APET
Association for Public Economic Theory

777wps777 11-00014

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Abstract

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Citation: Nguyen Bich, (2011) "Test", *777wps777*, 777wps777-11-00014.
Submitted: Jun 13 2011. **Published:** June 17, 2011.
URL:http://www.accessecon.com/Pubs/777wps777/777wps777-11-00014.pdf

Economics Bulletin

[Submission Number: EB−02C30001S]

Bootstrapping and Bartlett corrections in cointegrated VARs

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Abstract

We compare different methods for reducing the size distortion, when testing linear restrictions on the cointegration vectors (using the Maximum Likelihood framework). We compare the Bartlett correction as derived by Johansen (2000), the Bootstrap and the fast double bootstrap (Davidson, R., and J.G. MacKinnon (2000)). After a Monte Carlo study of a 5 variable DGP we conclude that (1) asymtptotic tests are so distorted, that they should never be used (2) Bartlett corrected tests do go a long way in correcting the size distortion, but do not eliminate it totally. It should therefore be used with care.(3) the bootstrap and fast double bootstrap do most to correct the size distortion and suffer from a marginally larger power loss than the Bartlett correction. They can definitely be used, especially in the many relevant cases, for which a Bartlett correction has not yet been derived.

We are grateful to Søren Johansen for his encouragement and suggestions, and Tor Jacobson, Bent Nielsen and Paolo Paruolo for helpful discussions; the usual disclaimers apply. Financial support of MURST is gratefully acknowledged. Correspondence to: pomtzigt@eco.uninsubria.it, s.fachin@caspur.it.
Submitted: January 18, 2002.

1. Introduction

The small sample properties of tests on long-run coefficients in cointegrated systems are still a matter of concern to applied econometricians. Since the asymptotic procedures proposed by Johansen (1991) have been shown to suffer from severe size distortion, two natural and complementary solutions have been proposed: (i) applying Bartlett corrections to the test statistics, in the hope that the corrected statistic will follow a small sample distribution closer to the asymptotic one, and thus bring actual sizes closer to the nominal sizes (Johansen, 2000); (ii), trying to estimate the actual small sample distribution by the bootstrap, a computer-intensive technique strictly linked with the Edgeworth expansion (Fachin, 2000, Gredenhoff and Jacobson, 2001, and references therein). For the time being, no definite solution has however appeared.The primary aim of this paper is precisely that of comparing size and power properties of the bootstrapped tests on the coefficients of the cointegration relations.

2. Bartlett-corrected and Bootstrap Tests on Cointegrating Coefficients

The idea behind the Bartlett correction is both simple and appealing. Suppose the aim is testing an hypothesis on a subset $\boldsymbol{\theta}$ of the parameters $\boldsymbol{\Theta}$, $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}^0$. In regular cases, the LR test statistic S has an expected value of $E\left[-2\ln(LR)\right] = h\left(1 + \frac{1}{T}g\left(\boldsymbol{\theta}\right)\right) + O\left(\frac{1}{T^2}\right)$,where h denotes the number of restrictions tested. Then dividing the test statistic S by $\left(1 + \frac{1}{T}g\left(\boldsymbol{\theta}\right)\right)$ we may obtain the modified test statistic S_B and expect the resulting distribution to be closer to a χ^2 distribution. This division is called a Bartlett correction and $\frac{1}{T}g\left(\boldsymbol{\theta}\right)$ will be referred to as the Bartlett factor. Johansen (2000) derived the Bartlett correction for three different kind of hypotheses on β: (1) $\beta = \beta^0$, a simple hypothesis on all the cointegration vectors; (2) $\beta_1 = \beta_1^0$ whereβ_1^0 are the first r_1 relations ($1 \leq r_1 < r$) and the other cointegration relations are unrestricted; (3) $\beta = H\varphi$ where H is a $(r \times s)$ matrix of full rank and $s < r$. This hypothesis implies the same restriction on all relations in β. The problem with the Bartlett correction is that it is extremely difficult to derive. At the opposite, the great advantage of the bootstrap is that in principle it can offer immediate solutions to new problems. However, in practice its ability to deliver good alternatives when reliable small sample parametric procedures are lacking must be accurately tested before its use may be recommended.

