E c o n o m i c s B u l l e t i n

Sense of impartiality

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Abstract

The distribution of indivisible good in a society has social implications on individual behavior. In this paper, I present a model of choice which permit the quantification of the sense of impartiality. This model has implication in the choice of a winner of an indivisible good among a group of eligible individuals

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1. INTRODUCTION

A problem, central to economic theory is that of specifying the most desirable distribution of goods and services in an economy. The problem of distributional equity and any basic principle of distributive justice underlie the traditional normative economics. A difficulty with much of this work in this area has been that is ultimately involved interpersonal comparisons of utility (subjective evaluations of the utility that someone else derives from a particular state of economy). Since Arrow's impossibility theorem, a considerable literature refusing interpersonal comparisons has appeared over the last 23 years. The aim of this literature concerns the "envy-free" allocation (Foley (1967)) which is the concept that has played the central role, that is, an allocation such that nobody prefers what someone else receives to what he receive. A few alternative criteria are proposed and compared to alternative approaches to no envy by Pazner and Schmeidler (1978) developed by Berliant, Dunz and Thomson (1992) namely "egalitarian equivalence". According to these authors, an allocation is egalitarian equivalent if there is a "reference bundle" that each agent find indifferent to the bundle to receive. The central idea underlying egalitarian equivalence namely that an allocation can be meaningfully evaluated by reference to hypothetical economies, has been generalized in a number of ways. However, tests based on reference to hypothetical economies in which all agents access to the same choice set have been formulated, leading to the notion "equal opportunity equivalence" (Thomson (1994) Fleurbaey (1995) Roemer (1996) and Bossert (1997)). The references to economies where all agent have the same preferences underlies the so called "identical preferences lower bound" or "identical preferences upper bound" varies on whether it gives a lower bound or a upper bound depends on the model, in particular whether public good are present; Moulin (1990) Bevia (1993, 1996). This last notion are meaningful in non classical economies such as economies with indivisible goods (Svensson (1983) Tadenuma and Thomson (1991)), economies with public goods, exchange production and economies in which what has to be divided is non homogeneous (time, land etc.). In other hand, criteria on reference to the agent preference (Moulin and Sprumont (1998) "welfare domination under preference replacement"), economies in which all agent's productivity are the same share also appeared (Fleurbaey and Maniquet (1999)). Other studies concerns the fairness of process as opposed to that of end results, this literature address the issue of accommodating talent or handicap (Fleurbaey (1991)), consequences, opportunities, capability and procedures.

My present study seeks to develop the concept of impartiality as a measure of the distributional equity of allocations in private indivisible goods. I consider a society in which few individuals require kidney transplant and assume that only one kidney is available. I suppose that all claimers have the same relevant characteristics (that is, they are the same age, have the same health status and have the similar family social responsibility, talent and compensation) and thus have equal claims to the goods. As Karni and Safra (2000) I beg the question of how the society ought to allocate the indivisible good according to individual's claim on it?. Instead, the prevalent attitude in economics is to model individual behavior as motivated solely by material self-interest. Yet, there are many remarkably stable institutions whose existence and functioning seems inconsistent with this model of human behavior. Disregarding the possibilities of disposal of the indivisible good or compensation, I present an axiomatic model of self-interest seeking moral individuals. More specifically, I consider a theory of individual choice behavior among random allocation procedures incorporating impartiality. This study involves interpersonal comparisons of the utility, of course the very notion of impartiality and preferences are difficult to define and difficult philosophical issues may quickly arise. Harsanyi (1955), Kolm (1972), Ralws (1971) Dworkin (1981), Arneson (1989) among others point to different ways in which these notions can be defined for the purpose of a fair distribution of allocation. In this paper, I will not advocate a particular definition of fairness (versus impartiality), and simply assume that a separation between the individual characteristics that lie in the self-interest preference and the fairness preference. Then assuming that such a separation is done, two main ethical principles seems to be at work. First the self-interest preference is inferred from individual actual choices whereas the fairness preference may be inferred from hypothetical choice among allocations procedure from behind a veil of ignorance. As Karni and Safra's (2000), my approach to modeling takes the impartiality and the preferences relations as primitives and derives the self-interest relation. My approach in the paper is analogous to that used by Pratt (1964), Arrow (1965) and Kimball (1990) in the development of measures of risk aversion and prudence in the theory of individual making choice under risk. As in the theory of risk aversion and prudence, the measure of the intensity of the sense of impartiality involve two distinct steps. The first is to define the sense in which one individual may be considered to have a stronger sense of impartiality than another does. Next, the second step is to characterize this relation in terms of the properties of the corresponding utility function. At this stage other difficulty is to take account of the multidimensionality of the choice space and the possible disagreement of the ordinal preferences of the individuals being compared (see Kihlstrom and Mirman (1974), Duncan (1977), Ambarish and Kallberg (1987) and Chalfant and Finkelstein (1991)). The issues of the ordinal and cardinal comparability of the sense of impartiality are considered by restricting the comparisons to individuals who agree on the definition of impartiality and whose selfish preferences are ordinally comparable. Finally, I show that the measure of the intensity of the sentiment of impartiality have intuitive appealing behavioral characterizations. Possessing stronger sense of impartiality is equivalent to having a lower gap between the fairer allocation procedures choosing without risk and in the presence of uncertainty.

