E C O N O M I C S B U L L E T I N

Reconsideration of the Crowding−out Effect with Non−linear Contribution Technology

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Abstract

In this paper we reconsider the completely crowding−out effect in a model of the private provision of public goods with non−linear technology for government contributions. Even though there are no free−riders, government contributions financed by lump−sum taxes crowd out private contributions only marginally. We also investigate the relationship between desirable government policies and country size (the number of individuals). We show that equilibrium government contributions are unaffected by the change in the number of individuals in a no−free−rider economy.

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1. Introduction

What should the government do in an economy in which public goods are voluntarily provided by private individuals? Warr (1982) and Roberts (1984) derived the well-known result that government contributions to public goods, financed by lump-sum taxes, will crowd out private contributions on a one-to-one basis. This '100% crowding-out theorem', however, runs counter to the empirical observations that government contributions can only incompletely crowd out private contributions (e.g., Abrams and Schmitz (1978)).[1]

By way of explaining this discrepancy, Andreoni (1988, 1989) suggested the model of 'impure altruism': individuals are assumed to care not only about aggregate contributions, but also about their own contributions. Bergstrom et al. (1986) showed that the crowding-out effect of government contributions would be partial in a case in which there were some non-contributors (or free-riders). In this paper, we propose another model to explain the discrepancy: the government is assumed to have a technology for providing public good that differs from that of private individuals.[2]

We consider a two-stage contribution game. In the first stage, the government decides the level of lump-sum taxes and the provision of public goods. In the second, stage private individuals simultaneously decide their contributions. We assume that the government has a non-linear (concave) production function for public goods. Consideration of the government as a group justifies this feature: the larger the group size, the lower its effectiveness. On this point, for example, Olson (1965, p.48) suggested that 'the larger the number of members in the group the greater the organization costs, and thus the higher the hurdle that must be jumped before any of collective good at all can be obtained.' This property of the government plays a key role in our analysis.

One application of our model concerns environmental problems, such as the treatment of waste or garbage. In recent years, voluntarily reducing and recycling wastes, along with an increasing awareness of the environment, has become popular in the private sector (households, firms, volunteer group). The government deals with waste treatment on a larger scale; for example, by constructing waste treatment plants. It is reasonable that the technology of such government activity is not linear because of constraints on land, the number of officials involved in policy making, and the disposal capacity. Considering environmental quality (or the amount of waste) as a public good (or bad), our analysis can be applied to these types of problems.

Within the above framework, we first investigate the impact of government contributions on aggregate contributions and on each individual's utility. We find that there can be

[1]If the government designs distorted tax (or subsidy) policies, the aggregate contributions can be changed. See, for example, Andreoni and Bergstrom (1996).

[2]Buchholz and Konrad (1995) and Ihori (1996) incorporated productivity differentials *among individuals* into the standard model to provide some interesting results. However, since their investigations focused on the context of provision of international public goods, the government contribution (or crowding-out property) was not examined.

government policies which maximize the aggregate contributions and contributor's utility. The theorem of complete crowding-out holds only in such a case; however, it holds only marginally.

Second, we consider the relationship between the equilibrium government policies and the number of individuals. How do the desirable government policies change as the size of the country grows? We answer this question by means of comparative statics, and show the property of 'size neutrality', which means that equilibrium government contributions are independent of country size.

The organization of this paper is as follows. Section 2 presents the model. We derive the backwards-induction outcome (subgame-perfect Nash equilibrium). Section 3 investigates the property of equilibrium government policies. Section 4 concludes the paper.

2. The model

We consider a model where there is one pure public good, one private good, a government, and n ($n \geq 2$) individuals. The utility of individual i is represented by a continuous and strictly quasi-concave function $U_i = U^i(x_i, G)$, where x_i is the private consumption of individual i and G is the total provision of the public good. We assume that the private and public goods are strictly normal. The budget constraint of individual i is given by $x_i + g_i = w_i - \tau$, where g_i is the voluntary contributions to the public good made by individual i, w_i is the exogenously given income of individual i, and τ is the lump-sum taxes imposed by the government. Aggregate contributions are given by

$$G = \sum_{i=1}^{n} \beta_i g_i + e, \tag{1}$$

where β_i is individual i's productivity for providing public good and e is the amount of government contributions. Notice that the contribution technology of each individual is linear.

The government collects lump-sum taxes from each individual and spends the cumulative amount collected from individuals for providing public good with non-linear technology,

$$e = f(R), \tag{2}$$

where R ($= n\tau$) denotes government revenue.

