E C O N O M I C S   B U L L E T I N

Linking Gini to Entropy : Measuring Inequality by an interpersonal class of indices.

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Abstract

The aim of this paper is to propose a new class of inequality indices. It is a generalisation of Gini coefficient based on his interpersonal expression. We inter into axiomatic properties of our indices and we show that they are relative, regular indices which, in particular, satisfy the Pigou−Dalton transfer principle.

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1- Introduction

Research studies on the measurement of economies inequality are dominated by the Gini index (or coefficient) and the entropy family of indices. Many studies have been devoted to the properties of these two categories of indices. Since the early works of Gini in 1916, the Gini index has been studied by several authors, nowadays it lends itself to axiomatic characterisation and at least to generalisation (Yitzhaki (1983), Chotikapanich and Griffiths (2001)). Its decomposition into sub-groups which previously was not very satisfactory has been improved by the recent works of Dagum (1997a,1997b) who proposes a new approach for solving the problem .More recently, S.Mussard (2005) proposed a simultaneous decomposition of the Gini index into sub-groups and sources of income etc.
The present study is in keeping with this area of research which it attempts to extend. We propose a family of inequality indices which generalise the Gini index, and which intersects the entropy family through the coefficient of variation squared. We analyse the axiomatic properties of our class of indices and we show in particular, that, it is a class of relative, regular indices which satisfy the Pigou-Dalton transfer principle. We study the consequences of a transfer from a richer to a poorer individual and we show that the effect of such a transfer is maximal at a central value (of the income distribution) which we define.
The paper is attacked as follows: Section 2 is devoted to notations and preliminaries; in particular, we define two real functions which will be used to analyse the utility function linked to the index. Section3 expose the new class of indices and its properties. We conclude in section 4.

2- Notations and preliminaries

Lets consider a population P in which we have an income distribution X with n income units x_1, x_2, $x_3 \ldots x_i \ldots x_n$ where CV^2, Var and μ are respectively the square of coefficient of variation, the variance and the mean on P.
for any real number α we define the following real functions:

$$D_\alpha(x) = \sum_{x_i \le x} (x - x_i)^\alpha - \sum_{x_i \ge x} (x_i - x)^\alpha = \sum_{x_i \le x} |x - x_i|^\alpha - \sum_{x_i \ge x} |x - x_i|^\alpha \quad (1)$$

$$H_\alpha(x) = \sum_{x_i \le x} (x - x_i)^\alpha + \sum_{x_i \ge x} (x_i - x)^\alpha = \sum_{i=1}^{n} |x - x_i|^\alpha \quad (2)$$

$D_\alpha(x)$ represents the sum of differentials (to the power α) relative to x of the income less than x minus the sum of differentials relative to x of the incomes which are greater than it.
$H_\alpha(x)$ represents the sum of differentials to the power α , relative to x of all the incomes of the population.

Properties of $D_\alpha(x)$ **and** $H_\alpha(x)$

1) Analysis of $D_\alpha(x)$

a) **If** $\alpha = 0$,

- $\forall x \in R,\ D_0(x) =$ (Number of x_i less or equal à x)-(Number of x_i greater or equal x)
- If we assume $x_1 < x_2 < x_3 < \ldots < x_n$,

$$D_0(x)=\begin{cases}-n & if\ \ x<x_1\\ 2i-n & if\ \ x_i<x<x_{i+1}\\ (2i-1)-n & if\ \ x=x_i\\ n & if\ \ x>x_n\end{cases} \tag{3}$$

- D_0 therefore is an increasing step function which becomes null at the median of X: if n is odd n=2p+1, the only point which cancels D_0 is $M_0=x_{P+1}$
  if n is even, n=2p, for all x such that $x_p<x<x_{p+1}$, $D_0(x)=0$. (4)

b) **if** $\alpha>0$

- D_α is strictly increasing from $-\infty$ to $+\infty$, continuous and differentiable (except at points x_1, x_2, x_3,…, x_n if $0<\alpha<1$) on R . There exist a unique point M_α >x_1 which nullifies D_α. $D_\alpha(x)$ is positive for any $x\geq M_\alpha$ and negative for any $x\leq M_\alpha$.
- In particular, $\forall x\in R,\ D_1(x)=nx-n\mu$ et $M_1=\mu$ = mean of X. (5)