In the next section I present the model, section 3 develop measure for interpersonal comparisons of additive utility of the sense of impartiality, then concluding remark appear in the end.

2 THE MODEL.

I use the analytical framework and main results in Karni and Safra. A society consisting of $N=(1,...,n)$, $n\geq 2$ eligible individuals faces the need to allocate among its members one unit of indivisible goods. Denote by e^i, the unit vector in R^n, the ex post allocation in which the individual i is assigned the good. Let $\{e^i/1\leq i\leq n\}$ be the set of ex post allocations and $P=\Delta(X)$, be the $(n-1)$ dimensional simplex representing the set of all probability distribution on X. P has the interpretation of the set of allocation procedures. Assumed that P is endowed with the R^{n-1} topology. Two transitive, complete and continuous binary relations on P represent an individual: the relation $\succeq$, representing his actual choice of behavior. The empirical meaning of $\succeq$ may be inferred from actual choice; it's the classical relation used in most economic field. The relation of impartiality $\succeq_I$, may be inferred from hypothetical choice among allocation procedures from behind of veil of ignorance or real choices among allocation procedures for a set of individuals which does not include the decision maker himself and among whose members he is impartial.

3.1 UTILITY REPRESENTATIONS

As Karni and Safra (2000), I take the preference relation and the impartiality relation as primitive, and I derive the selfish motive implicit in the individual choice behavior. An allocation procedure p is preferred over another allocation procedure q from selfish point of view, if the two allocation procedures are equally fair and p is preferred over q. let $\succeq_S$, denote the derived binary relation representing the self-interest component of the preference relation $\succeq$. Consider a set of axioms (including quasi-concavity of $\succeq_I$) that is equivalent to the existence of an affine function

$k : P \to R$, representing $\succeq_S$

a function

$s : P \to R$, strictly quasi-concave on int P representing the impartiality relation $\succeq_I$, and a utility function U, representing the preference relation $\succeq$ that is a function of a self-interest and the fairness representations. More specifically, we assumed the existence of a function

$$U':\{(k.p, s(p))/ \ p \in P\} \to R$$

such that for all allocation procedure $p, q \in P$,

$$p \succeq q \Leftrightarrow U'((k.p, s(p)) \geq U'(k.q, s(q)))$$

The function U', is additively separable in the self-interest and impartiality components.
Now assume that there are axioms that are equivalent to the existence of a monotonic increasing function/

