Monopolistic price discrimination and output effect under conditions of constant elasticity demand

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Abstract

The paper proves that monopolistic price discrimination increases output under conditions of constant demand elasticity. The demonstration is simpler than that of Formby, Layson and Smith (1983)

I am grateful to Rod Garratt for helpful comments. Financial support from Ministerio de Ciencia y Tecnología and FEDER (BEC2003-04430) is gratefully acknowledged.
Citation: Aguirre, Iñaki, (2006) "Monopolistic price discrimination and output effect under conditions of constant elasticity demand." *Economics Bulletin*, Vol. 4, No. 23 pp. 1-6
Submitted: June 23, 2006. **Accepted:** June 23, 2006.
URL: http://economicsbulletin.vanderbilt.edu/2006/volume4/EB-06D40012A.pdf

1 INTRODUCTION

A well-known result in the theoretical literature on the welfare effects of third degree price discrimination is that a move from uniform pricing to price discrimination reduces welfare if total output decreases.[1] So a focal point has been the analysis of the effects of price discrimination on output. It is known at least from Robinson (1933) that in the case of linear demands price discrimination does not change output (assuming that all markets are served under both pricing regimes). In the general non-linear case, the effect of price discrimination on output may be either positive or negative. Since Robinson (1933) a great many papers have addressed this issue, including Leontief (1940), Edwards (1950), Silberberg (1970), Greenhut and Ohta (1976), Löfgren (1977), Smith and Formby (1981), Schmalensee (1981), Shih, Mai and Liu (1988) and Cheung and Wang (1994), among others. However, one of the most popular demand families, constant elasticity demands, has proven particularly resistant to analysis. The criteria developed by the relevant literature are unable to answer the question ¿does price discrimination always increase total output under constant demand elasticity conditions?[2] Formby, Layson and Smith (1983) (FLS) using Lagrangean techniques, show that monopolistic price discrimination increases total output over a wide range of constant elasticities. In this paper, I provide a simpler proof by using the Bernoulli inequality.[3]

2 ANALYSIS

Consider a monopolist selling a good in n perfectly separated markets. The demand function in market i (i = 1,..., n) has constant elasticity and is given by $D_i(p_i) = a_i p_i^{-\varepsilon_i}$, where p_i is the price charged in that market, ε_i is the constant elasticity (in absolute value) of demand ($\varepsilon_i > 1$) and $a_i > 0$. Marginal cost, c, is assumed constant.

Under price discrimination, the optimal policy for the monopolist is given by

[1] Schmalensee (1981) proves this conjecture assuming nonlinear demand curves, perfectly separated markets and constant marginal cost. Varian (1985) extends the result by allowing imperfect arbitrage and by allowing marginal cost to be constant or increasing. Using a revealed-preference argument, Schwartz (1990) generalizes the result to the case in which marginal cost is decreasing.

[2] So, neither the "adjusted concavity" criterion (Robinson, 1933, Cheung and Wang, 1994) nor the "slope ratio" criterion (Edwards, 1950), nor the "mean-value theorem" criterion (Shih, Mai and Liu, 1988), among others, serve to answer the question.

[3] Galera and Zaratiegui (2006) use the Bernoulli inequality to show that under duopoly an increase in total output is not a necessary condition for welfare improvement (when one firm is more efficient than the other).

$$\frac{p_i^d - c}{p_i^d} = \frac{1}{\varepsilon_i} \qquad (i = 1, ..., n), \tag{1}$$

where p_i^d denotes the optimal price in market i. That is, the Lerner index (or price-marginal cost ratio) in each market is inversely proportional to its elasticity of demand. The optimal price in market i is given by

$$p_i^d = \frac{c}{1 - \frac{1}{\varepsilon_i}} \qquad (i = 1, ..., n), \tag{2}$$

and the monopolist, therefore, states a higher price in the market with the lower elasticity of demand. The quantity sold in market i, q_i^d, is given by:

$$q_i^d = D_i(p_i^d) = a_i \left(\frac{c}{1 - \frac{1}{\varepsilon_i}} \right)^{-\varepsilon_i} \qquad (i = 1, ..., n), \tag{3}$$

and total output under price discrimination, q^d, is

$$q^d = \sum_{i=1}^{n} q_i^d = \sum_{i=1}^{n} a_i \left(\frac{c}{1 - \frac{1}{\varepsilon_i}} \right)^{-\varepsilon_i} . \tag{4}$$

Under simple monopoly pricing, profits are maximized by charging all consumers a common price p^0. The Lerner index under uniform pricing is:

$$\frac{p^0 - c}{p^0} = \frac{1}{\varepsilon(p^0)} \ , \tag{5}$$

where $\varepsilon(p^0)$ is the elasticity of the aggregate demand at p^0. If $D(p) = \sum_{i=1}^{n} D_i(p)$ denotes the aggregate demand, that elasticity is the weighted average of the elasticities of each market

$$\varepsilon(p^0) = -\left(\sum_{i=1}^{n} D_i'(p^0) \right) \frac{p^0}{\sum_{i=1}^{n} D_i(p^0)} = \frac{\sum_{i=1}^{n} \varepsilon_i a_i [p^0]^{-\varepsilon_i}}{\sum_{i=1}^{n} a_i [p^0]^{-\varepsilon_i}} = \sum_{i=1}^{n} w_i(p^0) \varepsilon_i, \tag{6}$$

where the elasticity of market i is weighted by the "share" of that market at the optimal uniform price, $w_i(p^0) = D_i(p^0)/\sum_{i=1}^{n} D_i(p^0)$. Denote as q_i^0 and q^0 the quantity sold in market i, $q_i^0 = D_i(p^0)$ $(i = 1, ..., n)$, and the total output, $q^0 = \sum_{i=1}^{n} D_i(p^0)$, under uniform pricing, respectively. Following FLS (1983) I assume that the units of production are defined so that $p^0 = 1$. This assumption implies that:

$$\varepsilon(1) = \frac{\sum_{i=1}^{n} \varepsilon_i a_i}{\sum_{i=1}^{n} a_i}; \quad q_i^0 = a_i \quad (i = 1, ..., n) \text{ and } q^0 = \sum_{i=1}^{n} a_i. \tag{7}$$