2) Relationship between $D_\alpha(x)$ **and** $H_\alpha(x)$

a) $\forall\alpha>1$, D_α and H_α are two continuous and differentiable function on R, and we have :

$$D_\alpha^{'}(x)=\alpha H_{\alpha-1}(x)\text{ and }H_\alpha^{'}(x)=\alpha D_{\alpha-1}(x) \tag{6}$$

b) For any integer p greater than 1, and for any α >p, we set :

$$\alpha(\alpha-1)(\alpha-2)...(\alpha-p+1)=A_\alpha^P$$

If $D_\alpha^{(p)}$ and $H_\alpha^{(p)}$ are the p[th] derivatives of D_α and H_α respectively, we have :

$$D_\alpha^{(p)}(x)=\begin{cases}A_\alpha^p D_{\alpha-p}(x) & if\ p\,is\,even\\ A_\alpha^p H_{\alpha-p}(x) & if\ p\,odd\end{cases}\quad\text{and}\quad H_\alpha^{(p)}(x)=\begin{cases}A_\alpha^p H_{\alpha-p}(x) & if\ p\,is\,even\\ A_\alpha^p D_{\alpha-p}(x) & if\ p\,is\,odd\end{cases} \tag{7}$$

c) From properties 1)b and 2)b we deduce that :

a) For $\alpha\geq 1$, H_α is convex (strictly convex if $\alpha>1$) , decreases from $+\infty$ to $M_{\alpha-1}$ then increase from $M_{\alpha-1}$ to $+\infty$. In other word, $M_{\alpha-1}$ is the unique minimum of H_α. (8)

b) For $0<\alpha<1$, H_α is concave in each of interval $[x_i\ ,\ x_{i+1}]$, where it admits a maximum at e_{α_i-1} (i=2,3,…,n) and a vertical tangent at each point x_i . (9)

c) For $\alpha=0$, H_α is constant and equal to n.

3- The Gini index of order α and axiomatic properties

Definition

We call Gini index of order α $(\alpha>0)$ of X in P, the function I_G^α defined by :

$$I_G^{(\alpha)}(X)=\frac{1}{2n^2\mu^\alpha}\sum_{i=1}^n\sum_{j=1}^n\left|x_i-x_j\right|^\alpha=\frac{1}{2n^2}\sum_{i=1}^n\sum_{j=1}^n\left|y_i-y_j\right|^\alpha \tag{10}$$

$I_G^{(\alpha)}(X)$ is equal to half of the mean of differentials to the power α of the y_i $\left(y_i=\frac{x_i}{\mu}\right)$.

Lemma 1

1) If α *=1,* $I_G^{(\alpha)}$ *is equal to the standard Gini index* I_G *.*

2)If α *=2,* $I_G^{(\alpha)}$ *is equal to the coefficient of variation squared* CV^2 *.*

Proof: It is obvious that $I_G^{(1)} = I_G$. We are going to show that $I_G^{(2)}(X) = CV^2(X)$.

We know that $CV^2(X) = \dfrac{Var(X)}{\mu^2}$, it is therefore sufficient to show that

$Var(X) = \dfrac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^2$: Indeed,

$$\frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^2 = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(x_i - x_j\right)^2 = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(x_i^2 + x_j^2 - 2x_i x_j\right) \tag{11}$$

$$= \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i^2 + \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_j^2 - \frac{2}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i x_j \tag{12}$$

$$= \frac{1}{2n}\sum_{i=1}^{n}x_i^2 + \frac{1}{2n}\sum_{j=1}^{n}x_j^2 - \left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)\left(\frac{1}{n}\sum_{j=1}^{n}x_j\right) \tag{13}$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \mu^2 = Var(X) \qquad \square \tag{14}$$

While the literature tends to treat the Gini index and the entropy class of indices separately, the above lemma proves, in particular, that there exist a link between the Gini index and the coefficient of variation squared which belongs, to the entropy family.

