

Appendix and Supplemental material not intended for publication-Round 1

**Submission Number:EB-14-00364**

**Supplementary Tables for the US SCF** (See endnote 1 for explanation of the special cases of the dP<sup>2</sup>LN.)

Goodness-of-fit of PIDMs, AIC (differences from the corresponding values of the dPLN)

Model	No. of para.	1992	1995	1998	2001	2004	2007	2010	Ave.
SM	3	-5.5	11.7	6.3	0.0	-3.9	10.0	29.4	6.9
Da	3	6.1	-5.9	-3.3	-0.3	0.0	10.4	21.3	4.1
$\kappa$ G	3	-0.3	-11.3	-11.1	0.4	4.2	33.9	86.8	14.7
$I\kappa$ G	3	52.9	13.5	26.1	34.6	39.4	59.3	94.0	45.7
GB1	4	95.3	212.6	247.8	223.9	132.7	127.4	26.5	152.3
GB2	4	-3.6	-3.9	-1.5	1.1	-1.9	6.2	10.2	0.9
E $\kappa$ G1	4	37.9	55.5	42.1	10.8	3.4	17.0	66.4	33.3
E $\kappa$ G2	4	-9.9	-5.5	-5.6	-4.8	-11.7	-5.3	3.8	-5.6
IE $\kappa$ G1	4	-8.8	-9.9	-9.8	-6.9	-9.5	-2.3	4.6	-6.1
IE $\kappa$ G2	4	1.2	1.6	7.8	11.2	6.3	17.1	19.2	9.2
dPLN	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
GB	5	-1.6	-1.9	0.5	3.1	0.1	8.2	12.2	2.9
GdPLN	5	-2.3	-5.5	3.5	3.8	0.9	4.0	4.0	1.2
dP <sup>2</sup> LN $\mu$	5	-0.7	2.0	-3.2	-12.2	2.0	-17.9	-23.0	-7.6
dP <sup>2</sup> LN $\sigma$	5	-18.2	-19.4	-29.3	-26.0	-23.8	-19.4	-5.8	-20.3
dP <sup>2</sup> LN $r$	5	2.0	-0.3	-11.1	-4.3	2.0	2.0	-4.4	-2.0
dP <sup>2</sup> LN $\mu\sigma$	6	-34.4	-19.9	-70.7	-68.6	<u>-34.9</u>	<u>-67.2</u>	-38.9	-47.8
dP <sup>2</sup> LN $\mu r$	6	-32.5	-18.5	-53.4	-57.1	-27.6	-59.5	-37.0	-40.8
dP <sup>2</sup> LN $\sigma r$	6	-31.5	-19.5	-58.2	-45.4	-29.3	-50.3	-32.8	-38.1
dP <sup>2</sup> LN *	7	-38.3	-23.1	-72.8	<u>-68.8</u>	-33.0	-66.0	<u>-41.4</u>	<u>-49.1</u>
dP <sup>2</sup> LN'	6	<u>-39.6</u>	-21.4	<u>-74.0</u>	-66.2	-18.5	-53.4	-25.3	-42.6
dP <sup>2</sup> LN''	6	<u>-39.6</u>	<u>-25.1</u>	<u>-74.0</u>	-66.2	-18.5	-53.4	-25.3	-43.1

\* Constraint  $\sigma_L < \sigma_R$  is imposed.

