E C O N O M I C S B U L L E T I N

A useful graphical method under Cournot competition

Patrice Cassagnard
Université Montesquieu−Bordeaux IV−LAREefi

Abstract

This note proposes a graphical approach useful in game theory. This method consists in representing incentives to move strategically to graphical areas. The method can be used on several occasions; we apply it as an example to the model of Bouët (2001).

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1 The method

We consider a Cournot duopoly. Two agents A and B compete in terms of quantity. The commodities are homogeneous and the inverse demand function, in a linear model, can be written as

$$\begin{aligned} p(a+b) &= \theta - k \cdot (a+b) \\ \theta &> 0;\ k > 0 \end{aligned}$$

The marginal cost of production (c) is supposed constant; thus the payoff of A is

$$\pi = a \cdot [\theta - k \cdot (a+b)] - c \cdot a \tag{1}$$

implying a linear reaction function f whose slope is equal to (-2)

$$a = f(b)$$

Equation (2) can easily be obtained from the first order condition of (1).

$$\pi(\hat{a}, b) = k \cdot \hat{a}^2 \tag{2}$$

For each point which pertains to reaction function of A, there is a level of profit which depends only on the quantity produced by B (a is fixed on its optimal level $\hat{a}$, for each quantity produced by the agent B).

If we choose two unspecified points (δ and ε) of the reaction function $\mathcal{R}$, the variation of the profits obtained by A, between these two points[1] is written:

$$\pi_\delta - \pi_\varepsilon = k \cdot \left(a_\delta^2 - a_\varepsilon^2 \right)$$

then,

$$\pi_\delta - \pi_\varepsilon = k \cdot (a_\delta - a_\varepsilon) \cdot (a_\delta + a_\varepsilon) \tag{3}$$

The slope of the reaction function is in this case (-2), and consequently equation (3) becomes

$$\pi_\delta - \pi_\varepsilon = k \cdot \left(-\frac{1}{2} \cdot b_\delta + \frac{1}{2} \cdot b_\varepsilon \right) \cdot (a_\delta + a_\varepsilon)$$

[1] Subscript denotes the corresponding point.

$S = \frac{1}{2} \cdot (b_\varepsilon - b_\delta) \cdot (a_\delta + a_\varepsilon)$ is the surface of the trapezoide $(b_\varepsilon \varepsilon \delta b_\delta)$. This surface is shaded in figure 1.Then,

$$\pi_\delta - \pi_\varepsilon = k \cdot S$$

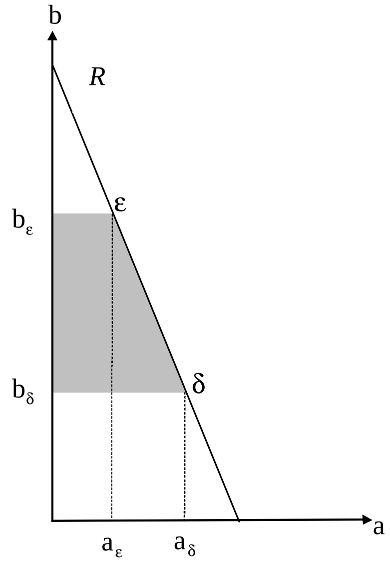

Figure 1: Graphical method: general case

2 Application to the model of Bouët (2001)

The model of Bouët (2001) corresponds to the framework clarified in the previous section, but applied to a North-South Cournot duopoly. First only the North can invest in a cost reducing R&D activity and then both compete in quantity. The issue of the investment is uncertain; on the one hand, $prob(c = c_b) = \alpha(r)$: the probability ($prob$) that the North obtains the low marginal cost (c_b) in the last stage of the game is a function of the volume of R&D investment r. On the other hand, $[1 - \alpha(r)]$ is the probability that the high marginal cost of production (c_h) is obtained. This probability is endogenously determined since it depends positively on the volume of investment in R&D (r). Depending on the success or the failure of the investment, the reaction function is respectively $\mathcal{R}_b$ or $\mathcal{R}_h$. A not very binding VER (voluntary export restraint) modifies only one of the equilibria: N_h is replaced by N_z, as depicted in Figure 2. The method, suggested in the first section indicates that, in case of failure, the VER induces an increase of the profit of the North. This increase reduces the incentive to invest in R&D activity, which is represented by S_1 in figure 2 ($k = 1$ in this case). By increasing only the profit corresponding to an unfavourable issue of the investment (the profit in case of success remains the same), the VER slows down the incentive to invest in R&D (proposition 2 of Bouët, 2001).

