E c o n o m i c s   B u l l e t i n

# Bayesian and DF−GLS unit root tests of real exchange rates over the current floating period

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## *Abstract*

In an analysis of purchasing power parity, the Phillips−Ploberger (1994) Bayesian model selection and unit root test procedure is applied to 53 real exchange rates over the current floating exchange rate period. The DF−GLS unit root test of Elliott, Rothenberg, and Stock (1996) is also applied. The Bayesian test provides very little support for the stationarity of the real exchange rates, while the DF−GLS test provides somewhat more support.

I thank Peter Phillips for helpful comments concerning the implementation of the Phillips−Ploberger procedure, but I remain responsible for any errors.
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## 1. Introduction

The time series properties of empirical real exchange rates in the current floating system have received a lot of attention. If relative purchasing power parity (PPP) holds, the real exchange rate should have a constant long-run mean (e.g., see Frankel and Rose, 1996, and Papell, 1997). Some researchers (e.g., Cheung and Lai, 1998) also assume that a deterministic linear trend in the real exchange rate could be consistent with PPP. The most commonly considered empirical alternative has been that real exchange rates are nonstationary, possessing a unit root and following a stochastic trend.

Many researchers have examined this by applying unit root tests to bilateral real exchange rates one at a time (e.g., Meese and Rogoff, 1988). A variation has been to test for cointegration among the nominal exchange rate and price components of individual real exchange rates (e.g., Taylor, 1988, MacDonald, 1993). Rogoff (1996) provides a survey of such efforts. For the current floating exchange rate period, he finds that in general the null hypothesis of nonstationarity is not rejected by these tests. Because the standard tests have relatively low power for the time span of the floating period, alternative approaches have been suggested.

Frankel and Rose (1996) and Papell (1997), among others, try to gain power by pooling bilateral data. The resulting panel unit root test results have generated some support for the proposition that real exchange rates are mean stationary. However, O'Connell (1998) points out that these tests are prone to over-reject the unit root null in the presence of contemporaneous correlation among the bilateral real exchange rates. When he adjusts the panel unit root tests to account for this, the unit root null can no longer be rejected. Also, Rogoff (1996) notes that when the panel tests generate a rejection, they cannot indicate which exchange rates are responsible for it. Sarno and Taylor (1998) and Taylor and Sarno (1998) address this problem by applying tests to a group of real exchange rates where rejection of the relevant null only occurs when all the series are stationary. They are able to generate such a rejection for the four U.S. real exchange rates with the other G5 countries. Unfortunately, the technique can only be applied to a limited number of exchange rates at one time. Because of this problem and those noted for the panel tests, unit root tests applied to individual real exchange rates remain valuable as a way of confirming, or not, the stationarity conclusion of the multivariate approaches.

Among all of the studies applying univariate unit root tests to individual real exchange rates, not one has yet applied a Bayesian test. Therefore, the present study attempts to fill the gap by applying the Bayesian unit root test procedure of Phillips and Ploberger (1994) to 51 bilateral OECD real exchange rates and two aggregate OECD real exchange rates over the floating exchange rate period. The Bayesian approach is of interest for two reasons. First, rather that acceptance or rejection and classical p-values, the Bayesian gives specific posterior odds in favor of or against the unit root. Thus, the strength of the evidence concerning the unit root is indicated in a different way than in past papers. Second, the procedure also uses a Bayesian information criterion to specify both autoregressive (AR) and moving average (MA) serial correlation orders, and to choose deterministic time polynomial terms.

Explicitly modeling MA serial correlation could be important, because standard tests of the Dickey-Fuller variety, which approximate MA processes with AR terms, can exhibit serious size distortions (rejecting too often under the null) when the moving average polynomial of the variable in first differences has a large negative root (Elliott, Rothenberg, and Stock, 1996; Ng and Perron, forthcoming). The Bayesian data-based choice of deterministic terms should also be useful, because other unit root tests do not provide clear procedures for this choice, which can influence the outcome of the test. Of course, test power could still be a problem for the Phillips-Ploberger procedure, but the data for the present paper cover a longer period than in any previous paper on unit roots and real exchange rates. Thus, power for unit root tests should be improving.