The general idea underlying bootstrap tests is to assess the value of the test statistic s obtained from the empirical analysis on the basis of the distribution of a large number of statistics s^* computed from suitably constructed pseudodata, with the null hypothesis of the former consistent with the data generating process (DGP) of the latter. To this end, H_0 may be imposed when generating the pseudodata, or, vice versa, the chosen DGP taken as the null hypothesis (as recommended by Hall, 1992). In both cases, H_0 is true for the pseudodata, and thus, assuming for simplicity a one-sided test, the proportion of s^* more extreme than s in the relevant direction is a natural estimate of the p-value of the test.With cointegrated VARs and some hypothesis on the long-run coefficients $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^0$, the two approaches entail respectively: (a) estimating a VAR constrained under $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^0$, generating the pseudodata on the basis of the estimated *constrained* coefficients and a set of random noises (we will discuss the choice of these below), and testing $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^0$ both on the original data and on the pseudodata; (b) estimating an unconstrained VAR, generating the pseudodata on the basis of the estimated *unconstrained* coefficients and a set of random noises, testing $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^0$ on the original data and $H_0^* : \boldsymbol{\beta} = \widehat{\boldsymbol{\beta}}$ (where $\widehat{\boldsymbol{\beta}}$ are the unconstrained estimates of $\boldsymbol{\beta}$) on the pseudodata. So far, approach (a) has been favoured with no exception in the applications of interest here. However, a point of crucial importance for testing in the maximum likelihood estimation of cointegrated VARs seems to have gone unnoticed: although both approaches are valid and asymptotically equivalent under H_0, this is not true any more when it is false[1]. Thus, we will consider bootstrap tests of type (b). Defining $\boldsymbol{\Theta}$ the entire parameter set of the VAR and assuming we are interested in running a one-sided test on a subset $\boldsymbol{\theta}$, with H_0: $\boldsymbol{\theta} = \boldsymbol{\theta}^0$, the general structure of the bootstrap test we shall implement is thus the following:

1. Estimate VAR on data $\mathbf{X}$; for given cointegrating rank obtain estimates $\widehat{\boldsymbol{\Theta}}$, residuals $\widehat{\boldsymbol{\varepsilon}}$, and test statistic s for the hypothesis $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}^0$

2. Construct pseudodata: $X^* = \phi(\widehat{\boldsymbol{\Theta}}, \boldsymbol{\varepsilon}^*), \boldsymbol{\varepsilon}^*$ drawn at random with replacement from $\widehat{\boldsymbol{\varepsilon}}$ or NID.

3. Estimate VAR on pseudodata X^*; obtain coefficients $\widehat{\boldsymbol{\Theta}}^*$ and test statistic s^* for the hypothesis H_0^*: $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$

4. *Repeat 2-3 a large number of times;*

5. Compute bootstrap p-value: $p^* = prop(s^* > s)$.

As mentioned in the introduction, Davidson and MacKinnon (2000) recently put forth a computationally cheap double bootstrap procedure which may deliver results superior to the standard bootstrap just outlined, and which we shall thus examine. We refer to their paper for a discussion of the fast double bootstrap p-value of type I (their (and our) prefered measure) and that of type II.

3. Monte Carlo Experiment

31 Design

On the basis of the simulation results reported by Gredenhoff and Jacobson (2001) and Fachin (2000), the key characteristics of the DGP to be controlled in the experiments are the dimension of the system, i.e. number of variables and lags, and its long-run structure, i.e. number of the cointegrating relationships and the speed at which the system adjusts to them. Estimation of systems of higher dimension (both in terms of number of variables and lags) demand more from the data, and thus it is (ex-post) not surprising to see that both