$h : R \to R$, such that for all p, q $\in P$

$$p \succeq q \Leftrightarrow h(k.p) + s(p) \geq h(k.q) + s(q)$$

this representation is unique in the sense that if $(\tilde{h}, \tilde{k}, \tilde{s})$ is another triple of functions corresponding to $\succeq, \succeq_S$ *and* $\succeq_I$ as above then $hok = b\tilde{h}o\tilde{k} + a,$ *and* $s = b\tilde{s} + c,\ b > 0.$

3.2 INTERPERSONAL COMPARISONS WITH ADDITIVE UTILITY

In this part, I study the meaning of the relation "possessing a stronger sense of impartiality", develop alternative representations of this relation, and depict its behavioral characterization. Assuming that U is twice differentiable and U''' exists. In contrast of Karni and Safra, who only used the Pratt and Arrow measures of risk aversion, I add the notion of prudence. Indeed, although the notion of risk aversion is very rich, it's not suitable for answering questions that depend on the effect of risk on marginal utility rather than total utility. My approach of impartiality is analogous to Kimball (1990) concept of prudence. This last notion offers an economic interpretation that is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to risk aversion which is how much one dislikes uncertainty and would turn away from uncertainty if one could. My approach to quantify the sense of impartiality is to measure the willingness to sacrifice one's material interest to attaint fairer allocation procedures. A formal definition of impartiality is to define an impartiality-premium, which require an explicit consideration of the utility representation. In the sequel, I define the notion of impartiality premium in the context of the additive utility models and show that it constitutes a criterion of interpersonal comparison of the sense of impartiality. I define the most impartial agent 's as one who makes the lowest gap choice between situation without uncertainty and the others involving uncertainty.

3.3 DEFINITION 1 AND NOTATIONS

Let the following sets for the preference relation $\succeq$: for each $p \in P,\ B(p) = \{q \in P / q \succeq p\}$, $W(p) = \{q \in P / p \succeq q\},\ I(p) = B(p) \cap W(p)$. Similarly, I define $B_I(p), W_I(p)$ *and* $I_I(p)$ for the impartiality relation $\succeq_I$. For another pair of preference relation and impartiality relation $(\hat{\succeq}, \hat{\succeq}_I)$. I denote the corresponding sets by $\hat{B}(p), \hat{B}_I(p)$ etc. The preference relation and impartiality relation$(\succeq, \succeq_I)$, $(\hat{\succeq}, \hat{\succeq}_I)$ on P are said to be comparable if they incorporate the same idea of impartiality and the same view of what constitutes self-interest. Next, we assume that $(\succeq, \succeq_I)$, $(\hat{\succeq}, \hat{\succeq}_I)$ are said to be comparable if $(\hat{\succeq}_I = \succeq_I)$ and $(\succeq_S = \hat{\succeq}_S)$.

DEFINITION 2

I admit analogous to Yaari's (1969) definition of the partial ordering "totally more risk averse than" in the theory of decision making under risk. Let the preference-impartiality relations $(\succeq, \succeq_I)$, $(\hat{\succeq}, \hat{\succeq}_I)$ be comparable, then the preference-impartiality relation $(\succeq, \succeq_I)$ is said to possess a stronger sense of impartiality than $(\hat{\succeq}, \hat{\succeq}_I)$ if, for every allocation procedure $p \in P$

$A(p) = B(p) \cap W_I(p) \subseteq \hat{A}(p) = \hat{B}(p) \cap \hat{W}_I(p)$. For comparable preference impartiality relations, I assume that one preference relation as displaying a stronger sense of impartiality than another if, given any allocation procedure p, every other allocation procedure that is less fair than p and is acceptable to the former is acceptable to the latter.