For comparison, and because two tests are probably better than one, the present study also applies the "DF-GLS" test of Elliott, Rothenberg, and Stock (1996). This is a more powerful version of the standard Dickey-Fuller test. Indeed, Cheung and Lai (1998) report rejections of the unit root null for 10 OECD real exchange rates (all country pairs for the G5) using this test. However, this test can exhibit the size distortions from MA serial correlation noted above. Therefore, in contrast to Cheung and Lai's (1998) effort, the present study uses an AR lag-selection criterion very recently developed by Ng and Perron (forthcoming) for the DF-GLS test. Ng and Perron show that this yields much better size characteristics than other lag-selection techniques for the MA serial correlation case, while maintaining power.

## 2. The Phillips-Ploberger Bayesian unit root test with data-based model selection

The Phillips-Ploberger procedure is now described. The first stage consists of model selection, which is accomplished by minimizing a criterion that Phillips and Ploberger (1994) call PIC. For comparison, here is the more familiar Schwarz (1978) Bayesian criterion, BIC, for dimension $k$:

$$\text{BIC}_k = \ln(\hat{\sigma}_k^2) + k\ln(n)/n. \tag{1}$$

In contrast,

$$\text{PIC}_k = (dQ_n^K / dQ_n^k)(\hat{\sigma}_k^2). \tag{2}$$

The term $(dQ_n^K / dQ_n^k)$ is the likelihood ratio for a reference model with $K$ regressors relative to a smaller model with $k$ regressors, and the PIC assigns equal prior odds to the two models. Specifically, one way to calculate this is

$$\text{PIC}_k = \left| (\hat{\sigma}_K^2)^{-1} A \right|^{-1/2} \exp\left\{ (2\hat{\sigma}_K^2)^{-1}(ss_k - ss_K) \right\}, \tag{3}$$

where $A = X_s'X_s - X_s'X_k(X_k'X_k)^{-1}X_k'X_s$, $ss$ = sum of squared residuals, and $\hat{\sigma}_K^2 = ss_K/(n-K)$. Subscript $k$ indicates the sum of squared residuals or variables in the smaller model, subscript $s$ indicates the remaining variables in the reference model, and subscript $K$ indicates the sum of squares in the reference model.

Implementation of the criterion for a model with possible AR, MA, and time polynomial components proceeds in a series of steps. The first is to choose the maximum AR, MA, and time polynomial orders $K$, $J$, and $L$, along with an AR order $\overline{K} \geq K$ for an initial reference model. Next, one estimates the time order $\hat{\ell}$ and AR order $\hat{k}$ by calculating the PIC values for a sequence of AR models of the form

$$\Delta y_t = \hat{a}_0 y_{t-1} + \sum_{i=1}^{k-1} \hat{\alpha}_i \Delta y_{t-i} + \sum_{j=0}^{\ell} \hat{b}_j t^j + residual \tag{4}$$

for ($k = K, \ldots, \bar{K}$ and $\ell = -1, 0, 1, \ldots, L$) (where -1 means no deterministic terms). Next, the residuals from selected model with time order $\hat{\ell}$ and AR order $\hat{k}$ are used to estimate MA order $\hat{q}$ along with AR order $\hat{p}$ for the final model. This is done by estimating a sequence of regressions of the form

$$\Delta y_t = \hat{a}_0 y_{t-1} + \sum_{i=1}^{k-1} \hat{\alpha}_i \Delta y_{t-i} + \sum_{j=0}^{\hat{\ell}} \hat{b}_j t^j + \sum_{s=1}^{q} \hat{c}_s \hat{\varepsilon}_{t-s} + residual \tag{5}$$

for ($k = 0, 1, \ldots, \bar{k} = \max(\hat{k}, K)$ and $q = 0, 1, \ldots, J$), where $\hat{\varepsilon}$ are the residuals from the long AR model with time order $\hat{\ell}$ and AR order $\hat{k}$. The final model is selected from the sequence of regressions using the PIC, with AR order $\bar{k}$ and MA order $J$ for the reference model.[1]

