E C O N O M I C S   B U L L E T I N

Some curiosites about the Engel method to estimate equivalence scales

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Abstract

This paper lends legitimacy to the food share as an indicator of welfare by demonstrating the conditions necessary in empirical work for the Engel method of estimating equivalence scales to provide an exact measure of welfare. In analogy to a money metric of utility, the Engel's food share is shown to be a "quantity metric of utility."

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1. Introduction

The main objective of the paper is to provide further evidence about the theoretical legitimacy of the food share as an indicator of welfare. We use the property of Equivalence Scale Exactness (ESE, Blackorby and Donaldson 1991)—also known as the property of independence from the base income level chosen for comparisons (IB, Lewbel 1991)—to illustrate the conditions for the Engel method of estimating equivalence scales to provide an exact measure of welfare in applied work. This exercise unveils interesting curiosities about the Engel method. The main result is highly restrictive, but provides the theoretical justification to use the Engel food share method to make comparisons of standards of living. The fact that the Engel method can be exactly rationalized by utility theory, however, does not imply that it is a "good" indicator of welfare (Anand and Harris 1994).

The election of the food share as an indicator of economic well-being is based on Engel's observation that the standard of living of a household varies with family size and is negatively related to the share of the household budget spent on food. This empirical regularity is often referred to as Engel's first law. When comparing families differing for their number of children, it is reasonable to expect that larger households need more resources to attain the same standard of living. This assertion constitutes the second empirical Engel's law. An Engel equivalence scale measures this cost difference between households of varying size from the budget shares devoted to food. Deaton and Paxson (1998) show that, at the same level of per capita expenditure, larger households spend less per head on food. This evidence poses serious doubts about the reliability of the Engel method to measure the cost of a child. While this empirical paradox still needs to be explained, the potential user of the method should be aware of the theoretical curiosities that are concealed by the apparent simplicity of the method. We pursue this by uncovering the analogy between the Engel method that compares levels of income providing the same food share (Engel 1895, Deaton 1981, Deaton and Muellbauer 1986, Hagenaars 1986, Conniffe 1992, Van Praag and Flik 1992, Banks and Johnson 1993, Murthi 1994, Lyssiotou 1997), and the condition of income-ratio comparability (Blackorby and Donaldson 1991) allowing one to compare levels of income yielding the same utility level. In analogy to a money metric of utility, Engel's food share is shown to be a "quantity metric of utility." Interestingly, this analogy provides a simple tool to estimate Engel scales with a minimum amount of information.

The paper first introduces the relevance of the food share as a measure of economic well-being and defines the properties of equivalence scale exactness or base independence and the implied property of income ratio comparability. The analysis then exploits these definitions to show that the Engel method, under restrictive but theoretically interesting conditions, is an exact measure of welfare.

2. The Food Share as a Measure of Economic Well-Being

The suitability of the food share as an indicator of welfare is usually maintained *a priori* by those like Eswaran and Kotwal (1993) who deem that the preference structure appropriate for poor countries is the one that ranks goods according to a hierarchy of needs. The assumption of a hierarchical preference structure implies that an individual's propensity to spend on non-food consumption is positive only after meeting a subsistence level of food consumption. The assumption that a large household and a small household having the same food share are equally well-off has been often exploited by many authors to construct equivalence scales mainly for its simplicity. We claim that Engel's method, under restrictions derived from the property of income ratio comparability, is not an approximate measure of welfare and can be legitimately used for interpersonal comparisons.

2.1 Income Ratio Comparability and Income Base Independence

Observable behavior usually reveals only how children affect utility indirectly by modifying the consumption patterns of the household. Let us represent preferences as $U^*(q,d)=\phi(u(q,d),d)$, where u is a well-behaved utility function, q is a vector of quantities, d is a vector of demographic characteristics and $\phi(u,d)$ is a monotonically increasing function. Then, it is possible to observe the same demand behavior, but the utility levels would change as the non-observable direct effects of d on $U^*(.)$ via $\phi(.)$ vary and the scales cannot be uniquely defined. To ensure comparability, Blackorby and Donaldson (1991) and Lewbel (1991) show that the class of cost functions $C(u,p,d)=C^*(u,p)m(p,d)$ for some functions $m(p,d)$ generated by restricting preferences to be conditional, i.e. $U^*(q,d)=u(q,d)$, gives equivalence scales that are independent of the base level of utility or income chosen for comparisons. Lewbel terms this condition the independence of base (IB) property, while Blackorby and Donaldson call it equivalence scale exactness (ESE). The ESE/IB property restricts comparability. Its behavioral counterpart implies a restriction on incomes.

