E C O N O M I C S B U L L E T I N

When do Cost Differentials among Privately Provided Public Goods make Income Transfer Policy Effective?

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Abstract

Some papers have disputed when cost differentials among privately provided public goods make income transfer policy effective. This paper clarifies the different assumptions underlying this disputation and shows that original cost equalization is a necessary and sufficient condition to hold the transfer neutrality.

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1. Introduction

This paper examines the condition under which the neutrality of income transfer policy on the allocation of goods holds (hereafter referred to as *the transfer neutrality*). It uses a model with privately provided public goods, under the presence of two types of cost differentials. Warr (1982, 1983) formulated the transfer neutrality theorem, which states that the income transfer is independent of the equilibrium allocation of goods in the standard model of the private provision of a public good. Following Warr, the theorem has been highlighted because it is important in the discussion of income transfer policy.[1] Some researchers derived the result that income transfer will not be completely neutral under some general assumptions.[2] In all such papers, it was implicitly assumed that contributors faced the same cost when they increased the amount of public goods marginally. However, contributors may face different costs of contributions. When agents contribute to the public good, costs usually differ between agents. It is important to analyze the effect of income transfers under this general situation. Following this point of view, some recent papers (Boadway et al., 1989; Buchholz and Konrad, 1995; and Ihori, 1996) focus on cost differentials among contributors.

However, the existing analysis of cost differentials is not sufficient to obtain general and clear results, as the neutrality result has not been consistent. Boadway et al. (1989) has shown that the transfer neutrality holds in the presence of the cost differentials. In contrast, Buchholz and Konrad (1995) and Ihori (1996) have shown that the neutrality will not hold. It is important to clarify the differences between these inconsistent conclusions so that the effect of income transfers can be clearly discussed.

The main difference underlying these seemingly inconsistent conclusions is the way that the cost differential is created. In the model developed by Boadway et al. (1989), government can subsidize the private provision of public goods and the cost differential is created by tax deductibility or matching grants. On the other hand, in the models of Buchholz and Konrad (1995) and Ihori (1996), a productivity differential exists before intervention by the government. It is important to clarify the characteristics of each cost differential, which is the purpose of this paper.

This paper is organized as follows. Section 2 presents the model and considers the non-cooperative Nash equilibrium of voluntary contributions to a pure public good. To compare existing models, section 3 derives the welfare effect of the income transfers and discusses the condition required to hold the neutrality.

2. The Model

Consider an economy with two types of heterogeneous regions, each of which

[1] See Bergstrom and Varian (1985) for the general theorem.

[2] For example, consider the case where some agents do not contribute at all or the case where altruism is not pure. See Bergstrom et al. (1986) for the case with non contributors and Andreoni (1989) for the case of impure altruism.

contributes to a pure public good, G.[3] We call this a *privately provided public good*. Therefore, the quantity of public goods in this economy becomes G.

Region i obtains utility from consumption, x^i, and public goods. Then the utility in region i is represented by the following utility function:

$$U^i = u^i(x^i, G), \tag{1}$$

where both x^i and G are assumed to be normal goods. Each local government, by setting the local lump sum tax, divides fixed endowments or incomes in each region into the contribution to pure public goods and consumption goods. It is assumed that each government in each region has a linear production frontier associated with public goods and consumption goods. Finally, each local government faces the following budget constraint:

$$x^i + c^i g^i = y^i - Tax^i, \tag{2}$$

where g^i is the level of public goods contributed by the local government in region i, c^i is an *original* cost, representing the productivity of public goods in region i before the intervention of the central government, y^i is exogenously given income in region i, and Tax^i is an effective tax burden. Since the public good contributed by the local government is pure, G becomes equal to the sum of the quantity of contributions in each region. That is:

$$G \equiv g^1 + g^2. \tag{3}$$

Next, consider the tax and subsidy schemes of the central government. The central government is assumed to control the income tax level and the subsidy rate on each contribution of public goods (or the grants tied to privately provided public goods). Then the effective tax burden in region i is represented as follows:

$$Tax^i = ty^i - \beta^i c^i g^i, \tag{4}$$

where t and β^i represent an income tax rate and a rate of subsidy to the public good provided by the local government i, respectively. We assume that $0 \le \beta < 1$. As shown below, when this subsidy rate is different between regions, the cost differential is created artificially.

Then the budget constraint of the central government becomes:

$$ty^1 - \beta^1 c^1 g^1 + ty^2 - \beta^2 c^2 g^2 = 0. \tag{5}$$

If the subsidy rate is assumed to be exogenous, this equation determines the level of the income tax endogenously.