Proposition 1: *The index* $I_G^{(\alpha)}$ *satisfies the following axiomatic properties:*

1) Relative invariance or Homogeneity of zero degree(RI) :

$\forall \lambda > 0,\ I_G^{(\alpha)}(\lambda X) = I_G^{(\alpha)}(X)$

2) Normalization (N):

If X is an egalitarian distribution: $X = (x, x, x, ..., x)$ *then* $I_G^{(\alpha)}(X)$ *=0*

3) Symmetry or Anonymity(S):

For any permutation ρ *in* $P = \{1,2,3,...,i,...n\}$, $I_G^{(\alpha)}(x_{\rho(1)}, x_{\rho(2)}, ...x_{\rho(n)}) = I_G^{(\alpha)}(X)$.

4) Dalton's population principle(DP)

$$I_G^{(\alpha)}(\underbrace{x_1, x_1, ...x_1}_{m\,times}; \underbrace{x_2, x_2, ...x_2}_{m\,times}; ...; \underbrace{x_n, x_n ...x_n}_{m\,times}) = I_G^{(\alpha)}(X)$$

Proof : Assertion 2) being obvious, we prove 1), 3) and 4).

1) $I_G^{(\alpha)}(\lambda X) = \dfrac{1}{2n^2(\lambda\mu)^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\lambda x_i - \lambda x_j\right|^\alpha = \dfrac{\lambda^\alpha}{2n^2\lambda^\alpha\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = I_G^{(\alpha)}(X)$

3) $I_G^{(\alpha)}(x_{\rho(1)}, x_{\rho(2)}, ...x_{\rho(n)}) = \dfrac{1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_{\rho(i)} - x_{\rho(j)}\right|^\alpha = \dfrac{1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = I_G^{(\alpha)}(X)$.

4) $I_G^{(\alpha)}(\underbrace{x_1,x_1,...x_1}_{m\,times};\underbrace{x_2,x_2,...x_2}_{m\,times};...;\underbrace{x_n,x_n...x_n}_{m\,times}) = \frac{1}{2(nm)^2\mu^\alpha}\sum_{k=1}^{nm}\sum_{l=1}^{nm}|x_k - x_l|^\alpha$

$$= \frac{m^2}{2(nm)^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|^\alpha = I_G^{(\alpha)}(X) \quad \square$$

Proposition 2: *For* $\alpha \geq 1$, $I_G^{(\alpha)}$ *satisfies the Pigou-Dalton transfer principle(PD) and it is therefore a relative, regular index.*

Proof and interpretation in term of social welfare

We know for $\alpha = 1$, $I_G^{(\alpha)}$ is equal the Gini coefficient and thus satisfies (see e.g. Sautory, 1996) Pigou-Dalton transfer principle. Let show the property for $\alpha > 1$:

The social welfare function associated with $I_G^{(\alpha)}(X)$ is:

$$W_\alpha(X) = -\,I_G^{(\alpha)}(X) = \frac{-1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|^\alpha = \frac{-1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|^\alpha \qquad (15)$$

where $y_i = \frac{x_i}{\mu}$ is the relative income of the individual i. We note $Y = (y_1, y_2, y_3,..., y_n)$ the distribution of relative income corresponding to X.

This function may be written as the sum of individual appreciation:

$$W_\alpha(Y) = \sum_{i=1}^{n} u_\alpha(y_i) \text{ where } u_\alpha(y) = \frac{-1}{2n^2}\sum_{j=1}^{n}|y - y_j|^\alpha = \frac{-1}{2n^2}H_\alpha(y) \qquad (16)$$

Where H_α is defined in (2).

In economic terms, the value of $u_\alpha(y_i)$ corresponds to the utility[1] associated with income y_i and the value of $W_\alpha(Y)$ to the social utility associated with the distribution of incomes $(y_1, y_2, y_3,..., y_n)$.