Engel's cost of a child is a special case of a more general method known as the *iso-prop* method (Browning 1992, Blackorby and Donaldson 1994). The corresponding iso-prop cost functions are of the form $C(u,p,d)=C^*(u,p)\, m(u,p_{-i},d)$, where p_{-i} is the vector of all prices except the i-th good, imply hicksian budget shares independent of demographic characteristics d. Iso-prop scales reduce to Engel scales when the function $m(u,p_{-i},d)$ is independent of prices, so that $m(u,d)$ does not vary across goods. If the cost function m is also independent of u such that $C(u,p,d)=C^*(u,p)m(d)$, then the Engel equivalence scales are also IB or exact. Blackorby and Donaldson (1991) state that if there exists an income $\hat{y}$ such that the members of a household enjoy the same level of well-being of a reference household, then any arbitrary common scaling of household incomes preserves the equality of well-being. They term this restriction on both preferences and inter-household comparisons Income-Ratio Comparability (IRC). As defined, for a comparison family 1, we have to determine $\{\hat{y}^1 \mid u = V(p,y^0,d^0) = V(p,\hat{y}^1,d^1)\}$ where the index 0 denotes a reference household. Then, assuming that the ESE/IB property is satisfied, we can write:

$$V(p,\lambda y^0,d^0) = V\left(p,\frac{\lambda y^0}{m(p,d^0)}\right) = V\left(p,\frac{\lambda \hat{y}^1}{m(p,d^1)}\right) = V(p,\lambda \hat{y}^1,d^1). \tag{1}$$

The ESE/IB property implies IRC. According to Blackorby and Donaldson (1993), utilities satisfy exactness if and only if they satisfy income-ratio comparability. In fact, by IRC $u=V(p, \hat{y}^1/y^0, d^1)=V(p,1,d^0)$. Since V is increasing in income, this expression can be solved for $\hat{y}^1/y^0$ to give $\hat{y}^1/y^0=m(p,d)$. Considering that $C(u,p,d^0)=C^*(u,p)$, we can write $\hat{y}^1 = \hat{c}(u,p,d^1) = y^0\, m(p,d) = C^*(u,p)\, m(p,d)$ thus demonstrating the claim.

It is relevant to observe that $C(u,p,d)$ is homogeneous of degree 1 if and only if the equivalence scale $m(p,d)$ is homogeneous of degree zero in p. Further, $C(u,p,d)$ satisfies the Slutsky conditions if and only if the Hessian of the reference household cost function $C^*(u,p)$ is symmetric negative semidefinite and the degenerate cost function $m(p,d)$ is also weakly concave (Blundell *et al.* 1998). According to Blackorby and Donaldson (1991), the fact that a restriction on behavior is also a restriction on comparability provides an axiomatic justification for the ESE/IB property and indicates that can be plausibly imposed *a priori*.

In general, the empirical relation linking income and the food share is linear. However, it is not always evident that the Engel curves for different household types distinct by an increasing number of children are at an equal distance at any level of total expenditure as implied by the ESE/IB property. In such parallel cases, the household equivalence scales remain constant independently of the income level at which the cost of a child is computed. The cost of a child inferred from a needs measure possessing the ESE/IB property would be the same for a rich and a poor household, or for a household living in an opulent region and a household located in a deprived area.