After model selection, the second stage of the Phillips-Ploberger procedure consists of calculating the Bayesian odds in favor of a unit root from the selected model. Rewriting the model selected from (5) as

$$\Delta y_t = \tilde{a}_0 y_{t-1} + Z\tilde{\delta} + \tilde{\varepsilon}\ , \tag{6}$$

one calculates

$$\mathrm{BLR}(\tilde{a}_0) = \left\{ (\hat{\sigma}_K^2)^{-1} y_{-1}' \bar{P}_Z y_{-1} \right\}^{-1/2} \exp\left\{ (2\hat{\sigma}_K^2)^{-1}\ \tilde{a}_0^2 (y_{-1}' \bar{P}_Z y_{-1}) \right\}, \tag{7}$$

where $M = \hat{p} + \hat{q} + \hat{\ell} + 1$, $\bar{P}_Z = I - (Z'Z)^{-1} Z'$, and $\hat{\sigma}_K^2 = \tilde{\varepsilon}'\,\tilde{\varepsilon}/(n-M)$. The Bayesian odds on favor of a unit root are given by $1/\ \mathrm{BLR}(\tilde{a}_0)$.[2]

## 3. The data and test results

The data consist of 45 quarterly bilateral real exchange rates for the U.S. and Germany with other OECD countries, as well as three more bilateral OECD exchange rates in order to include all 10 in Cheung and Lai (1998), and a final three Canadian rates. The IMF's real effective exchange rates (REER) for the U.S. and Germany are also included. Consumer price indexes are used for the price components of the real exchange rates. All data are from the IMF's *International Financial Statistics* on CD-ROM. Bilateral nominal exchange rates are period averages from line "rf", consumer prices are from line 64, and the real effective exchange rates are from line "re*u*."[3] All values are converted to logs. The estimation period begins in 1974:1, a

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[1] In their empirical application, Phillips and Ploberger (1994) actually applied the BIC in this step, because they had not yet completed the programming for the specification with possible MA components.

[2] The Gauss add-on COINT 2.0 contains the Phillips-Ploberger Bayesian unit root procedure with PIC model selection, but only for AR processes (with possible polynomial time trend). However, I have coded the complete procedure including the MA terms in TSP 4.5, and this code is available upon request. I am grateful to Peter Phillips for clarifying comments on implementing the complete procedure (of course, any errors are mine).

[3] I have filled in the mysterious 1982 gap in the IMF's consumer prices for Iceland with data from the Central Bank of Iceland.

few months after the advent of generalized floating. The estimation period ends in 1999:4 for most of the U.S. rates, and in 1998:4 for the German and other rates that became part of the Euro in 1999:1.

The Phillips-Ploberger procedure was implemented with the following maximum orders: $K = 8$, $\overline{K} = 10$, $J = 2$, and $L = 2$. The DF-GLS test with the Ng-Perron lag selection criterion was implemented in the constant-only and constant-with-linear-trend forms. Table I gives the results, including the selected orders for the Phillips-Ploberger procedure, and the number of lagged first differences for the DF-GLS test.

The Phillips-Ploberger procedure indicates odds not in favor of the unit root for only three of the 53 exchange rates. The median value for the odds in favor of the unit root is 22.08, which seems fairly strong. In comparison, the DF-GLS test rejects the unit root at the 0.05 level for seven of 53 cases with the constant-only specification, and for one with trend.[4] However, because the Phillips-Ploberger procedure never selects order one or higher for the time polynomial, it seems justified to focus on the constant-only DF-GLS results. The fact that the proportion of such DF-GLS rejections somewhat exceeds the stated significance level (0.13 > 0.05) could be taken as mild support for mean stationarity in real exchange rates. The failure of the Phillips-Ploberger results to confirm this could then be interpreted as a sign of low power. However, it might be expected that the two tests would agree at least in a few cases on the exchange rates most likely to be stationary. This does not occur, and so the DF-GLS results may just reflect sampling error. Another factor that weakens the force of the DF-GLS rejections is inconsistency regarding results for cross rates. Because the test generates a rejection for the U.S.-German rate, the cross-rate relationship means that any rejections for U.S. rates with other countries should be accompanied by rejections for German rates for those same countries, and vice versa. However, this only occurs in one of the six relevant situations (France).