Goodness-of-fit of PIDMs, L-RSSE

Model	No. of para.	1992	1995	1998	2001	2004	2007	2010	RMSE
SM	3	0.732	1.837	2.094	2.318	1.590	1.939	0.402	1.695
Da	3	0.249	1.472	1.769	1.959	1.072	1.513	0.338	1.350
$\kappa$ G	3	0.409	1.443	1.730	1.691	0.938	1.212	0.670	1.248
$I\kappa$ G	3	1.003	0.718	0.856	0.699	0.464	<u>0.534</u>	2.213	1.079
GB1	4	1.304	2.168	2.755	3.788	2.746	4.283	3.922	3.153
GB2	4	0.746	1.462	1.825	2.184	1.611	2.378	1.258	1.717
E $\kappa$ G1	4	1.622	1.776	1.629	1.488	0.595	1.173	1.940	1.519
E $\kappa$ G2	4	0.596	1.669	2.089	2.463	1.649	2.223	0.789	1.767
IE $\kappa$ G1	4	0.661	1.381	1.779	2.076	1.498	2.228	1.096	1.614
IE $\kappa$ G2	4	2.800	7.039	8.252	15.039	7.930	11.697	13.017	10.148
dPLN	4	0.633	1.343	1.648	1.941	1.421	2.017	0.868	1.490
GB	5	0.746	1.462	1.825	2.184	1.611	2.378	1.258	1.717
GdPLN	5	0.842	1.505	1.683	1.911	1.855	2.014	0.867	1.590
dP <sup>2</sup> LN $\mu$	5	0.582	1.343	1.212	2.306	1.421	3.496	0.295	1.823
dP <sup>2</sup> LN $\sigma$	5	0.356	0.940	1.358	1.443	0.955	1.514	0.550	1.098
dP <sup>2</sup> LN $r$	5	0.633	1.141	2.258	3.265	1.421	2.017	1.775	1.953
dP <sup>2</sup> LN $\mu\sigma$	6	1.561	<u>0.359</u>	3.128	5.601	1.266	6.551	6.137	4.241
dP <sup>2</sup> LN $\mu r$	6	5.115	0.883	n.a.#	n.a.#	n.a.#	n.a.#	0.389	n.a.
dP <sup>2</sup> LN $\sigma r$	6	0.410	0.630	0.652	2.741	<u>0.387</u>	2.618	7.101	3.069
dP <sup>2</sup> LN *	7	<u>0.104</u>	0.662	0.380	1.818	1.284	3.257	1.963	1.691
dP <sup>2</sup> LN'	6	0.122	0.735	<u>0.175</u>	<u>0.370</u>	0.554	0.790	<u>0.309</u>	<u>0.500</u>
dP <sup>2</sup> LN''	6	0.122	0.886	<u>0.175</u>	<u>0.370</u>	0.554	0.790	<u>0.309</u>	0.534

\* Constraint  $\sigma_L < \sigma_R$  is imposed. # Expectation is infinite.

Goodness-of-fit of PIDMs, absolute error of the Gini index

Model	No. of para.	1992	1995	1998	2001	2004	2007	2010	RMSE
SM	3	0.0108	0.0257	0.0305	0.0343	0.0241	0.0292	0.0045	0.0249
Da	3	0.0013	0.0228	0.0273	0.0289	0.0151	0.0204	0.0031	0.0198
$\kappa$ G	3	0.0079	0.0221	0.0277	0.0269	0.0155	0.0199	0.0083	0.0198
$l\kappa$ G	3	0.0181	0.0094	0.0105	0.0051	0.0072	<u>0.0075</u>	0.0368	0.0170
GB1	4	0.0095	0.0138	0.0257	0.0461	0.0328	0.0609	0.0555	0.0395
GB2	4	0.0109	0.0226	0.0282	0.0327	0.0244	0.0349	0.0171	0.0256
$E\kappa$ G1	4	0.0378	0.0365	0.0376	0.0171	<u>0.0029</u>	0.0136	0.0268	0.0277
$E\kappa$ G2	4	0.0097	0.0259	0.0330	0.0381	0.0260	0.0340	0.0107	0.0274
$IE\kappa$ G1	4	0.0104	0.0215	0.0282	0.0319	0.0232	0.0336	0.0153	0.0247
$IE\kappa$ G2	4	0.0716	0.1509	0.1752	0.2670	0.1692	0.2231	0.2282	0.1929
dPLN	4	0.0076	0.0206	0.0248	0.0281	0.0207	0.0283	0.0111	0.0215
GB	5	0.0109	0.0226	0.0282	0.0327	0.0244	0.0349	0.0171	0.0256
GdPLN	5	0.0140	0.0234	0.0257	0.0294	0.0291	0.0304	0.0132	0.0245
$dP^2LN\mu$	5	0.0059	0.0206	0.0039	0.0356	0.0207	0.0501	<u>0.0006</u>	0.0259
$dP^2LN\sigma$	5	0.0072	0.0143	0.0227	0.0236	0.0159	0.0248	0.0076	0.0179
$dP^2LNr$	5	0.0076	0.0165	0.0123	<u>0.0009</u>	0.0207	0.0283	0.0075	0.0159
$dP^2LN\mu\sigma$	6	0.0177	0.0025	0.0479	0.0816	0.0179	0.0916	0.0754	0.0582
$dP^2LN\mu r$	6	0.0308	0.0099	n.a.#	n.a.#	n.a.#	n.a.#	0.0054	n.a.#
$dP^2LN\sigma r$	6	0.0060	0.0085	0.0065	0.0361	0.0056	0.0334	0.0946	0.0406
$dP^2LN^*$	7	<u>0.0001</u>	<u>0.0095</u>	0.0055	0.0256	0.0182	0.0453	0.0256	0.0233
$dP^2LN'$	6	0.0008	0.0100	<u>0.0017</u>	0.0051	0.0060	0.0083	0.0037	<u>0.0060</u>
$dP^2LN''$	6	0.0008	0.0113	<u>0.0017</u>	0.0051	0.0060	0.0083	0.0037	0.0063