The conditions of IRC and ESE/IB[1] are necessary to compute the relative equivalence scale $\hat{y}^1_{|u}/y^0$ where $\hat{y}^1_{|u}=\{\hat{y}^1 \mid u = V(p,y^0,d^0) = V(p,\hat{y}^1,d^1)\}$ is the level of income that would give the comparison household the same level of utility of the reference household. Similarly, the same conditions are needed to estimate the relative equivalence scale $\hat{y}^1_{|w}/y^0$ based on the analogous Engel's method, where $\hat{y}^1_{|w} = \{\hat{y}^1 \mid w^0 (p, y^0,$

[1] Here thereafter, we will refer only to the ESE/IB property which implies the condition of income ratio comparability.

d^0) = w^1 $(p, \hat{y}^1, d^1)\}$ is the level of income that would give the comparison household the same food share of the reference household. So, the plan is to find the exact conditions under which $\hat{y}^1_{|u}=\hat{y}^1_{|w}$ and the utility and food share based relative equivalence scales are the same. We will show that this result is conditioned by choices made regarding the interaction between prices and demographic characteristics at the cost function level generating price dependent equivalence scales and different specifications of the underlying share equations.

If a) the ESE/IB property holds, and b) prices interact with demographic characteristics, then, the ratio of the income of the comparison household $\hat{y}^1_{|u}$ providing the same utility as the income y^0 of the reference household is:

$$\frac{\hat{y}^1_{|u}}{y^0} = \frac{C^*(u,p)\, m(p,d^1)}{C^*(u,p)\, m(p,d^0)} = \frac{m(p,d^1)}{m(p,d^0)} = m(p,d). \tag{2}$$

If prices do not interact with demographic characteristics, then the cost function $m(p,d)=m(d)$ is independent of prices. This situation corresponds to the condition of Engel equivalence exactness (Blackorby and Donaldson 1993).[2]

In general, as a consequence of ESE/IB, the budget shares underlying the cost structure incorporating interactions between prices and demographic variables decomposes additively into:

$$w_i(p,y,d) = w_i^*(p,y^*) + \ln \mu_i(p,d) = w_i^*\left(p, \frac{y}{m(p,d)}, d\right) + \frac{\partial \ln m(p,d)}{\partial \ln p_i}, \tag{3}$$

where $\mu_i(p,d)$ is the i-th good share translating function obtained as the derivative of the demographic deflator $m(p,d)$. Note that $\mu_i(p,d)$ is a vertical commodity-specific translation of the share, while ln $m(p,d)$ is the horizontal income translation which is commodity independent (Pendakur 1999). The vertical translation is also the elasticity with respect to prices of the logarithm of the household equivalence scale $m(p,d)$. This specification does not possess the Engel scale property because the price dependent demographic functions are good specific.

Equation (3) encompasses the following cases consistent with the Engel scale property requiring m to be independent of u, by the ESE/IB condition, and of p:

$$w_i(p,y,d) = w_i^*(p,y^*) \qquad for\ y^* = \frac{y}{m(d)}, \tag{4}$$

$$w_i(p,y,d) = w_i^*(p,y) + \ln \mu(d) \qquad for\ y^*=y\ and\ \mu_i(p,d) = \mu(d)\ \forall\ i. \tag{5}$$

Eq. (4) is an Engel curve nonlinear in y^* estimating the $m(d)$ cost component directly. Eq. (5) is a linear Engel curve estimating the demographic cost function $m(p,d)$ indirectly by integrating back ln $\mu(d)$. The question that we address now investigates whether it is possible to build a unique relation between food shares specifications described in equations (3), (4) and (5) and utility. Theorem 1 below deals with the nonlinear Engel curve specification (eq. 4), Theorem 2 with the linear one (eq.5), Theorem 3 with the more general specification described in eq. (3).

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[2] Note that by normalizing $m(d^0)=1$, then $m(d)=m(d^1)$ scales the expenditure function of the reference household $C^*(u,p)$ to obtain the level of income ensuring the same reference utility level.

2.2 The ESE/IB Property and the Engel Method

The ESE/IB property implies that the budget share equation for each good decomposes additively into a function of $w^*(p,y)$, which is the share for the reference household and is the same for all households having same income and prices, and a function $\mu(p,d)$ of prices and demographic characteristics alone. This property is exploited to investigate the relationship between the income ratio of two households sharing the same level of utility and the income ratio of two households having the same share of food.