From formulas (2) and (6), we deduce that:

If $\alpha > 1$, the derivative of u_α is : $u_\alpha'(y) = \frac{-\alpha}{2n^2}\left(\sum_{y_i \leq y}(y - y_i)^{\alpha-1} - \sum_{y_i \geq y}(y_i - y)^{\alpha-1}\right) = \frac{-\alpha}{2n^2}D_{\alpha-1}(y)$

And it follows that (cf. 1)b of preliminaries) u_α' is strictly decreasing, u_α is thus concave and consequently $I_G^{(\alpha)}$ satisfies the Pigou-Dalton transfer principle. $I_G^{(\alpha)}$ is relative regular index because it satisfies (RI), (DP), (S) and (PD) $\square$

If $\alpha < 1$, $I_G^{(\alpha)}$ does not satisfies the Pigou-Dalton transfer principle although some transfers may reduce the value of $I_G^{(\alpha)}$. It is for instance the text book case:

X= 23 ,45 ,67 ,43.5 ,123, 78, 45, 89, 213, 90, 23, 45, 67, 43.5, 123, 78, 45, 89, 213, 90 and $\alpha = 0.3$.

For which we have $I_G^{(0.3)}(X) = 0 .368$ when individual 2 transfer 10 units to individual 1, the index increases to 0.37201. When individual 5 transfers 23 units to individual 7, the index decreases to 0.3674.

[1] We note that a utility function is defined up to an increasing monotonic transformation.

Corollary 1 : *The maximum value of* $I_G^{(\alpha)}$ *, for* $\alpha \geq 1$*, is equal to* $\frac{(n-1)}{n} n^{\alpha-1}$ *which is obtained with the perfect inegalitarian* X *distribution where only one individual holds the entire resource.*

Proof: The fact that the maximum value of $I_G^{(\alpha)}(X)$ can be obtained with the perfectly unequal distribution X_e is a direct consequence of The Pigou-Dalton transfer principle. If r represents the individual who holds the entire resource in X_e and x the total resource held by r, then :

$$I_G^{(\alpha)}(X_e) = \frac{1}{2n^2 \mu^\alpha} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| x_i - x_j \right|^\alpha = \frac{1}{2n^2 \left(\frac{x}{n} \right)^\alpha} \left(\sum_{j=1}^{n} \left| x_r - x_j \right|^\alpha + \sum_{i \neq r}^{n} \left| x_i - x_r \right|^\alpha \right) \quad (17)$$

$$= \frac{1}{2n^2 \left(\frac{x}{n} \right)^\alpha} \left((n-1)x^\alpha + (n-1)x^\alpha \right) = \frac{n^\alpha (n-1)}{n^2} = \frac{n-1}{n} n^{\alpha-1} \quad \square \quad (18)$$

This result shows in particular that there is no upper limit for inequality; it depends on the size of the population and the parameter α. If $\alpha > 1$ and n exceed 10, the upper value is greater than 1. However, it is interesting to note that :

$J_G^{(\alpha)}(X) = \frac{I_G^{(\alpha)}(X)}{(n-1)n^{\alpha-2}} = \frac{1}{2(n-1)n^\alpha \mu^\alpha} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| x_i - x_j \right|^\alpha$, which is obtained from $I_G^{(\alpha)}$ by reduction, takes on its values in the interval $[0 , 1]$.

Corollary 2: *If* $\alpha \geq 1$*, the variation* $dI_G^{(\alpha)}(Y)$ *of the index, consecutive to an infinitesimal transfer* dh *from a rich j to a poor i implies a decrease in the index equal to:*

$$dI_G^{(\alpha)}(Y) = \frac{\alpha dh}{2n^2} \left(D_{\alpha-1}(y_i) - D_{\alpha-1}(y_j) \right) \quad (19)$$

Where D_α *is the function defined by the expression (2)*

Proof: Indeed, $dI_G^{(\alpha)}(Y) = dh \left(\frac{\partial I_G^{(\alpha)}(Y)}{\partial y_i} - \frac{\partial I_G^{(\alpha)}(Y)}{\partial y_j} \right) = dh \left(u'_\alpha(y_j) - u'_\alpha(y_i) \right)$

where $u_\alpha(y)$ is defined in (11)

$$= \frac{\alpha dh}{2n^2} \left(D_{\alpha-1}(y_i) - D_{\alpha-1}(y_j) \right) \quad \square$$