On the other hand, marginal cost, given condition (5), is

$$c = \frac{\sum_{i=1}^{n} a_i(\varepsilon_i - 1)}{\sum_{i=1}^{n} a_i \varepsilon_i}. \tag{8}$$

FLS (1983) address the question by using Lagrangean techniques. In particular they minimize the difference between q^d and q^0 with respect to the sub-market elasticities subject to the restriction that the changes must leave the elasticity of aggregate demand (at the simple monopoly price) unchanged. Next, I shall use the Bernoulli inequality to provide a simpler proof.

In order to prove that price discrimination increases total output, I use a generalization for real exponents of the Bernoulli inequality which says that "if $0 \neq x > -1$ and $a > 1$ are real values, then $(1 + x)^a > 1 + ax$."[4] It is useful to rewrite this inequality as $(y)^a > 1 + a(y - 1)$, where $y = x + 1$.

Note that as $\varepsilon_i > 1$ the Bernoulli inequality implies that

[4] See, for example, Mitrinovic (1970). The generalization for real exponents of Bernoulli inequality is straightforward with the use of calculus. Consider the function $f(x) = (1 + x)^a - (1 + ax)$. The first derivative is given by $f'(x) = a(1+x)^{a-1} - a$. Notice that if $a > 1$ then $f''(x) = a(a-1)(1+x)^{a-2} > 0$, so the function is strictly convex and therefore for any $0 \neq x > -1$ and $a \in R$, such that $a > 1$, the inequality is satisfied.

$$\left(\frac{\varepsilon_i c}{\varepsilon_i - 1}\right)^{-\varepsilon_i} = \left(\frac{\varepsilon_i - 1}{\varepsilon_i c}\right)^{\varepsilon_i} > 1 + \varepsilon_i \left(\frac{\varepsilon_i - 1}{\varepsilon_i c} - 1\right). \tag{9}$$

Given condition (8) we have

$$\frac{\varepsilon_i - 1}{\varepsilon_i c} - 1 = \frac{\varepsilon_i \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_i \varepsilon_i}{\varepsilon_i \sum_{i=1}^{n} a_i(\varepsilon_i - 1)}. \tag{10}$$

Substituting (10) into (9) we obtain

$$\left(\frac{\varepsilon_i c}{\varepsilon_i - 1}\right)^{-\varepsilon_i} = \left(\frac{\varepsilon_i - 1}{\varepsilon_i c}\right)^{\varepsilon_i} > 1 + \frac{\varepsilon_i \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_i \varepsilon_i}{\sum_{i=1}^{n} a_i(\varepsilon_i - 1)}. \tag{11}$$

By multiplying both sides of inequality (11) by a_i, we have

$$q_i^d = a_i \left(\frac{\varepsilon_i c}{\varepsilon_i - 1}\right)^{-\varepsilon_i} = a_i \left(\frac{\varepsilon_i - 1}{\varepsilon_i c}\right)^{\varepsilon_i} > a_i + a_i \frac{\varepsilon_i \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_i \varepsilon_i}{\sum_{i=1}^{n} a_i(\varepsilon_i - 1)}. \tag{12}$$

By summing i in (12) from 1 to n we obtain

$$q^d = \sum_{i=1}^{n} q_{id} = \sum_{i=1}^{n} a_i \left(\frac{\varepsilon_i c}{\varepsilon_i - 1}\right)^{-\varepsilon_i} > q^0 + \sum_{i=1}^{n} \left(a_i \frac{\varepsilon_i \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_i \varepsilon_i}{\sum_{i=1}^{n} a_i(\varepsilon_i - 1)} \right). \tag{13}$$

To complete the proof we have only to check that the last term in (13) is non-negative. It is easy to check that, in particular, it is zero since

$$\sum_{i=1}^{n}\left(a_i\frac{\varepsilon_i\sum\limits_{i=1}^{n}a_i-\sum\limits_{i=1}^{n}a_i\varepsilon_i}{\sum\limits_{i=1}^{n}a_i(\varepsilon_i-1)}\right)=\frac{1}{\sum\limits_{i=1}^{n}a_i(\varepsilon_i-1)}\left[\sum_{i=1}^{n}a_i\varepsilon_i\left(\sum_{j\neq i}a_j\right)-\sum_{i=1}^{n}a_i\left(\sum_{j\neq i}a_j\varepsilon_j\right)\right] \quad (14)$$

and $\sum\limits_{i=1}^{n}a_i\varepsilon_i\left(\sum\limits_{j\neq i}a_j\right)-\sum\limits_{i=1}^{n}a_i\left(\sum\limits_{j\neq i}a_j\varepsilon_j\right)=0$ from the associative law of addition. So $q^d>q^0$ and price discrimination therefore increases output.

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