3.4 IMPARTIALITY-PREMIUM

Assume that $(\succeq, \succeq_I)$ and $(\hat{\succeq}, \hat{\succeq}_I)$ be comparable preference-impartiality relations with corresponding functions (h, k, s) and $(\tilde{h}, \tilde{k}, \tilde{s})$ respectively. Comparability means that there exist monotonic increasing functions f *and* g satisfying $\hat{h} = fog$ *and* $\hat{s} = gos$. To define the impartiality in this model, I fix an initial allocation p and suppose that is perturbed by a variation v. Let $E(p) \subset R^n$, be the set of variation v satisfying $p + v \in P$. I define the impartiality-premium y, as satisfying:

$$y : \{(p, v) / p \in R^n \text{ and } v \in E(p)\} \to R,$$

and for which the preference-impartiality relations $(\succeq, \succeq_I)$, y verifies the relation:

$$h'(k.(p+v) - y(p,v)) + \nabla s(p) = h'(k.(p+v)) + \nabla s(p+v)$$

3.5 THE CONCEPT OF IMPARTIALITY AND IT'S RELATIONSHIP WITH FAIRNESS.

Consider the Karni and Safra's fairness-premium function

$$p : \{(p, v) / p \in R^n \text{ and } v \in E(p)\} \to R$$

and for which the preference-fairness relation $(\succeq, \succeq_F)$, is defined by

$$p(p, v) = k.(p+v) - k.q$$

and they assume that

$$h(k.(p+v) - p(p,v)) + s(p) = h(k.(p+v)) + s(p+v)$$

before proceeding, I there exists a connected path $p(.)$ satisfying $p(0) = p$ and $p(1) = q$, for all $b \in [0, 1]$ we have:

$$h(k.(p(b)+v) - p(p(b),v)) + s(p(b)) = h(k.(p(b)+v)) + s(p(b)+v) \qquad (3.1)$$

Now, differentiate (3.1) with respect to $p(b)$, and using the impartiality definition, I obtain

$$h'(k.(p(b)+v) - p(p(b),v)) \cdot \left(k - \frac{\partial p(p(b),v)}{\partial p(b)}\right) + \nabla s(p(b)) \cdot p'(b) =$$

$$h'(k.(p(b)+v) - y(p(b),v)) + \nabla s(p(b)) \cdot p'(b)$$

in fact, it's easy to show by the above equality that $h'(k.(p(b)+v)-p(p(b),v))\cdot\left(k-\frac{\partial p(p(b),v)}{\partial p(b)}\right) = h'(k.(p(b)+v)-y(p(b),v))$

from p is closely related to y by the equality:

$$\begin{cases} y(p(b),v)=p(p(b),v) \ \ if \ \left(k-\frac{\partial p(p(b),v)}{\partial p(b)}\right)=1 \\ y(p(b),v)\geq p(p(b),v) \ \ if \ \left(k-\frac{\partial p(p(b),v)}{\partial p(b)}\right)\geq 1 \\ y(p(b),v)\leq p(p(b),v) \ \ if \ \left(k-\frac{\partial p(p(b),v)}{\partial p(b)}\right)\leq 1 \end{cases}$$

DEFINITION 3

An allocation procedure q is a path-utility increasing reduction in impartiality relative to p given (h,k,s) if there exists a connected path $p(.)$ satisfying $p(0)=p$ and $p(1)=q$, for all $b\in[0,1]$ we have:

$$h''(k.p(b))\cdot k.p'(b)db+s''(p(b))\cdot p'(b)db \geq 0$$

and

$$s''(p(b))\cdot p'(b)db \leq 0$$

LEMMA 1

Let $(\succeq,\succeq_I)$ and $(\hat{\succeq},\hat{\succeq}_I)$ be comparable preference-impartiality relation with corresponding functions (h,k,s) and $(\tilde{h},\tilde{k},\tilde{s})$ respectively. Suppose that the functions f *and* g defined by $\hat{h}=fog$ *and* $\hat{s}=gos$ satisfy for every p; $f''(h(k.p))\geq g''(s(p))$. If q is a path-utility increasing reduction in impartiality relative to p given (h,k,s) then it's utility increasing reduction in impartiality relative to p given $(\tilde{h},\tilde{k},\tilde{s})$.