Consequence of a transfer

The result of corollary 2, though given at the nearest increasing monotonic transformation, permit to study the behaviour of $dI_G^{(\alpha)}(Y)$ as a function of incomes y_i and y_j. Here we give the particular cases for $\alpha = 1, 2$ and $\alpha \geq 3$

1) If $\alpha = 1$,

$$dI_G^{(\alpha)}(Y) = \frac{dh}{2n^2} \left(D_0(y_i) - D_0(y_j) \right) = \frac{dh}{2n^2} \left[(2rank(y_i) - n - 1) - ((2rank(y_j) - n - 1) \right]$$

$$= dh\frac{rank(y_i) - rank(y_j)}{n^2} = \frac{dh}{n^2}(i-j) \text{ if } y_1 < y_2 < ... < y_n \quad (20)$$

$dI_G^{(\alpha)}(Y)$ depends on the rank of individuals and not on their incomes: the index gives the same importance to the inequality among the poor as among the rich. This is a well-known result concerning the Gini coefficient.

If $\alpha = 2$, $dI_G^{(\alpha)}(Y) = \frac{2dh}{2n^2}\left(D_1(y_i) - D_1(y_j)\right)$ and by using formula (5) ,

$$= \frac{dh}{n^2}\left[\left(ny_i - n\right) - \left(ny_j - n\right)\right] = \frac{dh}{n}\left(y_i - y_j\right) \quad (21)$$

Again we find that, for the coefficient of variation squared, the decrease is independent of the income level of individuals, but depends only on the differential between these incomes: this index therefore lends the same importance to inequality among the poor as among the rich.
If $\alpha \geq 3$, then $\alpha - 2 \geq 1$ and we know (cf. (8)) that $H_{\alpha-2}$ is convex and admits a minimum $M_{\alpha-3}$. Consequently, the second derivative of u_α, which is equal to $u_\alpha''(y) = -\frac{\alpha(\alpha-1)}{2n^2}H_{\alpha-2}(y)$ is concave, and admits a maximum at $M_{\alpha-3}$. This means that the index lends more importance to inequality among individuals who have an income closed to the 'central' value $M_{\alpha-3}$.It is worth noting that, if $\alpha = 3$, $M_{\alpha-3}$ is the median (cf. (4)) population income and if $\alpha = 4$, $M_{\alpha-3}$ is the mean income of the population (cf. (5)).

Proposition 3 : *For any distribution X, one and only one of the following properties is verified:*

1) $I_G^{(\alpha)}(X)$ is a decreasing function of α which tends towards a real constant when α tends towards $+\infty$

2) There exist an α_0 for which we have: $\alpha > \alpha' \geq \alpha_0 \Rightarrow I_G^{(\alpha)}(X) > I_G^{(\alpha')}(X)$; in this case $I_G^{(\alpha)}(X)$ tends towards $+\infty$ when α tends towards $+\infty$.

Proof : Consider the distribution X and all the possible relative differentials $\left|\frac{x_i - x_j}{\mu}\right|$ i=1,2,…,n ; j=1,2,…,n. Represent by $a_1, a_2, \ldots, a_p$ those of the differentials which are strictly greater than 0 and smaller or equal to 1, and by $b_1, b_2, \ldots, b_q$ the differentials which are strictly greater than 1. It is obvious that:

$$I_G^{(\alpha)}(X) = f(\alpha) = \frac{1}{2n^2}\left(\sum_{k=1}^{p} a_k^\alpha + \sum_{k=1}^{q} b_k^\alpha\right).$$

The first and second derivative of f are respectively :

$$f'(\alpha) = \frac{1}{2n^2}\left(\sum_{k=1}^{p}\ln(a_k)a_k^\alpha + \sum_{k=1}^{q}\ln(b_k)b_k^\alpha\right) \text{and } f''(\alpha) = \frac{1}{2n^2}\left(\sum_{k=1}^{p}\ln^2(a_k)a_k^\alpha + \sum_{k=1}^{q}\ln^2(b_k)b_k^\alpha\right).$$

This expression proves that f'' is strictly positive and consequently f' is strictly increasing in the interval $[0\,;+\infty[$.