Consider the simplest case of Engel scales, that are not commodity specific, obtained by constraining the translation of the intercept with demographic effects to be the same for all commodities. The objective is then to investigate the empirical relationship between the food share and utility when preferences are ESE/IB and are restricted to Engel scales. We pursue this objective by analyzing the consequences on comparability of the cost structure described in the Engel curves in eq. (3), (4) and (5) through the following theorems. We focus our attention to rank two Engel curves belonging to the PIGLOG family based on the empirical evidence that food Engel curves are linear in y.

Theorem 1: *Assume a) the ESE/IB property holds, b) prices and demographic characteristics do not interact, and c) preferences are PIGLOG. Then, the ratio of the incomes of two households sharing the same level of utility is the ratio of the incomes of two households facing the same prices and having the same food share.*

Proof: Assumption (a) implies that the cost function is separable from the demographic variables, while assumption (b) implies that the demographic function is independent of p, as $\ln C(u,p,d)=\ln(C^*(u,p)\, m(d))$. The underlying marshallian shares take the general form $w_f(p,y,d)=w_f^*(p,y^*)$, where $y^*=C^*(u,p)=y/m(d)$, and the demographic function $m(d)$ is not share specific. By assumption (c), the Engel function takes the following non linear Working (1943) and Leser (1963) form:

$$w_f = \alpha_f + \beta_f \ln y^* = \alpha_f + \beta_f\left(\ln y - \ln m(d)\right), \tag{6}$$

with $\beta \in (-\infty, 0]$, where the limit situation $\beta=0$ is the homothetic case, and $\ln y^*=\ln y-\ln m(d)$. Then, using the definition of Engel's method to find $\{\hat{y}^1 \mid w_0(y^0,d^0) = w_1(\hat{y}^1,d^1)\}$, we obtain:

$$\ln\left(\frac{\hat{y}^1_{|w}}{y^0}\right) = \ln\frac{m(d^1)}{m(d^0)}. \quad \textit{By the ESE/IB property,} \tag{7}$$

$$\frac{\hat{y}^1_{|w}}{y^0} = \frac{m(d^1)}{m(d^0)} = \frac{\hat{y}^1_{|u}}{y^0}, \ \textit{as claimed.}\| \tag{8}$$

The horizontal distance between the logarithm income levels ensuring the same food share or utility level is the demographic function $m(d^1)$ considering that $m(d^0)=1$ for the reference family.

Corollary 1.1 *Given the assumptions in Theorem 1, the difference of the reference and comparison share evaluated at y^0 equals the logarithm of the Engel equivalence scale multiplied by the slope coefficient:*

$$w(y^0,d^0) - w(y^0,d^1) = \beta \ \ln\frac{m(d^1)}{m(d^0)}, \tag{9}$$

that is, the Engel equivalence scale m_E is a comparable income ratio equal to the exponential of the share difference raised to $1/\beta$:

$$m_E = \frac{\hat{y}^1_{|w}}{y^0} = \frac{m(d^1)}{m(d^0)} = \frac{\hat{y}^1_{|u}}{y^0} = \exp\left(w_0(y^0,d^0) - w_1(y^0,d^1)\right)^{\frac{1}{\beta}} . \| \tag{10}$$

The log equivalence scale $\ln m_E = (\ln \hat{y}^1_{|w} - \ln y^0)$ corresponds to the food share difference corrected by the scaling factor $1/\beta$. The vertical distance $w_0(y^0,d^0) - w_1(y^0,d^1)$ is equal to the horizontal distance scaled by the income coefficient, that is $\beta \ln m(d^1)$ for $m(d^0)=1$.

Let us now contrast this result with the case linear in y described in equation (4) in which the demographic function translates the food share.

Theorem 2: *Assume a) the ESE/IB property holds, b) prices interact with demographic characteristics, c) the function $m(p, d)$ is linear in $\ln p$, d) $m(p, d)=1$, and e) preferences are PIGLOG. Then, the ratio of the incomes of two households sharing the same level of utility is the ratio of the incomes of two households facing the same prices and having the same food share.*