[1] To see this, consider the case of a test $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^0$ in a model without lags and just one cointegration vector. If this vector is misspecified, then $\boldsymbol{\beta}^{0\prime} X_{t-1}$ is clearly an $I(1)$ process, whereas ΔX_t is $I(0)$. The only congruent values for the loading factors α are therefore zero. Hence all the element of the matrix $\hat{\Pi} = \hat{\alpha}\boldsymbol{\beta}^{0\prime}$ equal zero (asymptotically) and the rank of such a matrix is 0 not 1. If one were to use this matrix for the Bootstrap DGP, one would generate just random walks without any cointegration.

the asymptotic test and the bootstrap test proposed by Gredenhoff and Jacobson (2001) perform better in smaller systems. A crucial remark here is that the simple bivariate DGPs employed in virtually all simulation studies do suffer from loss of generality, a fact not suspected so far. The experimental design adopted here will thus generalize to a multivariate system the classical DGP used by a number of studies starting with Engle and Granger (1987), which allows an easy control of the speed of adjustment. We shall consider systems including $p = 5$ random variables and with $r = 1$ or 2 cointegrating relationships. Let $\mathbf{x}_t = [x_{1t} \ldots x_{kt}]'$ be the column vector of the realizations of the random variables of interest at time $t = 1, \ldots, T$, $\mathbf{u}_t = [u_{1t} \ldots u_{kt}]'$ the errors, $\epsilon_t = [\varepsilon_{1t} \ldots \varepsilon_{kt}]'$ the noise, whose stochastic structure will be discussed in detail below, and τ a time trend. Our DGP is then given by

$$\mathbf{G}\mathbf{x}_t + \boldsymbol{\rho}\tau = \mathbf{u}_t \tag{1a}$$

$$\Phi\mathbf{u}_t^{'} = \epsilon_t \tag{1b}$$

with
$\mathbf{G} = [\boldsymbol{\gamma}_1 \ldots \boldsymbol{\gamma}_r]'$, $\boldsymbol{\gamma}_j = \left[\gamma_{j1} \ldots \gamma_{jk}\right]$,
$\Phi = diag(\boldsymbol{\phi})$, $\boldsymbol{\phi} = \left[\begin{array}{ccccc} \phi_1(L) & \phi_2(L) & \phi_3(L) & \phi_4(L) & \phi_5(L) \end{array}\right]$.

Although the Bartlett corrections do depend on the parameters of the system, in order to keep the size of the experiment within manageable dimensions in the size simulations the cointegrating coefficients will be kept fixed across trials to either zero or 1, with the vectors resembling quite closely those used by Haug (1996), while in the power simulations we shall consider a few values in the range $[0.5, 1.5]$. Given that we are using a full-information method we do not need to worry about endogeneity; we shall thus consider a very simple structure, with one stochastic trend (X_p) transmitted to the first r variables of the system, while the remaining $p - r - 1$ follow independent random walks. The details in the two cases are as follows:

(a) $r = 1$

$\boldsymbol{\gamma}_1 = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & \beta_{15} \end{array}\right]; \boldsymbol{\gamma}_2 = \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \end{array}\right]; \boldsymbol{\gamma}_3 = \left[\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \end{array}\right];$
$\boldsymbol{\gamma}_4 = \left[\begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \end{array}\right]; \boldsymbol{\gamma}_5 = \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \end{array}\right];$
$\boldsymbol{\rho} = \left[\begin{array}{ccccc} 0.01 & 0 & 0 & 0 & 0 \end{array}\right]';$
$\phi_1(L) = (1, \varphi_1 L, \ldots, \varphi_k L^k);$
$\phi_2(L) = \phi_3(L) = \phi_4(L) = \phi_5(L) = (1, -L).$

(b) $r = 2$

$\boldsymbol{\gamma}_2 = \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & \beta_{25} \end{array}\right]$;all the other $\boldsymbol{\beta}'s$ as in case (a).
$\boldsymbol{\rho} = \left[\begin{array}{ccccc} 0.01 & 0.01 & 0 & 0 & 0 \end{array}\right]';$
$\phi_1(L) = \phi_2(L) = (1, \varphi_1 L, \ldots, \varphi_k L^k);$
$\phi_3(L) = \phi_4(L) = \phi_5(L) = (1, -L).$