- If there are no differentials strictly greater than 1, then all the differentials fall between 0 and 1 and f' is strictly negative since it increases from $\frac{1}{2n^2}\sum_{k=1}^{p}\ln(a_k)$ to 0. In this case the function $f(\alpha)$ is strictly decreasing and assertion 1) of the proposition is verified.

- If on the other hand, there exist differentials which are strictly greater than 1, the function f' increases from $f'(0)=B=\frac{1}{2n^2}\left(\sum_{k=1}^{p}\ln(a_k)+\sum_{k=1}^{q}\ln(b_k)\right)$ to $+\infty$. If $B\geq 0$, f' is positive and f is strictly increasing. By taking $\alpha_0=1$, assertion 2) of the proposition is verified. If $B<0$, In accordance with the intermediate value theorem, there will exist a unique real r which nullifies the function f' and by taking $\alpha_0 = Max(r\ ,1)$, assertion 2) of the proposition is verified. □

Economic interpretation and choice of parameter α

The value of the index $I_G^{(\alpha)}(X)$ is defined as the mean of the relative differentials $\left|\frac{x_i - x_j}{\mu}\right|^{\alpha}$.

Now some of differentials $\left|\frac{x_i - x_j}{\mu}\right|$ may be smaller or equal to 1whereas others are strictly greater than 1. Taking the power of these differentials has the effect of amplifying them in case they are greater than 1 and reducing them in case they are less than 1. It results from this that, relative to the Gini index, the large differentials will contribute more to the final value of the index, while the differentials less than 1 will have their contribution reduced. Since this phenomenon takes on increasing significance with the value of α, the problem of choosing the appropriate value of α will emerge. As in the case of the family of entropy indices, this problem strictly speaking, does not have a solution. In practice, economists simply prefer the first integer values (1 or 2) of parameter β of the entropy. In the case of the class of indices $I_G^{(\alpha)}$, α= 1 or 2 correspond to the Gini index or to the square of the coefficient of variation which are among the indices widely used by practitioner..

Moreover in the case of $I_G^{(\alpha)}$, an approach for solving the problem of choosing parameter α may be proposed from the proposition 3 above. In effect, in the light of this proposition, income distributions are partitioned into two categories; the first one of which is made up of variables X which all have differentials $\left|\frac{x_i - x_j}{\mu}\right|$ less than or equal to 1 and the second with variables X having at least one differential $\left|\frac{x_i - x_j}{\mu}\right|$ greater than 1:

- If income distribution X is in the first category, $I_G^{\alpha}(X)$ will be a decreasing function of α which tends forward a real constant as α tends towards infinity. In this case we will choose α =1 in order not to have a very low value index and in order not to completely cancel the contribution of the very small differentials to the final value of $I_G^{\alpha}(X)$.
- If income distribution X is in the second category, $I_G^{\alpha}(X)$ tends towards infinity as α tends towards infinity and according to proposition3, there will exist α_0 for which $I_G^{\alpha}(X)$ will become an increasing function of α : $\alpha_1 > \alpha_2 \geq \alpha_0 \Rightarrow I_G^{\alpha_1}(X) > I_G^{\alpha_2}(X)$

Hence, $\alpha \geq \alpha_0$ will be interpreted as a parameter of aversion to inequality, and it seems natural to choose $\alpha = \alpha_0$ (or closed to α_0). This choice is also justified by the fact that before α_0, $I_G^{\alpha}(X)$ is a decreasing function of α, and after α_0, the contribution of the large differentials start being exceedingly amplified.

4- Conclusion

The class of indices we have proposed simply generalises the Gini coefficient. These indices possess most the most of axiomatic properties actually required for a good inequality index. It thus presents other possibilities for measuring inequality appropriately. It creates a link between the Gini index and the entropy family of indices, since it also contains the coefficient of variation squared. Nevertheless, others proprieties as subgroup and income source decomposition have to be studied.

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