Using the DF-GLS test, Cheung and Lai (1998) reject the unit root at the 0.05 level for three of the 10 G5 real exchange rates they consider. The present DF-GLS results also show three G5 rejections (although not the same three). However, the present results also show that this proportionate level of support for stationarity does not extend beyond the G5 to the much longer list of exchange rates also considered here. Jorion and Sweeney (1996), Papell (1997), and Papell and Theodoridis (1998) conclude that German real exchange rates more often generate unit root rejections than U.S. rates. This is also contradicted by the present results, where the majority of the rejections that do occur are generated by the U.S. real exchange rates.

## 4. Conclusions

In application to 53 real exchange rates, the Phillips-Ploberger Bayesian unit root test provides little support for stationarity in real exchange rates over the current floating period. The DF-GLS test results provide somewhat more support for the stationarity proposition, but because of several inconsistencies this support probably should be treated with caution. Overall, then, these two univariate tests provide only very modest support of recent stationarity conclusions from various multivariate tests.

[4] There are more rejections at the 0.10 level, particularly for the constant-only specification (for which the critical value is -1.62). Whether or not to consider them as evidence against the unit root is, of course, a matter of judgment in the balancing of Type I and II errors. In view of the possible heteroskedasticity from the fluctuating influence of the European Exchange Rate Mechanism and other sources, and the size distortion that heteroskedasticity can introduce into unit root tests (Hamori and Tokihisa, 1997), the use of the conservative 0.05 level seems defensible.