Proof: Assumption (a) implies that the reference cost function C^* is demographically separable as:

$$\ln C(u,p,d) = \ln\left(C^*(u,p)\ m(p,d)\right).$$

By assumptions (b) and (c), the underlying uncompensated shares take the following general form:

$$w_f(p,y,d) = w_f^*(p,y^*) + \ln \mu(d),$$

where the demographic translating function, $\ln \mu(d) = \partial \ln m(p_i,d) / \partial \ln p_i$ is the same for each good i. Choose a function $m(p,d)$ such that:

$$m(p,d) = 1 \text{ when either } \begin{cases} 1)\ p_i = 1\ \forall\ i,\ or \\ 2)\ d_j = 1\ \forall\ j,\ or \\ 3)\ \text{for some constant } p \mid m(p,d) = 1, \end{cases} \tag{11}$$

where $i=1,..,N$ indexes prices and goods and $j=1,..,J$ indexes demographic variables. An example of such a function, frequently used in the empirical analysis of demographically modified demand systems, is $m(p,d) = \prod_i p_i^{\ln \mu(dj)}$ and the demographic function $\mu(d) = \prod_j d_j^{\tau j}$ is an exponential function in the parameters τ. Therefore, by assumption (d) case 3 applies and $y^* = C^*(u,p) = y/m(p,d) = y$. By assumption (e), the Engel function takes the Working-Leser linear form:

$$w_f = w_f^*(y) + \ln \mu(d) = \alpha_f + \beta_f \ln y + \ln \mu(d), \tag{12}$$

with $\beta \in (-\infty, 0]$, where the limit situation $\beta=0$ is the homothetic case. Then, using the definition of Engel's method to find $\{\hat{y}^1 \mid w_0(y^0,d^0) = w_1(\hat{y}^1,d^1)\}$ we obtain:

$$\beta_f \ln\left(\frac{\hat{y}^1_{|w}}{y^0}\right) = \ln \frac{\mu(d^0)}{\mu(d^1)} \Rightarrow \frac{\hat{y}^1_{|w}}{y^0} = \left(\frac{\mu(d^0)}{\mu(d^1)}\right)^{\frac{1}{\beta_f}}. \tag{13}$$

Equating the Engel curve specifications in eq. (12) and eq. (6), we have:

$$\ln \mu(d) = -\beta_f \ln m(d). \tag{14}$$

Therefore,

$$\frac{\hat{y}^1_{|w}}{y^0} = \left(\frac{\mu(d^0)}{\mu(d^1)}\right)^{\frac{1}{\beta_f}} = \frac{m(d^1)}{m(d^0)} = \frac{\hat{y}^1_{|u}}{y^0}, \text{ as claimed.} \| \tag{15}$$

The difference between Theorem 1 and Theorem 2 is the assumption related to the independence of the equivalence scale $m(p,d)$ from prices associated with the lack of interaction between prices and demographic characteristics. In Theorem 1, the price independent equivalence scale $m(d)$ is directly estimated from eq.(6); in Theorem 2, the price independent equivalence scale $m(d)$ is estimated indirectly from $\mu(d)$ using eq. (14). However, in Theorem 2 the structure of preferences is not price independent otherwise it would not be possible to justify the specification of eq. (12) and its use for welfare analysis. Restrictions *(c)* and (*d*) impose *ex-post* the condition of price independence associated with the equality of $\mu(d)$ across goods.[3]

Corollary 2.1: *Given the assumptions in Theorem 2, the difference of the reference and comparison shares evaluated at* y^0 *equals the log of the ratio of the demographic functions evaluated at* d^0 *and* d^1*:*

$$w(y^0,d^0) - w(y^0,d^1) = \ln \frac{\mu(d^0)}{\mu(d^1)}. \tag{16}$$

Therefore, the Engel equivalence scale is equal to the exponential of the share difference raised to 1/β*:*

$$m_E = \frac{\hat{y}^1_{|w}}{y^0} = \left(\frac{\mu(d^0)}{\mu(d^1)}\right)^{\frac{1}{\beta_i}} = \exp\left(w_0(y^0,d^0) - w_1(y^0,d^1)\right)^{\frac{1}{\beta}}. \| \tag{17}$$

Comparison of Corollary 1.1 and 2.1 reveals that the vertical distance $w_1(y^1,d^1)$-$w_0(y^0,d^0)$=ln $\mu(d^1)$=β ln m(d^1). While in Theorem 1, the demographic component of the Engel curve specification is represented by the income deflating term $m(d)$, in Theorem 2 demographic heterogeneity is described by the share translating term $\mu(d)$. We now turn our attention to a specification that includes both $m(p, d)$ and $\mu(d)$ which is consistent with eq. (3) and with the translating specification (Pollak and Wales 1981, Lewbel 1985).