Substituting equation (4) into (2), we obtain:

$$x^i + c^i g^i = y^i - (ty^i - \beta^i c^i g^i). \tag{2'}$$

Rewriting this, we have:

$$x^i + (1 - \beta^i) c^i g^i = (1 - t) y^i, \tag{6}$$

[3] Although we present an inter-regional transfer in the model, the same results would hold for a transfer between agents or an international transfer.

where $(1-\beta^i)c^i$ is the effective and actual cost of the public good, which is constructed by two costs, the *artificial* cost, $1-\beta^i$, and the *original* cost, c^i. Further, this equation means that the effective cost of the public good is artificially lowered by the subsidy from the central government. Using equation (3), the above equation can be rewritten as:

$$x^i + (1-\beta^i)c^i G = (1-t)y^i + (1-\beta^i)c^i g^{-i}.$$

Then each local government determines the public good provision and the level of consumption in each region, treating the effective cost of the privately provided public good, the income tax rate and the contribution in the other region, as given.[4]

In order to describe the Nash equilibrium model, the utility maximization behavior of each region is defined by using the following expenditure function:

$$\begin{aligned} &Minimize\, E^i \equiv x^i + (1-\beta^i)c^i G \\ &Subject\ \ to\ \ u^i(x^i, G) \geq U^i \end{aligned},$$

Then, the expenditure level is represented as the function of the effective cost and utility as follows:

$$E^i = E^i(U^i, (1-\beta^i)c^i).$$

Noting normalities of goods, the expenditure function has the following sign and characteristics under the framework of this paper:

$$E_U^i = x_U^i + (1-\beta^i)c^i G_U^i > 0 、 G_U^i > 0 、 x_U^i > 0.$$

Here $G^i(U^i, (1-\beta^i)c^i)$ and $x^i(U^i, (1-\beta^i)c^i)$ are the compensated demand function for the public good and consumption in each region, respectively. Using the definition of E^i, equation (6) becomes:

$$(1-t)y^i = E^i - (1-\beta^i)c^i G^i + (1-\beta^i)c^i g^i.$$

Then the budget constraints in each region are formulated as follows:

$$(1-t)y^1 = E^1 - (1-\beta^1)c^1 G^1(U^1, (1-\beta^1)c^1) + (1-\beta^1)c^1 g^1 \qquad (7\text{-}1)$$

$$(1-t)y^2 = E^2 - (1-\beta^2)c^2 G^2(U^2, (1-\beta^2)c^2) + (1-\beta^2)c^2 g^2. \qquad (7\text{-}2)$$

Since $G = G^i(U^i, (1-\beta^i)c^i) = g^1 + g^2$, equation (7-2) gives:

$$g^1 = \frac{1}{(1-\beta^2)c^2}(E^2 - (1-t)y^2)$$

Substituting this into equation (7-1) and subtracting g^1, we have:

$$(1-t)y^1 = E^1 - (1-\beta^1)c^1 G^1 + \frac{(1-\beta^1)c^1}{(1-\beta^2)c^2}(E^2 - (1-t)y^2),$$

which becomes:

$$(1-\beta^2)c^2(1-t)y^1 + (1-\beta^1)c^1(1-t)y^2 = (1-\beta^2)c^2 E^1 + (1-\beta^1)c^1 E^2 - (1-\beta^1)(1-\beta^2)c^1 c^2 G^1.$$

In addition, from equations (2)' and (5), we have the following resource constraint:

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[4] In order to focus on the role of cost differentials in the transfer neutrality theorem, we assume the case of the inner solutions, where all agents are contributors. In the case of the corner solution, where there exists non-contributor in the model, transfer neutrality does not necessarily hold. See Bergstrom et al. (1986).

$$y^1 + y^2 - x^1 - x^2 - c^1 g^1 - c^2 g^2 = 0 .$$

Noting that $G^i = g^1 + g^2$ and $g^2 = \frac{1}{(1-\beta^1)c^1}(E^1 - (1-t)y^1)$, we obtain:

$$\begin{aligned} & y^1 + y^2 - x^1(U^1,(1-\beta^2)c^1) - x^2(U^2,(1-\beta^2)c^2) \\ & - c^1 G^1(U^1,(1-\beta^2)c^1) + \frac{c^1 - c^2}{c^1(1-\beta^1)}(E^1(U^1,(1-\beta^1)c^1) - (1-t)y^1) = 0. \end{aligned}$$

Finally, the model reduces to the following three equations:

$$\begin{aligned} & (1-\beta^2)c^2 E^1(U^1,(1-\beta^1)c^1) + (1-\beta^1)c^1 E^2(U^2,(1-\beta^2)c^2) \\ & - (1-\beta^1)(1-\beta^2)c^1 c^2 G^1(U^1,(1-\beta^1)c^1) = (1-\beta^2)c^2(1-t)y^1 + (1-\beta^1)c^1(1-t)y^2 \end{aligned} \quad (8\text{-}1)$$

$$G^1(U^1,(1-\beta^1)c^1) = G^2(U^2,(1-\beta^2)c^2) \quad (8\text{-}2)$$

$$\begin{aligned} & y^1 + y^2 - x^1(U^1,(1-\beta^2)c^1) - x^2(U^2,(1-\beta^2)c^2) \\ & - c^1 G^1(U^1,(1-\beta^2)c^1) + \frac{c^1 - c^2}{c^1(1-\beta^1)}(E^1(U^1,(1-\beta^1)c^1) - (1-t)y^1) = 0 \end{aligned} . \quad (8\text{-}3)$$

