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Appendix 1: The Closed Input–Output Model

In the open input–output model, all components of final demand are treated as exogenous, that is, disconnected from the technologically determined interdependence among productive sectors. This specification captures only the direct and indirect relationships across sectors. However, to account for the induced effects generated by income creation and household consumption, it is necessary to employ the closed model.

To this end, household consumption and labor income are endogenized in the model through the inclusion of an additional column (consumption coefficients) and an additional row (income coefficients). Thus, the interregional closed input–output model can be represented, following Miller and Blair (2022), as:

$$\begin{bmatrix} \mathbf{x} \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{h}_c \\ \mathbf{h}_r & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_{n+1} \end{bmatrix} + \begin{bmatrix} \mathbf{y} \\ y_{n+1} \end{bmatrix} \quad (\text{A1})$$

where \mathbf{h}_c is a column vector of consumption coefficients, and \mathbf{h}_r is a row vector of labor-income coefficients.

Assuming that

$$\begin{bmatrix} \mathbf{x} \\ x_{n+1} \end{bmatrix} = \bar{\mathbf{x}} \quad \begin{bmatrix} \mathbf{A} & \mathbf{h}_c \\ \mathbf{h}_r & 0 \end{bmatrix} = \bar{\mathbf{A}} \quad \begin{bmatrix} \mathbf{y} \\ y_{n+1} \end{bmatrix} = \bar{\mathbf{y}} \quad (\text{A2})$$

We can rewrite the basic input–output equation as:

$$\bar{\mathbf{x}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1} \bar{\mathbf{y}} = \bar{\mathbf{B}} \bar{\mathbf{y}} \quad (\text{A3})$$

in which $\bar{\mathbf{B}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1}$.

Equation A3 represents the closed model for households. The coefficients in the matrix $\bar{\mathbf{B}}$ are larger than those obtained in the open Leontief model. The differences between the coefficients of the inverse matrices in the two models capture the induced impact on sectoral output resulting from the expansion of household consumption.