Theorem 3: *Assume a) the ESE/IB property holds, b) prices interact with demographic characteristics, c) the function m*(*p*, *d*) *is linear in ln p, d) prices are constant, and e) preferences are PIGLOG. Then, the ratio of the incomes of two households sharing the same level of utility is the ratio of the incomes of two households facing the same prices and having the same food share.*

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[3] Technically, we would need to recover a constant of integration K to obtain $m(p, d)$=K $m(d)$. The size of the constant of integration K measures the dimension of the approximation of the demographic translating function $\mu(d)$ with respect to the welfare object of interest $m(p, d)$.

Proof: Assumption (a) implies that the reference cost function C^* is demographically separable as:

$$\ln C(u,p,d) = \ln\left(C^*(u,p)\ m(p,d)\right).$$

By assumptions (b) and (c), the underlying uncompensated shares take the following general form:

$$w_f(p,y,d) = w_f^*(p,y^*) + \ln\mu(d),$$

where the demographic translating function, $ln\ \mu(d)=\partial\ ln\ m(p_i,d)/\ \partial ln\ p_i$ is the same for each good $i=1,...,N$ and $y^*=C^*(u,p)=y/m(p,d)$. As in Theorem 2, we choose a function $m(p,d)=\prod_i p_i^{\ln \mu T(dj)}$ such that $\mu^T(d)=\prod_j d_j^{\tau j}$. By assumption (d) prices are constant so that, if we further assume for simplicity that prices are equal to the same positive constant $k\neq 1$, we have that $\ln m^T(d)=K\ ln\ \mu^T(d)$ with $K=N\ lnk$. By assumption (e), the Engel function takes the translated Working-Leser linear form:

$$w_f = w_f^*(y^*) + \ln\mu(d) = \alpha_f + \beta_f\left(\ln y - \ln m^T(d)\right) + \ln\mu^T(d), \tag{18}$$

with $\beta\in(-\infty, 0]$. Then, using the definition of Engel's method $\{\hat{y}^1 \mid w_0(y^0,d^0) = w_1(\hat{y}^1,d^1)\}$ we obtain:

$$\beta_f \ln\left(\frac{\hat{y}^1_{|w}}{y^0}\right) = \ln\frac{\mu^T(d^0)}{\mu^T(d^1)} + \beta_f \ln\left(\frac{m^T(d^1)}{m^T(d^0)}\right). \tag{19}$$

The Engel equivalence scale $m_E = (\hat{y}^1_{|w}/y^0)$ can then be expressed as:

$$\frac{\hat{y}^1_{|w}}{y^0} = \left(\frac{\mu^T(d^0)}{\mu^T(d^1)}\right)^{\frac{1}{\beta_f}}\left(\frac{m^T(d^1)}{m^T(d^0)}\right) = \phi(d)\left(\frac{m^T(d^1)}{m^T(d^0)}\right) = \phi(d)\left(\frac{\hat{y}^1_{|u}}{y^0}\right) \tag{20}$$

where the function $\phi(d)$:

$$\phi(d) = \left(\frac{\mu^T(d^0)}{\mu^T(d^1)}\right)^{\frac{1}{\beta_f}} \tag{21}$$

is monotonically increasing in d. Equating the Engel curve specifications in eq. (18), (12) and (6), we obtain a measure of the vertical distance $w_1(y^0,d^1)$-$w_0(y^0,d^0)$:

$$\ln\mu^T(d) - \beta_f \ln m^T(d) = -\beta_f \ln m(d) = \ln\mu(d). \tag{22}$$

Therefore,

$$\frac{\hat{y}^1_{|w}}{y^0} = \left(\frac{\mu^T(d^0)}{\mu^T(d^1)}\right)^{\frac{1}{\beta_f}}\left(\frac{m^T(d^1)}{m^T(d^0)}\right) = \frac{m(d^1)}{m(d^0)} = \frac{\hat{y}^1_{|u}}{y^0},\ \textit{as claimed.}\| \tag{23}$$