Assumption *$f : \mathbb{R}^+ \mapsto \mathbb{R}^+$ is strictly increasing, strictly concave, and twice continuously differentiable in R with $f(0) = 0$, $f' > 0$, $f'' < 0$, and $f'(0) > \max\{\beta_1, \beta_2, \cdots, \beta_n\}$.*

Assumption 1 represents that the productivity of the government in contributing public good satisfies the low of diminishing marginal productivity; the larger the government size, the lower its effectiveness for providing public good. The assumption $f'(0) > \max\{\beta_1, \beta_2, \cdots, \beta_n\}$ requires that the marginal productivity of the government is higher than any individual's productivity in the neighborhood of $R = 0$. If we assume $f'(0) < \min\{\beta_1, \beta_2, \cdots, \beta_n\}$, it is obviously better for the government to do nothing. Therefore this assumption represents the necessary condition for justifying government activities.[3]

The game runs as follows. In the first stage, the government chooses τ and e so as to maximize aggregate contributions and the individual's utility. In the second stage, each individual observes τ, and then simultaneously chooses g_i and x_i so as to maximize his/her utility subject to the budget constraint.

2.1 Nash equilibrium at the second stage

The game is solved by backward induction. First, we derive a Nash equilibrium at the second stage. Substituting (1) into the budget constraint, we have

$$x_i + \frac{G}{\beta_i} = w_i - \tau + \frac{G_{-i}}{\beta_i}, \tag{3}$$

where $G_{-i} = \sum_{j \neq i}^{n} \beta_j g_j + e$ is the total contribution of others. The individual i's problem is to maximize his/her utility subject to (3), given G_{-i} and τ.

Definition *An individual $i \in C$ is said to be a **contributor** if and only if $g_i > 0$ in some equilibrium, and C is the set of contributors. An individual $j \notin C$ is said to be a **free-rider** if and only if $g_j = 0$ in some equilibrium.*

The first-order conditions for contributor $i \in C$ are

$$\frac{\partial U^i(x_i, G)}{\partial x_i} = \beta_i \frac{\partial U^i(x_i, G)}{\partial G} \quad \text{and} \tag{4a}$$

$$G = \beta_i(w_i - \tau - x_i) + G_{-i}. \tag{4b}$$

Since (4a) implicitly defines x_i as a function of G, we rewrite it as $x_i = \psi^i(G)$, where $\psi^i : \mathbb{R}^+ \mapsto \mathbb{R}^+$ is strictly increasing with $\psi^i(0) = 0$. From (4b), we have

$$G + \beta_i \psi^i(G) = \beta_i(w_i - \tau) + G_{-i} \quad \forall i \in C. \tag{5a}$$

Because free-rider $j \notin C$ contributes nothing ($g_j = 0$), we have

$$G = G_{-j} \quad \forall j \notin C. \tag{5b}$$

[3] Olson (1965, p.7) pointed out that "there is obviously no purpose in having an organization when individual, unorganized action can serve the interests of the individual as well as or better than an organization."

Summing (5a) and (5b), aggregate contributions in the Nash equilibrium at the second stage (G^*), contributor i's indirect utility ($V_{i\in C}$), and free-rider j's indirect utility ($V_{j\notin C}$) are characterized as follows:

$$G^* + \sum_{i\in C}^{n} \beta_i \psi^i(G^*) = \sum_{i\in C}^{n} \beta_i (w_i - \tau) + f(n\tau), \tag{6a}$$

$$V_i = U^i(\psi^i(G^*),\, G^*) \quad \forall i \in C, \quad \text{and} \tag{6b}$$

$$V_j = U^j(w_j - \tau,\, G^*) \quad \forall j \notin C. \tag{6c}$$

Since we assume that both the private and public goods are strictly normal, a unique Nash equilibrium can be shown to exist.[4]

2.2 The choice of the government

We next turn to the government problem at the first stage. The purpose of the government is to maximize the total provision of public good and the utility of each individual by adjusting the level of lump-sum taxes. Differentiation of (6a), (6b), and (6c) with respect to τ yields the following Proposition 1.

Proposition 1 *If the government sets τ^* and e^* which satisfy*

$$f'(n\tau^*) = \frac{\sum_{i\in C}^{n} \beta_i}{n} \quad \textit{and} \tag{7a}$$

$$e^* = f(n\tau^*), \tag{7b}$$

then G^* and $V_{i\in C}$ are maximized by these policies.