The values taken by the two free parameters β_{15} and β_{25} in the various experiments will be detailed below. The order k of the autoregressive polynomial governing the dynamic structure of the noise in the cointegrating relationships will be set to either 2 or 4; in the main block of experiments the sum of the coefficients (on which depends the spectral mass at zero frequency, governing the speed of adjustment to long-run equilibrium) will be kept fixed at $\phi = 0.7$ so to examine the performances of the tests in rather unfavourable conditions at the same avoiding regions too close to non-stationarity. The individual coefficients of the lag polynomial will be fixed at the following values, chosen so to have a large part of the adjustment taking place in the first periods:

(*i*) $k = 2 : \phi_j(L) = (1, -\frac{\varphi}{2}L, -\frac{\varphi}{2}L^2)$;

(*ii*) $k = 4$: $\phi_j(L) = (1, -\frac{\varphi}{2}L, -\frac{\varphi}{3}L^2, -\frac{\varphi}{16}L^3, -\frac{\varphi}{16}L^4)$;

where $j = 1, \ldots, r$.

Some simple considerations will allow great simplification of the design as far as the ε's are concerned. First of all, in previous work on the related topic of stationary VARs (Fachin and Bravetti, 1996) one of the authors of this paper found that the shape of the distribution of the shocks does not appear to have a significant impact on the performances of asymptotic procedures. Further, the expectation that with a full-information method, their covariance structure should not matter either has been confirmed in the case of a simple bivariate DGP by Fachin (2000). We shall thus assume $\varepsilon = [\varepsilon_1 \ldots \varepsilon_p] \sim MNID(0, \mathbf{I}_p)$. The last aspect to be discussed is sample size. In order to shed some light on both the performances which can be expected in empirical work and on the asymptotic properties of the tests we shall consider a base case $T = 100$, with a control experiment replicated with $T = 400$. Finally, the number of both Monte Carlo replications and bootstrap redrawings has been fixed to 500: on the basis of previous work and some pilot experiments we concluded that the gain in precision deliver by higher numbers of either was not worth the higher computing costs and longer calendar time required. At 0.05 the Monte Carlo standard error will thus be about 0.010.

Although in principle both Wald and LR tests might be used, we shall limit the experiments to the latter in order to facilitate comparisons with other published results. The tests will be applied to the hypothesis that one or more of the cointegrating vectors are known.

The cointegrating vectors will be fixed in the DGP ($\boldsymbol{\beta}_i$) and in the null hypothesis H_0 ($\boldsymbol{\beta}_i^0$) according to the following scheme:

- size simulations:

$r = 1$, one tested vector: $\boldsymbol{\beta}_1^0 = \boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0.01 \end{bmatrix}$;

$r = 2$, one tested vector: $\boldsymbol{\beta}_1^0 = \boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0.01 \end{bmatrix}$; $\boldsymbol{\beta}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0.01 \end{bmatrix}$;

$r = 2$, two tested vectors: $\boldsymbol{\beta}_1^0 = \boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0.01 \end{bmatrix}$, $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_2^0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0.01 \end{bmatrix}$;

- power simulations (main block; a power curve will also be computed for a specific case, see below):

$r=1$, one tested vector: $\boldsymbol{\beta}_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0.01 \end{bmatrix}, \boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 & 0.01 \end{bmatrix}$

$r=2$, one tested vector: $\boldsymbol{\beta}_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0.01 \end{bmatrix}, \boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 & 0.01 \end{bmatrix}$; $\boldsymbol{\beta}_2 = [\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0.01 \end{bmatrix}]$;

$r=2$, two tested vectors: $\boldsymbol{\beta}_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0.01 \end{bmatrix}, \boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 & 0.01 \end{bmatrix}$; $\boldsymbol{\beta}_2^0 = [\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0.01 \end{bmatrix}], \boldsymbol{\beta}_2 = [\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0.01 \end{bmatrix}]$