\* Constraint  $\sigma_L < \sigma_R$  is imposed. # Expectation is infinite.

**Supplementary Tables for the Italian SHIW** (See endnote 1 for explanation of the special cases of the  $dP^2LN$ .)

Goodness-of-fit of PIDMs, AIC (differences from the corresponding values of the dPLN)

Model	No. of para.	2000	2002	2004	2006	2008	2010	2012	Ave.
SM	3	8.8	14.7	23.7	15.7	32.5	12.2	25.5	19.0
Da	3	24.7	42.1	36.5	27.5	50.5	21.5	32.2	33.6
$\kappa$ G	3	35.0	61.6	103.9	59.1	118.3	41.0	60.5	68.5
$l\kappa$ G	3	113.5	146.2	127.6	110.4	151.4	95.1	102.6	121.0
GB1	4	277.8	226.9	211.5	443.7	162.8	257.1	331.5	273.1
GB2	4	10.4	15.5	13.5	17.6	10.6	13.3	24.6	15.1
$E\kappa$ G1	4	10.7	13.7	16.3	17.3	16.0	14.0	27.0	16.4
$E\kappa$ G2	4	15.8	28.9	24.9	23.1	31.8	19.5	32.9	25.3
$IE\kappa$ G1	4	17.1	25.7	18.7	26.8	17.2	25.4	47.6	25.5
$IE\kappa$ G2	4	11.4	13.6	14.5	19.1	9.2	9.6	16.5	13.4
dPLN	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
GB	5	12.4	17.5	15.5	19.6	12.6	13.7	23.6	16.4
GdPLN	5	-38.3	-48.9	-21.4	-43.8	-11.8	-33.3	-153.7	-50.2
$dP^2LN\mu$	5	-39.7	-63.5	-43.0	-88.6	-22.0	-42.3	-104.2	-57.6
$dP^2LN\sigma$	5	0.6	1.9	1.9	1.9	1.9	-0.2	-5.2	0.4
$dP^2LNr$	5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
$dP^2LN\mu\sigma$	6	-54.9	-63.5	-43.6	-96.6	-21.1	-40.4	-103.0	-60.4
$dP^2LN\mu r$	6	-42.2	-61.7	-41.6	-86.8	-20.8	-52.5	-157.5	-66.3
$dP^2LN\sigma r$	6	-41.4	-64.5	-12.9	-70.3	-26.1	-52.3	-158.9	-60.8
$dP^2LN$	7	<u>-96.2</u>	-91.8	<u>-53.2</u>	-108.8	<u>-34.0</u>	-66.5	<u>-170.6</u>	<u>-88.7</u>
$dP^2LN'$	6	-94.3	<u>-92.4</u>	-38.5	<u>-110.0</u>	-33.9	<u>-67.1</u>	-167.3	-86.2
$dP^2LN''$	6	-94.3	<u>-92.4</u>	-38.5	-98.3	-33.9	<u>-67.1</u>	-167.3	-84.6