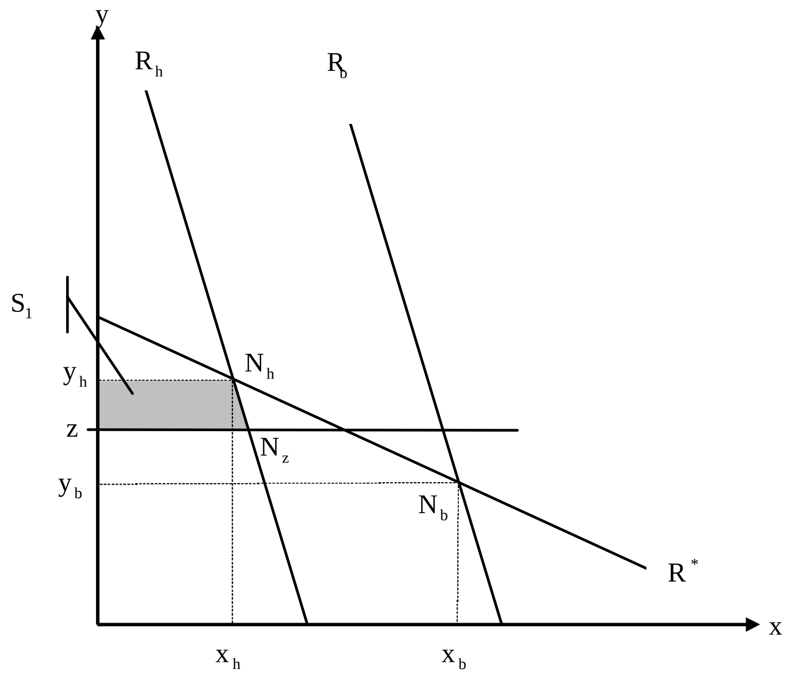

Figure 2: Application to proposition 2 (Bouët, 2001: 328)

Conversely, in the presence of a specific tariff, the reaction function of the South moves from $\mathcal{R}^*$ to $\mathcal{R}^{\tau *}$. Both free trade Nash equilibria are now modified. The first one implies a decrease of the incentive to innovate (S_2), whereas the second one induces an increase of the incentive to innovate (S_3).

$S_3 > S_2$, then proposition 3 of the paper of Bouët is quickly found.

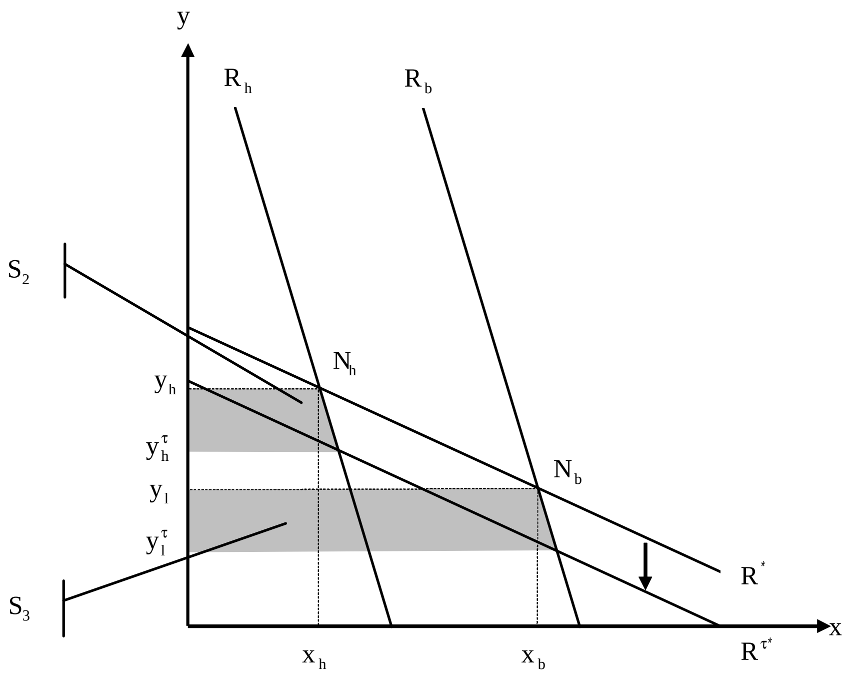

Figure 3: Application to proposition 3 (Bouët, 2001: 332)

References

[1] Bouët, A., 2001, "Research and Development, Voluntary Export Restriction and Tariffs", European Economic Review 45, 323-336