Finally, in order to keep the mass of results within manageable limits we shall report results relative to a few specific cases for the case of rank = 1, one tested vector only (in other terms, the combination rank = 1, one tested vector, $T = 100$, $\phi = 0.7$, $k = 2$ will be taken as the reference case). More specifically, we will test the effect of a higher speed of adjustment ($\phi = 0.4$),larger sample size ($T = 400$) and richer dynamic structure of the VAR ($k = 4$) as well as compute a power curve considering for $\beta_{15} \in [0.5, 1.50]$, with $\beta_{15}^0 = 1$ as usual.

32 Results

The simulations lead to the following picture (some of the results are collected in the tables 1-6 at the end of the paper):

1. The size correction in the standard case of none of the three methods is perfect: the horrible asymptotic size of 66% (at the nominal level of 5%) gets reduced to between 26% and 35% in the case of a simple hypothesis on the whole space (1 cointegration vector). In practical circumstances the second experiment (1 of the 2 vectors known) is more interesting. The size corrections work better in this case. There is some power loss in all cases, but power keeps up very well.

2. When the sample size increases to 400 all small sample corrections work properly, whereas the aysmptotic size is still 11%

3. Increasing the VAR length greatly increases all problems.

4. Faster adjustment to equilibrium improves all procedures.

5. All methods have very steep power curves, which is highly desirable

6. If we were to base the Bartlett correction and Bootstrap on the restricted model, then the power of the tests declines dramatically. Note that the Bartlett correction is not even defined if one of the restricted roots is explosive, which happens in up to 34% of the cases.

4. Conclusions

We have compared different variants of bootstrap and Bartlett-corrected tests in a DGP which is relatively unfavourable, but reproduces some features of real life empirical applications: a relatively large system (5 variables and 2 or 4 lags), and rather slow adjustment to long-run equilibrium. With such a complex DGP the caveats common to all simulation studies are even more important than usual. Further, the type of tests examined assumes full knowledge of the tested cointegrating vectors, a rare event in practice: however, they are the only tests for which the Bartlett correction is available. With all these caveats, our recommendations are the following:

(i) Asymptotic tests should be used in no circumstance;

(ii) Bartlett-corrected tests may be used provided considerable caution is exercised, as their Type I error is often much larger than the nominal size;

(iii) Bootstrap tests, with a somehow lower size distortion than the Bartlett corrected tests accompanied by limited power losses, may also be used; the fast double bootstrap delivers the best performance, and thus it appears to be a powerful tool for applied work, especially in the many cases when the Bartlett correction is not available.

Furthermore we stress that all procedures should be based on the *unrestricted* estimates.

Table 1
Size and Power: summary results
1 to 2 cointegration vectors, test on 1 to 2 vectors
$\phi = 0.7, T = 100, k = 2$

rank, tested vectors	1,1		2,1		2,2	
Test	*size*	*power*	*size*	*power*	*size*	*power*
Asymptotic	66.0	99.0	39.2	97.6	68.6	98.2
Bartlett	35.8	92.2	15.8	79.2	33.2	82.2
Bootstrap	32.0	86.0	15.2	77.2	28.2	74.6
FDB_1	26.2	76.0	13.4	68.0	20.0	62.2
FDB_2	27.8	81.8	14.2	71.4	23.6	68.2

nominal significance level: 5%; FDB_i: Fast Double Bootstrap type i
power simulations:
case (1,1) $H_0 : \boldsymbol{\beta}_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix}$, DGP: $\boldsymbol{\beta}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 \end{bmatrix}$
case (2,1): as case (1,1) with DGP: $\boldsymbol{\beta}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix}$
case (2,2): as case (2,1) with $H_0 : \boldsymbol{\beta}_2^0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

Table 2
Increasing the sample size
1 cointegrating vector, test on 1 vector
$\phi = 0.7, T = 100$ and $400, k = 2$