## Table I: Test results

| | U.S. exchange rates | | | German exchange rates | | |
|---|---|---|---|---|---|---|
| | Odds ( $\hat{p},\hat{q},\hat{\ell}$ ) | DF-GLS ($d$) | | Odds ( $\hat{p},\hat{q},\hat{\ell}$ ) | DF-GLS ($d$) | |
| Australia | 78.93 (1,1,-1) | -0.53 (1) | -1.96 (0) | 17.86 (2,0,-1) | -1.23 (1) | -2.13 (0) |
| Austria | 481.54 (2,0,-1) | -1.63 (1) | -2.15 (1) | 18.96 (2,1,-1) | 0.62 (5) | 0.66 (5) |
| Belgium | 7.66 (2,0, 0) | -1.89 (1) | -1.91 (1) | 2316.59 (2,0,-1) | -1.34 (1) | -1.68 (1) |
| Canada | 125.91 (4,1,-1) | -0.85 (3) | -1.98 (4) | 29.97 (2,0,-1) | -1.46 (1) | -2.19 (1) |
| Denmark | 389.24 (2,0,-1) | -1.88 (1) | -2.00 (1) | 777.54 (1,0,-1) | -0.42 (0) | -1.62 (0) |
| Finland | 3.60 (2,0, 0) | -1.88 (1) | -2.08 (1) | 315.58 (2,0,-1) | -1.39 (7) | -1.42 (8) |
| France (G5) | 374.07 (2,0,-1) | ***-2.15*** (1) | -2.23 (1) | 1.15 (1,0, 0) | ***-2.10*** (3) | -1.90 (0) |
| Germany (G5) | 96.97 (2,0,-1) | ***-2.01*** (1) | -2.02 (1) | | | |
| Greece | 11.40 (1,0, 0) | -1.75 (5) | -1.88 (5) | 1.82 (1,0, 0) | -1.71 (1) | -1.87 (1) |
| Iceland | 14.88 (2,1, 0) | -1.90 (1) | -2.14 (1) | 871.64 (1,0,-1) | -1.90 (0) | -2.33 (0) |
| Ireland | 71.08 (2,0,-1) | -1.83 (1) | -2.21 (0) | 325.56 (1,0,-1) | -0.87 (4) | -1.41 (4) |
| Italy | 3.02 (2,0, 0) | ***-2.16*** (1) | -2.44 (1) | 3.79 (1,1, 0) | -1.45 (4) | -1.74 (1) |
| Japan (G5) | 486.10 (1,1,-1) | -0.83 (1) | -2.50 (1) | 628.12 (2,0,-1) | -0.28 (5) | -1.63 (6) |
| Mexico | ***0.50*** (2,0, 0) | -1.64 (0) | -1.93 (0) | ***0.00*** (4,0, 0) | -1.46 (10) | ***-3.35*** (1) |
| Neth. | 115.38 (2,0,-1) | ***-2.09*** (1) | -2.11 (1) | 113.24 (9,0,-1) | -0.35 (12) | -1.51 (12) |
| N. Zeal. | 3.65 (2,0, 0) | -1.95 (1) | -2.18 (1) | 3.20 (1,0,-1) | -1.89 (0) | -2.08 (0) |
| Norway | 4.28 (2,0, 0) | ***-2.10*** (1) | -2.14 (1) | 565.61 (2,0,-1) | -1.05 (10) | -1.03 (10) |
| Portugal | 1054.63 (1,1,-1) | -1.30 (2) | -1.50 (2) | 1659.10 (4,0,-1) | -1.01 (10) | -2.41 (1) |
| Spain | 17.66 (2,1, 0) | -1.60 (1) | -1.98 (1) | 4.41 (2,0, 0) | -1.25 (1) | -1.99 (1) |
| Sweden | 18.22 (2,1, 0) | -1.56 (1) | -1.83 (1) | 330.65 (2,0,-1) | ***-1.99*** (1) | -2.69 (1) |
| Switz. | 54.13 (1,1,-1) | -1.63 (1) | -2.32 (1) | 47.63 (2,2,-1) | 0.38 (6) | -0.80 (7) |
| Turkey | 8.86 (1,0, 0) | -0.95 (0) | -1.52 (0) | 4.09 (1,0, 0) | -1.18 (0) | -2.45 (0) |
| UK (G5) | 1.90 (2,0, 0) | -1.58 (1) | -2.05 (0) | 2.64 (2,0, 0) | -1.38 (1) | -2.11 (1) |
| REER | 15.98 (2,0, 0) | -1.44 (1) | -1.51 (1) | 22.08 (1,0, 0) | -1.20 (7) | -1.54 (7) |

| | Other exchange rates | | |
|---|---|---|---|
| | Odds ( $\hat{p},\hat{q},\hat{\ell}$ ) | DF-GLS ($d$) | |
| UK-Fra. (G5) | 2.63 (1,0, 0) | -1.34 (2) | -2.04 (0) |
| UK-Jap. (G5) | ***0.90*** (1,1, 0) | -1.97 (1) | -2.88 (1) |
| UK-Can. | 4.89 (2,0, 0) | -0.66 (1) | -2.53 (1) |
| Jap.-Fra. (G5) | 394.42 (2,0,-1) | -0.85 (4) | -2.16 (0) |
| Jap.-Can. | 180.93 (1,1,-1) | -0.45 (3) | -2.73 (1) |
| Fra.-Can. | 253.58 (2,0,-1) | -1.42 (1) | -2.49 (1) |

Notes: "Odds ( $\hat{p},\hat{q},\hat{\ell}$ )" gives the Bayesian odds in favor of a unit root with the PIC-selected AR, MA, and time orders in parentheses. The first column of "DF-GLS (*d*)" gives the results with constant only, and the second column gives the results with constant and trend; (*d*) gives the number of first differences. Odds not in favor of the unit root, and DF-GLS statistics significant at the 0.05 level (<-1.98) are highlighted in boldface-italic. "G5" indicates exchange rates where both countries are G5 members.

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