The result of Theorem 3 is invariant to the specification of the Engel curve as it was the case for Theorem 2. The invariance come from the constancy of prices ensuring that the demographic functions $\mu^T(d)$ and

$m^T(d)$ do not vary across goods. The set of assumptions is the same as for Theorem 2 with exception for the condition restricting the equivalence scale $m(p,d)=1$. Note that $\mu^T(d)<\mu(d)$ and because $\phi(d)>0$, then $m^T(d)<m(d)$. In presence of sufficient price variability, however, the price dependent equivalence scale $m(p,d)$ would be the same as $m^T(p,d)$.

Corollary 3.1: *Given the assumptions in Theorem 3, the difference of the reference and comparison shares evaluated at y^0 equals the log of the ratio of the demographic functions evaluated at d^0 and d^1:*

$$w(y^0,d^0) - w(y^0,d^1) = \ln \frac{\mu^T(d^0)}{\mu^T(d^1)} + \beta_f \ln\left(\frac{m^T(d^1)}{m^T(d^0)}\right). \tag{24}$$

Therefore, the Engel equivalence scale is equal to the exponential of the share difference raised to $1/\beta$:

$$m_E = \left(\frac{\hat{y}^1_{|w}}{y^0}\right) = \left(\frac{\mu^T(d^0)}{\mu^T(d^1)}\right)^{\frac{1}{\beta_f}}\left(\frac{m^T(d^1)}{m^T(d^0)}\right) = \exp\left(w_0(y^0,d^0) - w_1(y^0,d^1)\right)^{\frac{1}{\beta}}.\| \tag{25}$$

The vertical distance $w_1(y^0,d^1)-w_0(y^0,d^0)=\ln \mu^T(d^1) - \beta_f \ln m^T(d^1)$ is now decomposed into a direct effect and an indirect income effect. The horizontal distance $\ln m_E=(\ln\hat{y}^1_{|w} - \ln y^0)=(-1/\beta_f) \ln \mu^T(d^1) + \ln m^T(d^1)$ is obtained dividing the vertical distance by the scaling factor $(-1/\beta_f)$. In fact, the ratio of the vertical and horizontal distance is the slope of the parallel Engel curves.

The following, somewhat curious, remarks reconcile Theorem 1, 2 and 3.

Remark 1: *Consider the special case $\beta = -1$. Then, the vertical and horizontal distance are the same and the Engel equivalence scale m_E is the inverse of the exponential of the difference of the shares evaluated at y^0.*

$$m_E = \frac{\hat{y}^1_{|w}}{y^0} = \frac{\mu(d^1)}{\mu(d^0)} = \frac{m(d^1)}{m(d^0)} = \frac{\mu^T(d^1)}{\mu^T(d^0)}\frac{m^T(d^1)}{m^T(d^0)} = \frac{\hat{y}^1_{|u}}{y^0} = \left(\exp\left(w_0(y^0,d^0) - w_1(y^0,d^1)\right)\right)^{-1}.\| \tag{26}$$

This result follows immediately if we realize that, in the special case of $\beta = -1$, PIGLOG observable demands have the same functional form linear in y with and without interactions between prices and demographic characteristics at the cost function level as specified in theorems 1 and 2. The same equivalence scale is consistent with apparently different preference structures. The remark also shows that the vertical distance corresponding to the share difference evaluated at the same income level $w_1(y^0,d^1)-w_0(y^0,d^0)$ equals the horizontal distance both in share ($\ln \hat{y}^1_{|w} - \ln y^0$) and utility space ($\ln \hat{y}^1_{|u} - \ln y^0$).

Remark 2:

$$\frac{\hat{y}^1_{|w}}{y^0} = \frac{m(d^1)}{m(d^0)} = \left(\frac{\mu(d^1)}{\mu(d^0)}\right)^{1/\beta_f} = \left(\frac{\mu^T(d^0)}{\mu^T(d^1)}\right)^{\frac{1}{\beta_f}}\left(\frac{m^T(d^1)}{m^T(d^0)}\right) = \left(\exp\left(w_0(y^0,d^0) - w_1(y^0,d^1)\right)\right)^{1/\beta_f} = \frac{\hat{y}^1_{|u}}{y^0}. \tag{27}$$

In general, for a known β, the horizontal distance ($\ln \hat{y}^1_{|w} - \ln y^0$) equals the vertical distance between the food shares computed at the same income level $w_0(y^0,d^0)-w_1(y^0,d^1)$ multiplied by the constant of proportionality $1/\beta$.