Goodness-of-fit of PIDMs, L-RSSE

Model	No. of para.	2000	2002	2004	2006	2008	2010	2012	RMSE
SM	3	0.113	0.130	0.185	0.263	0.258	0.104	0.169	0.185
Da	3	0.413	0.590	0.486	0.203	0.743	0.449	0.556	0.515
$\kappa$ G	3	0.205	0.304	0.411	0.210	0.572	0.347	0.446	0.377
$l\kappa$ G	3	1.215	1.432	1.449	0.974	1.655	1.242	1.550	1.376
GB1	4	0.546	0.407	0.506	0.588	0.354	0.505	0.614	0.510
GB2	4	0.112	0.107	0.201	0.269	0.088	0.135	0.224	0.174
$E\kappa$ G1	4	0.111	0.104	0.186	0.273	0.143	0.091	0.150	0.162
$E\kappa$ G2	4	0.168	0.284	0.202	0.178	0.369	0.290	0.455	0.295
$IE\kappa$ G1	4	0.111	0.114	0.206	0.274	0.093	0.169	0.261	0.188
$IE\kappa$ G2	4	2.203	1.885	2.492	2.357	1.957	2.489	2.807	2.332
dPLN	4	0.109	0.097	0.196	0.272	0.075	0.110	0.160	0.159
GB	5	0.112	0.107	0.201	0.269	0.088	0.104	0.192	0.166
GdPLN	5	0.121	0.141	0.187	0.322	0.183	0.337	0.212	0.228
$dP^2LN\mu$	5	0.385	0.305	0.190	<u>0.085</u>	0.118	0.397	0.438	0.304
$dP^2LN\sigma$	5	0.098	0.101	0.199	0.275	0.075	0.097	0.160	0.158
$dP^2LNr$	5	0.109	0.097	0.196	0.272	0.075	0.110	0.160	0.159
$dP^2LN\mu\sigma$	6	0.460	0.314	0.199	8.727	0.120	0.404	0.419	3.314
$dP^2LN\mu r$	6	0.534	0.318	0.218	0.089	0.120	0.123	0.112	0.263
$dP^2LN\sigma r$	6	0.297	0.199	0.175	0.225	0.127	0.122	0.113	0.190
$dP^2LN$	7	<u>0.062</u>	<u>0.051</u>	<u>0.086</u>	0.247	<u>0.047</u>	<u>0.029</u>	0.053	<u>0.107</u>
$dP^2LN'$	6	0.126	0.071	0.145	0.249	0.078	0.037	<u>0.045</u>	0.127
$dP^2LN''$	6	0.126	0.071	0.145	0.199	0.078	0.037	<u>0.045</u>	0.114