Proof

Let q be a path-utility increasing reduction in impartiality relative to p given (h,k,s) and let $p(.)db$ is an infinitesimal utility increasing reduction in impartiality relative to $p(b)$ given (h,k,s). Then invoking the fundamental theorem of calculus, I have:

$$\left[\hat{h}'(k.q)+\hat{s}'(q)\right]-\left[\hat{h}'(k.p)+\hat{s}'(p)\right]=$$
$$[f'(h(k.q(b)))h'.(k.q(b))k+g'(s(q(b)))\nabla s(q(b))]\cdot q'(b)-$$

$$[f'(h(k.p(b)))h'.(k.p(b))k+g'(s(p(b)))\nabla s(p(b))]\cdot p'(b)$$

$$= \int_0^1 \left[f''(h(k.p(b)))[h'.(k.p(b))k]^2 + f'(h(k.p(b)))h''.(k.p(b))k^2 + g''(s(p(b)))\nabla s(p(b)) + \right] (p'(b)db)^2 +$$

$$\int_0^1 [g'(s(p(b)))s''(p(b))](p'(b)db)^2 +$$

$$\int_0^1 [f'(h(k.p(b)))h'(k.p(b))k + g'(s(p(b)))\nabla s(p(b))]p''(b)db$$

rearranging the terms I have

$$= \int_0^1 g''(s(p(b))) \left[\frac{f''(h(k.p(b)))}{g''(s(p(b)))} [h'.(k.p(b))k]^2 + [\nabla s(p(b))]^2 \right] (p'(b)db)^2 +$$

$$\int_0^1 f'(h(k.p(b))) \left[h''.(k.p(b))k^2 + \frac{g'(s(p(b)))}{f'(h(k.p(b)))} s''(p(b)) \right] (p'(b)db)^2 +$$

$$\int_0^1 f'(h(k.p(b))) \left[[h'(k.p(b))k] + \frac{g'(s(p(b)))}{f'(h(k.p(b)))} \nabla s(p(b)) \right] p''(b)db$$

collecting the results, one obtains:

$$\leq \int_0^1 g''(s(p(b))) \left[[h'.(k.p(b))k]^2 + [\nabla s(p(b))]^2 \right] (p'(b)db)^2 + \quad (A)$$

$$\int_0^1 f'(h(k.p(b))) \left[h''.(k.p(b))k^2 + s''(p(b)) \right] (p'(b)db)^2 + \qquad (B)$$

$$\int_0^1 f'(h(k.p(b))) [[h'(k.p(b))k] + \nabla s(p(b))] p''(b)db \qquad (C)$$

This last inequality follows from

$g''(s(p(b))) \geq f''(h(k.p(b)))$ *for A*, $f'(h(k.p(b))) \geq g'(s(p(b)))$ *for B and* $g'(s(p(b))) > 0$ *C*

Finally it's easy to see that

$$\int_0^1 g''(s(p(b))) \left[[h'.(k.p(b))k]^2 + [\nabla s(p(b))]^2 \right] (p'(b)db)^2 +$$

$$\int_0^1 f'(h(k.p(b))) \left[h''.(k.p(b))k^2 + s''(p(b)) \right] (p'(b)db)^2 +$$

$$\int_0^1 f'(h(k.p(b))) [[h'(k.p(b))k] + \nabla s(p(b))] p''(b)db \leq 0$$

then

$$\left[\hat{h}'(k.q) + \hat{s}'(q) \right] - \left[\hat{h}'(k.p) + \hat{s}'(p) \right] \leq 0$$

□

CONCLUSION

In this paper I presented an axiomatic model of choice depicting the choice behavior. The behavior is representable by an utility function that decomposes into linear function representing the material self-interest and a function representing the individual sense of impartiality. Without regarding particular definition of impartiality or fairness I propose in analogy with prudence, an impartiality-premium which involves a choice of the path in the space of allocation procedure. To choose the winner, impartial society or the decision-maker must consider individuals sacrifices of self-interest in situation without and involving uncertainty.

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