Eq. (26) and (27) show the functional relationship linking a difference in expenditure shares, a *"quantity metric,"* and an income ratio, a *"money metric,"* through an *"utility metric,"* represented by the ratio of demographic functions. The quantity metric of utility is useful because it can be translated into a difference of nutritional requirements, thus showing the differential nutritional needs of households with the same incomes but different composition and stock of capabilities. This is especially true, when the reference income is the poverty or indigence line. The logarithm of the household equivalence scale is an increasing function of the marginal propensity to consume β.

The equalities shown in eq. (26) and (27) hold independently of the level of income chosen as the basis for comparisons because of the IB property. The set of theorems discussed so far share also the common property of estimating the same price independent equivalence scale $m(d)$. The intuition underlying this condition is that we are trying to learn something about a system object $C(u,p,d)=C^*(u,p)m(p,d)$ using commodity specific information derived from the food share. Therefore, information at the cost level can be retrieved if the specific effects are the same across goods. These considerations lead to the final theorem.[4]

Theorem 4: *Assume a) the food share is monotonically decreasing in total expenditure, b) the ESE/IB property holds, and c) the equivalence scale is independent of the price of food, then the food share w_f is an exact ordinal indicator of well-being.*

Proof: Denote the compensated food share as:

$$\tilde{w}(p,\phi(u),d) = \tilde{w}^0(p,\phi(u),d^0). \tag{28}$$

Condition (a) ensures uniqueness, so that if two households have the same food share then they have the same
utility. Condition (b) imposes that $\phi(.)$ is independent of d, while condition (c) eliminates the price elasticity and condition (d) imposes that the vertical distance is the same as the horizontal distance. Given (a), (b), and either (c) or (d), then the compensated share can be inverted to get

$$\phi(u) = \tilde{w}_f^{-1}(p,w_f,d^0) \tag{29}$$

which is a restatement of Engel's law. Since utilities u are equated when $y=y/m$ by condition (b), we get that any household with a given observed food share $\hat{w}_f$ has the same utility level $\hat{u}$, and, for any two households with food shares $\hat{w}_f$ and $\bar{w}_f$, $\hat{w}_f < \bar{w}_f \Leftrightarrow \hat{u} > \bar{u}$. ‖

Theorem 4 lends theoretical legitimacy to Engel's food share method as a "true" method to operate comparisons of standards of living. This legitimacy as an exact ordinal indicator of well-being comes at the cost of assuming that the ESE/IB property holds and the scales across goods are constant, but it is not restricted to PIGLOG preferences. As Theorems 2 and 3 show, there exists a clear analogy between Engel's method and the condition of income-ratio comparability. The first compares levels of income giving the same food shares; the latter compares levels of income yielding the same utility level. In analogy to a money measure of utility, Engel's food share is a "quantity metric of utility." Expression (25), where the

[4] This theorem was suggested by a referee.

exponential of the share difference is raised to $1/\beta$, offers a simple tool to estimate Engel equivalence scales with a minimum amount of information. The food shares are usually truthfully revealed at a low cost and the Engel slopes can be easily estimated. Therefore, it would be an inexpensive exercise to compute Engel scales.

4. Conclusions

This study shows that the Engel method of estimating equivalence scales provides an exact ordinal indicator of well-being if the properties of ESE/IB and price independent equivalence scales are assumed. The result is invariant to some of the Engel curve specifications commonly used in applied work that are also theoretically plausible. The theoretical and applied curiosities revealed by the analysis show that the Engel method of estimating equivalence scales may not be considered an approximation, but, more simply, a very restrictive method. The analysis leads to the definition of a simple technique to estimate Engel equivalence scales using a limited amount of information. These attributes are particularly desirable in developing countries where rapid and inexpensive household surveys for conducting policy analyses, target schemes, and assessments of poverty and living conditions are often required.

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