Equation (8-1) represents the budget constraint in region 1. In particular, the right hand side of this equation represents the income, taking into consideration the externality effect in each region, referred to as *an effective income*. Equations (8-2) and (8-3) represent the public good constraint and the resource constraint, respectively. From these three equations, the utility level in each region and the level of the income tax rate are determined. [5]

3. Welfare effects of disposable income transfers

We examine the welfare effects of disposable income transfers. The income tax rate is assumed to be endogenous to hold the budget constraint. Totally differentiating (8-1), (8-2) and (8-3) gives:

$$\begin{bmatrix} (1-\beta^2)c^2 x_U^1 & (1-\beta^1)c^1 E_U^2 & (1-\beta^2)c^2 y^1 + (1-\beta^1)c^1 y^2 \\ G_U^1 & -G_U^2 & 0 \\ -x_U^1 - c^1 G_U^1 & -x_U^2 & \frac{c^1 - c^2}{c^1(1-\beta^1)} y^1 \end{bmatrix} \begin{bmatrix} dU^1 \\ dU^2 \\ dt \end{bmatrix} = \begin{bmatrix} (1-\beta^2)c^2 - (1-\beta^1)c^1 \\ 0 \\ \frac{c^1 - c^2}{c^1(1-\beta^1)} \end{bmatrix} dT \quad (9)$$

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[5] Note that budget constraint in region 2 is omitted by the Warlas Law.

where dT is defined as $d(1-t)y^1 = -d(1-t)y^2$, which represents the transfer of disposable income from region 2 to region 1, and the determinant of $\begin{bmatrix} (1-\beta^2)c^2x_U^1 & (1-\beta^1)c^1E_U^2 & (1-\beta^2)c^2y^1+(1-\beta^1)c^1y^2 \\ G_U^1 & -G_U^2 & 0 \\ -x_U^1-c^1G_U^1 & -x_U^2 & \frac{c^1-c^2}{c^1(1-\beta^1)}y^1 \end{bmatrix}$, which is defined as Δ^1, becomes negative when we assume $c^1 \geq c^2$ without loss of generality.

Summarizing each term of equation (10), we have:fs

$$\frac{dU^1}{dT} = \frac{G_U^2}{\Delta^1}(y^1+y^2)(c^1-c^2) \quad (10)$$

[6]

The artificial cost differential does not appear in equation (10) and the sign of equation (10) crucially depends on original cost differentials. The neutrality holds as long as an original cost differential does not exist. This corresponds to the result of Boadway et al. (1989). In addition, this corresponds to the results of Buchholz and Konrad (1995) and Ihori (1996), which prove that the income transfer is effective in the presence of the original cost differential.

First, the intuition of the reason why the original cost differential affects is straightforward. If $c^1 > c^2$, the income transfer from region 2 to region 1 means the resource allocation from the low cost region to the high cost region inefficiently. Therefore, the utility changes (decreases).

Next, to understand the intuition of the reason why the artificial cost differential does not affect, we first examine the effect of the transfer on the tax rate. Setting $c^1 = c^2$, we have:

$$\frac{dt}{dT} = \frac{\beta^1-\beta^2}{(1-\beta^2)y^1+(1-\beta^1)y^2}, \quad (11)$$

which shows that the sign of the effect on the tax rate directly depends on the sign of $\beta^1-\beta^2$. Assuming that $\beta^1 > \beta^2$, then the effective income in the economy increases by the transfer from region 2 to region 1. Therefore, the provision of public goods increases, which in turn increases the subsidy amounts. Through the budget constraint, the income tax rate increases in order to keep revenue constant. Since this increase of the income tax rate completely offsets the increase of the effective income in the economy, the effect of the artificial cost differential vanishes.

We can summarize the results as follows.

Proposition 1

Even if the actual mixed cost which contributors face is the same among contributors, the transfer may not be neutral. Original cost equalization, that is, productivity equalization

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[6] Similarly, we have $\frac{dU^2}{dT} = \frac{G_U^1}{\Delta^1}(y^1+y^2)(c^1-c^2)$.

before intervention by the central government ($c^1 = c^2$), is essential to hold the transfer neutrality.

4. Conclusion

This paper has examined the condition to hold the transfer neutrality in an economy with privately provided public goods under the presence of both artificial and original cost differential. In the standard model of private provision of a public good, if the cost is the same, the transfer neutrality holds. However, when there exists two types of cost differentials (artificial cost differential and original cost differential), the neutrality is ambiguous and should be examined. In this point of view, this paper shows that even if the actual mixed cost that contributors face is the same among contributors, the transfer may not be neutral, and original cost equalization is essential to hold the transfer neutrality. The results of this paper indicate that the central government should focus only on the original cost differential, rather than on the actual cost. The policy of income redistribution should be reexamined, considering the relationship between the original cost and the artificial cost.

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