Proof See Appendix.

The left-hand side of (7a) is the marginal productivity of the government in providing public good and the right-hand side is the average productivity among contributors.[5]

The subgame-perfect Nash equilibrium of this game consists of the equations (6a), (6b), (6c), (7a), and (7b).

[4] This can be shown by applying the proof found in Bergstrom et al. (1986, 1992).

[5] If the equilibrium level of lump-sum taxes is larger than the income of the poorest individual (i.e., $\tau^* \geq \min\{w_1, w_2, \cdots, w_n\}$), the taxation is impossible to be levied. We exclude such a case for the sake of simplicity.

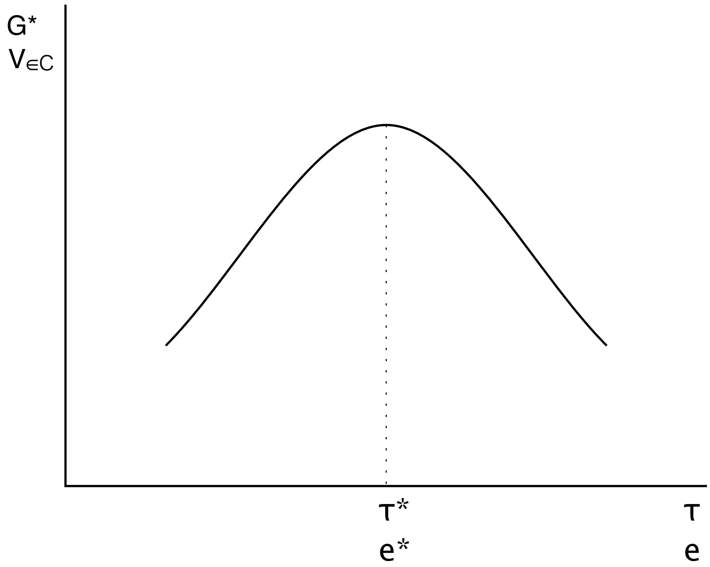

Figure 1: Marginal crowding-out effect

3. Equilibrium analysis

In this section we investigate the government policies in the subgame-perfect Nash equilibrium derived in Section 2.

First, we investigate whether the theorem of complete crowding-out holds.

Proposition 2 *Suppose that all individuals are positive contributors. Then the government contributions crowd out private contributions* ***only marginally****.*

This result is almost obvious since Proposition 1 shows $\partial G^*/\partial\tau^* = \partial V_{i\in C}/\partial\tau^* = 0$.

We depict the relationship between τ (or e) and $V_{i\in c}$ (or G^*) in Figure 1. At the point $\tau = \tau^*$ (or $e = e^*$), a marginal increase in τ (or e) is offset dollar-for-dollar by marginal reductions in private contributions. If the government sets $\tau < \tau^*$ (or $e < e^*$), the crowding-out effect is less than 1. Similarly, if the government sets $\tau > \tau^*$ (or $e > e^*$), this policy reduces not only aggregate contributions but also each individual's utility.

Bergstrom et al. (1986, Theorem 6) clearly show that government contributions result in a 'dollar for dollar' reduction in private contributions when all individuals are contributors. However, in our model, the crowding-out effect is only marginal even if there are no free-riders. This feature of this model may explain the discrepancy between theoretical results and empirical results regarding the 100% crowding-out effect.

Second, we investigate the relationship between the equilibrium government policies and the number of individuals.

Proposition 3 *Suppose that all individuals are positive contributors and they all have the same productivity; i.e.,* $g_i > 0$ *and* $\beta_i = \beta \ \ \forall i$*. Then*

1. *the equilibrium contributions by the government is unaffected by a change in the number of individuals (that is,* $\mathbf{e}^*$ ***is independent of n****), and*
2. *the equilibrium level of lump-sum taxes decreases as the number of individuals increases, and vice versa.*

Proof See Appendix.

Proposition 3-1 can be interpreted as the property of *size neutrality*, which implies that government size is independent of country size. Intuition is as follows. When an individual increases, the government expects its revenue to rise by τ^*. At the same time, from concavity of the government production function, the government also expects that its marginal productivity will become lower than that of the individual. To satisfy condition (7a), the government lowers the level of lump-sum taxes. Thus, an increase in n is offset by a decrease in τ, and government contributions are unchanged. In other words, the government should not respond to a change in tax revenue by changing its contribution level, but by adjusting the lump-sum taxes.