Goodness-of-fit of PIDMs, absolute error of the Gini index

Model	No. of para.	2000	2002	2004	2006	2008	2010	2012	RMSE
SM	3	0.0003	0.0011	<u>0.0002</u>	0.0038	0.0029	0.0018	0.0024	0.0022
Da	3	0.0061	0.0092	<u>0.0067</u>	0.0018	0.0115	0.0075	0.0092	0.0080
$\kappa$ G	3	0.0021	0.0040	0.0051	<u>0.0004</u>	0.0084	0.0057	0.0073	0.0054
$l\kappa$ G	3	0.0190	0.0232	0.0231	<u>0.0158</u>	0.0268	0.0206	0.0251	0.0222
GB1	4	0.0010	0.0023	0.0030	0.0021	0.0011	0.0028	0.0018	0.0021
GB2	4	<u>0.0000</u>	0.0006	0.0026	0.0039	0.0005	0.0023	0.0035	0.0024
$E\kappa$ G1	4	0.0001	0.0003	0.0019	0.0040	0.0008	0.0015	0.0019	0.0019
$E\kappa$ G2	4	0.0020	0.0042	0.0013	0.0017	0.0053	0.0050	0.0076	0.0044
$IE\kappa$ G1	4	0.0001	0.0008	0.0027	0.0039	0.0004	0.0029	0.0042	0.0027
$IE\kappa$ G2	4	0.0410	0.0359	0.0473	0.0455	0.0372	0.0473	0.0519	0.0441
dPLN	4	0.0003	0.0005	0.0027	0.0041	0.0007	0.0019	0.0022	0.0022
GB	5	0.0000	0.0006	0.0026	0.0039	0.0005	0.0016	0.0027	0.0022
GdPLN	5	0.0000	0.0015	0.0028	0.0054	0.0026	0.0049	0.0024	0.0033
$dP^2LN\mu$	5	0.0063	0.0053	0.0031	0.0012	0.0021	0.0070	0.0080	0.0053
$dP^2LN\sigma$	5	0.0002	0.0006	0.0028	0.0042	0.0006	0.0014	0.0024	0.0022
$dP^2LNr$	5	0.0003	0.0005	0.0027	0.0041	0.0007	0.0019	0.0022	0.0022
$dP^2LN\mu\sigma$	6	0.0080	0.0057	0.0034	0.1529	0.0022	0.0071	0.0077	0.0581
$dP^2LN\mu r$	6	0.0093	0.0056	0.0037	0.0016	0.0008	0.0010	0.0008	0.0044
$dP^2LN\sigma r$	6	0.0029	0.0015	0.0018	0.0023	0.0018	0.0010	<u>0.0002</u>	0.0018
$dP^2LN$	7	0.0008	<u>0.0000</u>	0.0009	0.0033	<u>0.0002</u>	<u>0.0004</u>	0.0007	<u>0.0014</u>
$dP^2LN'$	6	0.0018	0.0006	0.0011	0.0033	0.0008	0.0007	0.0004	0.0016
$dP^2LN''$	6	0.0018	0.0006	0.0011	0.0027	0.0008	0.0007	0.0004	0.0014

*Endnote 1:*

dP<sup>2</sup>LN $\sigma$ : the dP<sup>2</sup>LN with constraints  $\mu_L = \mu_R$  and  $r = \alpha/(\alpha + \beta)$ ; dP<sup>2</sup>LN $\mu$ : the dP<sup>2</sup>LN with constraints  $\sigma_L = \sigma_R$  and  $r = \alpha/(\alpha + \beta)$ ; dP<sup>2</sup>LN $r$ : the dP<sup>2</sup>LN with constraints  $\mu_L = \mu_R$  and  $\sigma_L = \sigma_R$ ; dP<sup>2</sup>LN $\mu\sigma$ : the dP<sup>2</sup>LN with constraint  $r = \alpha/(\alpha + \beta)$ ; dP<sup>2</sup>LN $\mu$ : the dP<sup>2</sup>LN with constraint  $\sigma_L = \sigma_R$ ; dP<sup>2</sup>LN $\sigma$ : the dP<sup>2</sup>LN with constraint  $\mu_L = \mu_R$ ; dP<sup>2</sup>LN': the dP<sup>2</sup>LN with constraint  $\beta = 1.01$  and  $\mu_L > \mu_R$ ; dP<sup>2</sup>LN'': the dP<sup>2</sup>LN with constraint  $\beta = 1.01$  and  $\sigma_L < \sigma_R$ .

*See the main article for the (other) abbreviations of the model names.*

*Endnote 2:*

The best value among the PIDMs is underlined for each wave or the average/RMSE over all waves.

