 2004. Economic Growth, Second Edition. MIT Press, Cambridge, MA.

[9] Demidova, S., Rodríguez-Clare, A., 2013. The simple analytics of the Melitz model in a small economy. Journal of International Economics 90, 266–272.

[10] Dinopoulos, E., Unel, B., 2011. Quality heterogeneity and global economic growth. European Economic Review 55, 595–612.

[11] Estevadeordal, A., Taylor, A. M., 2013. Is the Washington Consensus dead? growth, openness, and the Great Liberalization, 1970s-2000s. Review of Economics and Statistics 95, 1669–1690.

[12] Feenstra, R. C., 2016. Advanced International Trade, Second Edition. Princeton University Press, Princeton.

[13] Felbermayr, G., Jung, B., Larch, M., 2013. Optimal tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model. Journal of International Economics 89, 13–25.

[14] Fukuda, K., 2016. The effect of globalization in an endogenous growth model with firm heterogeneity, international spillover, and trade: a note. unpublished.

[15] Gustafsson, P., Segerstrom, P., 2010. Trade liberalization and productivity growth. Review of International Economics 18, 207–228.

[16] Grossman, G. M., Helpman, E., 1991. Innovation and Growth in the Global Economy. MIT Press, Cambridge, MA.

[17] Krugman, P., 1980. Scale economies, product differentiation, and the pattern of trade. American Economic Review 70, 950–959.

[18] Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71, 1695–1725.

[19] Naito, T., 2017. Growth and welfare effects of unilateral trade liberalization with heterogeneous firms and asymmetric countries. Journal of International Economics 109, 167–173.

[20] Ourens, G., 2016. Trade and growth with heterogeneous firms revisited. Journal of International Economics 100, 194–202.

[21] Perla, J., Tonetti, C., Waugh, M. E., 2015. Equilibrium technology diffusion, trade, and growth. unpublished.

[22] Rivera-Batiz, L. A., Romer, P. M., 1991. Economic integration and endogenous growth. Quarterly Journal of Economics 106, 531–555.

[23] Sampson, T., 2016. Dynamic selection: an idea flows theory of entry, trade, and growth. Quarterly Journal of Economics 131, 315–380.

[24] Wacziarg, R., Welch, K. H., 2008. Trade liberalization and growth: new evidence. World Bank Economic Review 22, 187–231.