T	100		400	
Test	*size*	*power*	*size*	*power*
Asymptotic	66.0	99.0	11.0	100.0
Bartlett	35.8	92.2	5.6	100.0
Bootstrap	32.0	86.0	6.2	100.0
FDB_1	26.2	76.0	5.6	100.0
FDB_2	27.8	81.8	5.8	100.0

nominal significance level: 5%
power simulations: see Table 1

Table 3
Increasing the VAR length
1 cointegrating vector, test on 1 vector
$\phi = 0.7, T = 100, k = 2$ and 4

lags	2		4	
Test	*size*	*power*	*size*	*power*
Asymptotic	66.0	99.0	85.4	99.4
Bartlett	35.8	92.2	53.2	91.0
Bootstrap	32.0	86.0	39.2	82.0
FDB_1	26.2	76.0	32.4	68.8
FDB_2	27.8	81.8	35.6	74.4

nominal significance level: 5%
power simulations: see Table 1

Table 4
Increasing the speed of adjustment
1 cointegrating vector, test on 1 vector
$\phi = 0.7$ and $0.4, T = 100, k = 2$

ϕ	0.7		0.4	
Test	*size*	*power*	*size*	*power*
Asymptotic	66.0	99.0	33.0	99.6
Bartlett	35.8	92.2	17.0	97.6
Bootstrap	32.0	86.0	14.2	96.8
FDB_1	26.2	76.0	10.8	94.4
FDB_2	27.8	81.8	11.8	95.6

nominal significance level: 5%
power simulations: see Table 1

Table 5: Power curve
$\phi = 0.7, T = 100, k = 2$

Test / β_{15}	*Asymptotic*	*Bartlett*	*Bootstrap*	FDB_1	FDB_2
0.5	99.0	92.2	86.0	76.9	81.8
0.6	99.0	91.8	85.8	76.6	81.8
0.7	99.0	91.2	86.6	76.6	81.6
0.8	99.0	91.4	86.4	76.0	81.6
0.9	98.8	88.8	83.6	73.8	78.0
0.92	98.8	86.4	80.8	72.0	75.6
0.94	98.0	83.2	75.6	65.2	70.2
0.96	93.8	72.2	66.4	54.6	58.6
0.98	81.4	50.6	43.4	35.2	37.8
1.0	*66.0*	*35.8*	*32.0*	*26.2*	*27.8*
1.02	81.6	50.6	45.4	38.0	41.8
1.04	94.4	71.6	63.6	56.6	58.8
1.06	98.6	81.2	76.0	65.6	69.0
1.08	99.2	84.6	79.6	70.4	74.2
1.1	99.0	86.6	81.8	72.2	77.4
1.2	99.4	89.6	85.8	76.8	80.4
1.3	99.4	90.6	86.8	77.8	81.4
1.4	99.6	91.0	87.6	77.4	82.2
1.5	99.6	91.0	88.0	78.6	82.2

nominal significance level: 5%

Table 6: Power curve
Bootstrap data constructed from the constrained VAR estimates
$\phi = 0.7, T = 100, k = 2$

Test / β_{15}	*Asymptotic*	*Bartlett*	*Bootstrap*	FDB_1	FDB_2	Explosive Roots *(% of simulations)*
0.5	99.0	11.4	15.8	10.2	12.2	34.8
0.6	99.0	14.0	17.8	9.6	12.6	30.4
0.7	99.0	20.2	18.0	10.0	12.2	23.4
0.8	99.0	23.2	18.4	10.6	14.0	12.6
0.9	98.8	15.8	22.2	12.2	14.4	0.2
1.0	*66.0*	*18.0*	*8.6*	*5.4*	*6.6*	*0*
1.1	99.0	16.4	21.4	12.8	14.6	0.6
1.2	99.4	23.0	18.4	10.4	13.8	9.2
1.3	99.4	21.6	18.4	9.2	13.2	20.4
1.4	99.6	18.4	16.6	9.8	12.0	28.8
1.5	99.6	15.4	16.8	8.8	12.0	32.2

nominal significance level: 5%

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