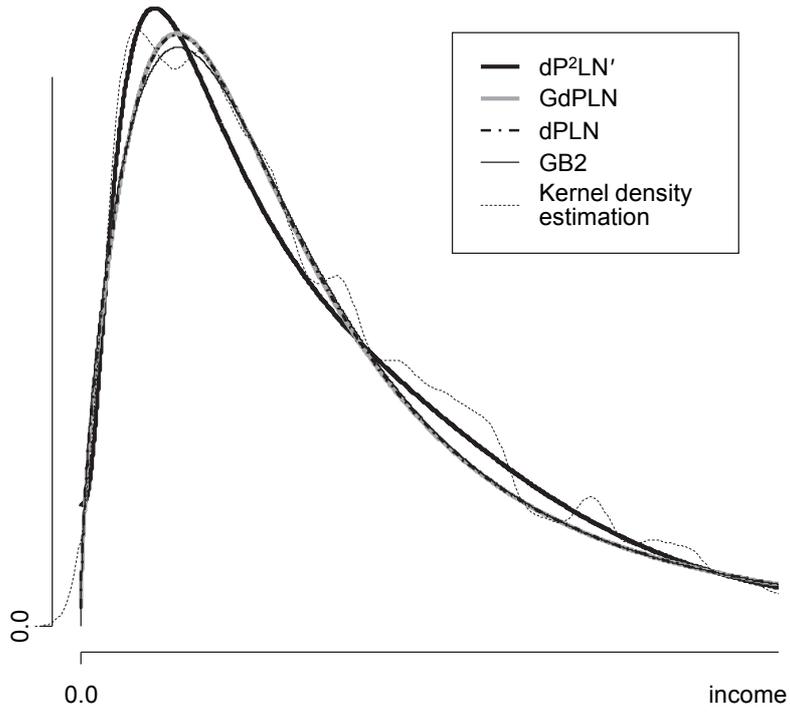


Figure 1A. PDFs of fitted PIDMs and kernel density estimation, USA, 2001

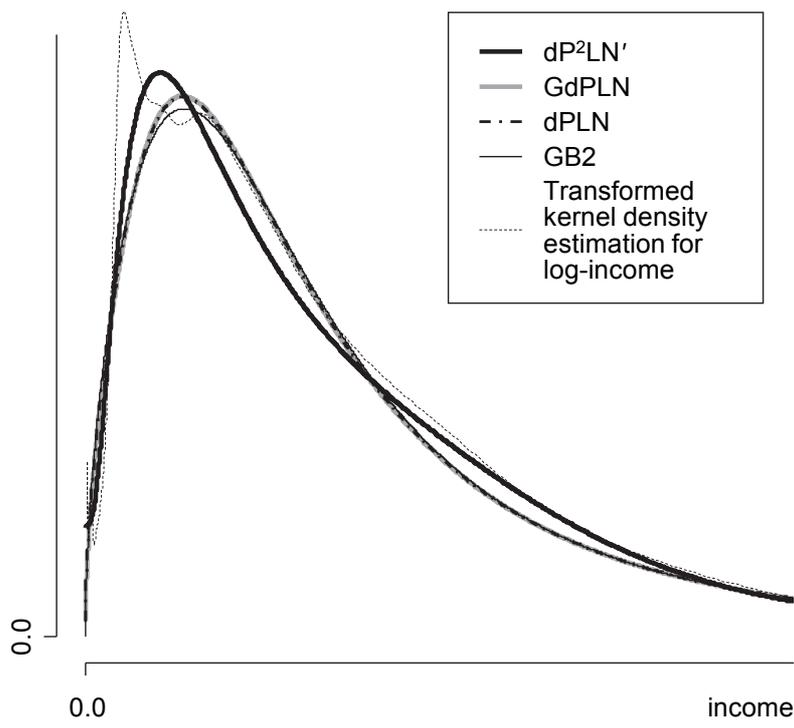


Figure 1B. PDFs of fitted PIDMs and transformed kernel density estimation for log-income, USA, 2001