Third, we take the free-rider into account in order to investigate the effects of a change in the number of free-riders on government policy.

Proposition 4 *An increase in the number of free-riders induces an increase in* τ^* *and* e^**.*

Proof See Appendix.

This result shows that an increase in the number of free-riders increases both τ^* and e^*. In other words, the government intervention is increasing as the number of free-riders increases. In addition, an increase in τ^* and e^* will induce contributors to become free-riders. Therefore, an increase in free-riders will cause an additional increase in the number of free-riders in an economy through an increase in tax burden.

4. Concluding Remarks

In this paper we incorporate the non-linearity of government contribution technology into the standard model of the private provision of public goods in order to reconsider the crowding-out effect. We show that even if all individuals are contributors, government contributions crowd out private contributions only marginally. Therefore, this feature may be a factor in the discrepancy between theoretical and empirical results regarding the crowding-out effect. In the sense that government contributions should not exceed the level represented by Proposition 1, this result also indicates a limitation on government policies that intend to improve the welfare of its citizens through government contributions.

Next, we examine the relationship between equilibrium government policies and country size. We show that if all individuals are contributors and have equal productivity, government contributions are determined independently of the number of individuals. This also indicates that government contributions should not be decided based on the number of individuals or government revenues, but on the government contribution technology.

One conceivable extension of our analysis is modification of the objectives of the government. In this paper we have assumed that the government intends to maximize not the utility of the free-rider, but of the contributor. Such government policies cannot be supported in an economy in which there are a great many free-riders. By using an appropriate social welfare function, we can investigate the properties of government policies more comprehensively.[6]

Appendix

Proof of Proposition 1

We rewrite (6a) as $G^* = \Omega(\tau, C)$, where $\Omega : \mathbb{R}^2_+ \mapsto \mathbb{R}_+$. The first-order condition for a maximum of G^* is

$$\Omega_\tau = \frac{\partial \Omega(\tau, C)}{\partial \tau} = \frac{n\, f'(R) - \sum_{i \in C}^n \beta_i}{1 + \sum_{i \in C}^n \beta_i \psi'(G^*)} = 0, \tag{8}$$

and the second-order condition is

$$\Omega_{\tau\tau} = \frac{\partial^2 \Omega(\tau, C)}{\partial \tau^2} = \frac{n^2 f''(R)}{1 + \sum_{i \in C}^n \beta_i \psi'(G^*)} < 0.$$

Equation (8) yields the condition (7a).

Then, from equation (6b), the first-order condition for a utility maximum for contributor $i \in C$ is

$$\frac{\partial V_i}{\partial \tau} = U_x^i \psi^{i'} \Omega_\tau + U_G^i \Omega_\tau = \Omega_\tau (U_x^i \psi^{i'} + U_G^i) = 0.$$

Thus, the condition (7a) also maximizes the contributor's utility. On the other hand, the first-order condition for a utility maximum for free-rider $j \notin C$ is

$$\frac{\partial V_j}{\partial \tau} = -U_x^j + U_G^j \Omega_\tau = 0.$$

[6] For example, Itaya et al. (1997) investigates the relationship between income distribution and social welfare. Myles (2000) also considers the question of whether public goods should be provided by the government or thorough private contribution in terms of social welfare.

Thus, the level of lump-sum taxes that maximizes the free-rider's utility is lower than τ^*; the government policy which satisfies condition (7a) cannot maximize the free-rider's utility. **Q.E.D.**

Proof of Proposition 3

Supposing that all individuals are in the set C and $\beta_i = \beta \ \forall i$, equation (7a) can be rewritten as

$$f'(n\tau^*) = \beta.$$

Since the variable β is constant, an increase in n is offset by a decrease in τ^*, and the left-hand side does not change as a whole. Therefore, the equilibrium government contribution e^* does not change. **Q.E.D.**

Proof of Proposition 4

Differentiating (7a) with the number of free-riders ($n_{\notin C}$) and rearranging it yields

$$\frac{d\tau^*}{dn_{\notin C}} = -\frac{f'(R) + R\,f''(R)}{n^2 f''(R)} > 0,$$

where $f'(R) + R\,f''(R) > 0$ comes from concavity of the government production function. Given that the function f indicates monotonicity increasing in n and τ^*, $de^*/dn_{\notin C} > 0$ is easily derived. **Q.E.D.**

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