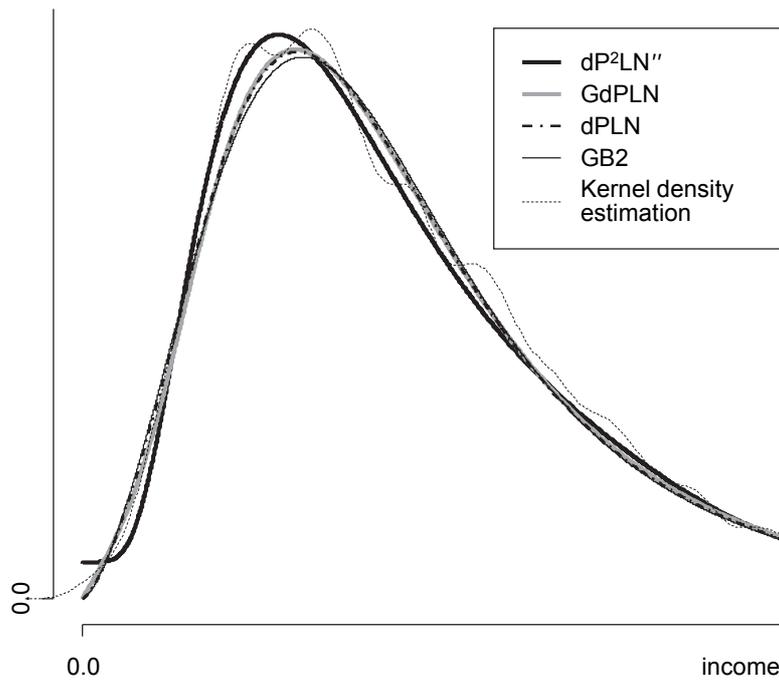


Figure 2A. PDFs of fitted PIDMs and kernel density estimation, Italy, 2006

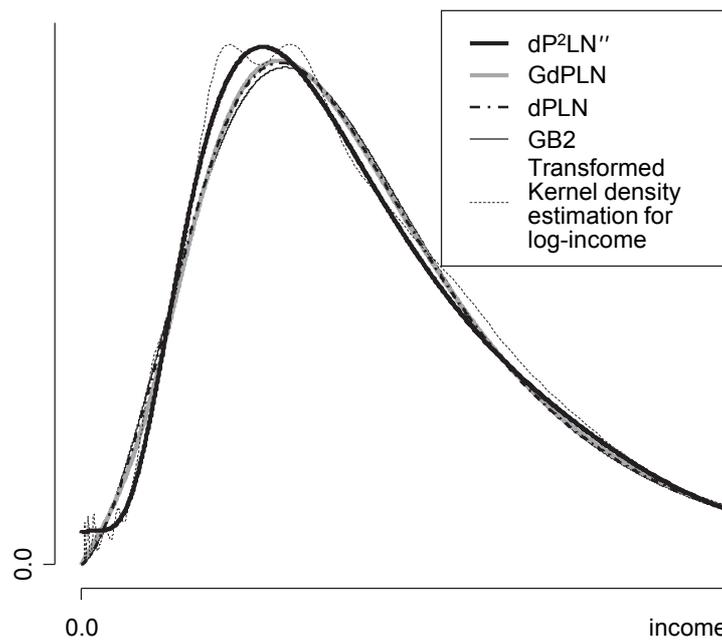


Figure 2B. PDFs of fitted PIDMs and transformed kernel density estimation for log-income, Italy, 2006

*Note.* The Gaussian kernel function was applied with the bandwidth of about  $0.0436 \times$  the average income for USA, 2001 (Figure 1A) and about  $0.0543 \times$  the average income for Italy, 2006 (Figure 2A). The same kernel function was applied to the log-incomes with the bandwidth of about  $0.152 \times$  the average log-income for USA, 2001 (Figure 1B) and about  $0.0913 \times$  the average log-income for Italy, 2006 (Figure 2B). These bandwidths were determined by the Shether-Jones method. Although the transformed kernel density estimations are shaped windingly at the lower tails, the actual densities near the lower ends can be properly inferred from the estimations.