# When Inertia Generates Political Cycles

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# Abstract

In this note, we propose a simple infinite horizon of elections with two candidates. We suppose that the government policy presents some degree of inertia, i.e. a new government cannot completely change the policy implemented by the incumbent. When the policy inertia is strong enough, no party can win the elections a consecutive infinite number of times.

#### 1 Introduction

In modern democracies, the alternation of political parties in power is a frequent phenomenon. Why isn't there a greater persistence of parties in power? How can one explain the turnover of parties in government? How can one explain political cycles? In a dynamic setting, we introduce two imperfections to the model of Downs (1957). As other scholars (see Casamatta and De Donder 2005), I consider a programs inertia assumption. As Heckelman (2000), I consider that parties have (different) fixed programs. The second, we call policy inertia represents imperfections in changing the government policy. This effect has various origins in real political life. Indeed, a majority of policies cannot be changed without any cost. In the model, I distinguish the effective policy, which is the objective state of the policy, and the qovernment policy, which is the policy of the incumbent during his legislature. We suppose that the effective policy at time t is a convex combination of the effective policy at date t-1 (past policy) and of the government policy at time t. This assumption captures the fact that the government cannot freely change the policy implemented in the past. I argue that this can be a reason why similar countries with different histories implement very different policies. I say that the model exhibits political cycles when no party can win the elections a consecutive infinite number of times. In this situation, parties indefinitely alternate (not necessary regularly) in power.

The main result of this note states that if the *policy inertia* is high enough, political cycles appear. Furthermore, I show that the dynamic can be history dependent. Indeed, if the median voter is indifferent between the two candidates programs at an election, the future dynamic will be dramatically affected. Scholars generally explain political cycles with psychological arguments (see Goertzel 2005 for a review of the American voters mood changes literature). Schlesinger (1949, 1986, 1992) considers that the electorate is inevitably disappointed by the party or the ideology that is in power. Klinberg (1952) suggests that American mood in public opinion balances between introversion and extroversion. This could explain why domestic and foreign concerns alternate through time and parties turnover in power. The main explanation is certainly disappointment. The "Negativity effect" theory (see Aragones 1997 for a survey) is build on the following remark: voters' decisions are based on the incumbent's past performance and negative pieces of information have a greater impact on voters decisions than positive pieces of information. In the light of the negativity effect, Aragones (1997) obtains a result of systematic alternation of two parties implementing different policies. In our analysis, there is no uncertainty and electorate decisions are not based on the government past performance, but as usually in political models, for their preferred program at each election. I propose a simple model suggesting that political cycles can be generated by inertia only.

### 2 The model

The policy setting: Two candidates L and R compete in an infinite horizon elections setting. The set of policies is the interval P = [-1, 1]. Each citizen is represented by a bliss point  $\widehat{\alpha}_i$  in the set of policies and voters bliss points are distributed over P. according to the cumulative distribution function F. The utility function of voter i is defined over the set of effective policies,  $u^i(p_t) = -|\widehat{\alpha}^i - p_t|$  where  $p_t$  is the effective policy at time t. We suppose that candidates programs are fixed. Candidate j proposes a policy  $z^j$ . We assume that the median voter preferred policy,  $\widehat{\alpha}^m$  is such that:  $z^L < \widehat{\alpha}^m < z^R$ .

**Policy inertia:** We suppose that a policy implemented at date t has an influence on the effective policy at date t+1. Let  $p_{t+1}$  be the effective policy at date t+1. We suppose this policy results from the government policy at date t+1 and from the effective policy at date t. Let  $\delta \in [0,1]$  be the "inertia" degree of past policies. Let  $z^{W(t+1)}$  denotes the program implemented in period t+1 by the elected party  $W(t+1) \in \{L,R\}$ . The effective policy at date t+1 is then:

$$p_{t+1} = (1 - \delta) z^{W(t+1)} + \delta p_t,$$

In the case without policy inertia ( $\delta = 0$ ) there is no linkage between the successive elections. In the second polar case with full policy inertia ( $\delta = 1$ ), the policy is completely fixed and voting has no influence on the policy implemented. In the following, we will consider that  $\delta < 1$ . Between the two polar situations, a new government will have to face an inertia force  $\delta p_t$ , which can be interpreted in many ways.

Although the preferred effective policy of a voter is fixed, his preferred program changes from an election to the following. To illustrate the dynamic, let compute voter i preferred program (noted  $p_{t+1}^i$ ) at election t+1:

$$p_{t+1}^i = \frac{\widehat{\alpha}^i - \delta p_t}{1 - \delta},$$

Then, the more the effective policy of the previous period was leftist, the more the median voter will move to the right, and the more the effective policy of the previous period was rightist, the more the median will move to the left. This intuition underline the swing of the voters.

**Policy history:** We suppose that the first election take place in period 1, the influence of period 0 depend on a degree of inertia  $\delta_0$  not necessarily equal to  $\delta$  and a past policy  $p_0$ . These parameters can represent different histories. For example  $\delta_0 = 0$  could refer to a revolution preceding the first democratic election in t = 1 and be interpreted as the fact that past policy is completely removed.

# 3 Political Cycles

We say that the set of parameters  $(\delta, \widehat{\alpha}^m, z^L, z^R)$  exhibits *Political cycles* if no party can win the elections an infinite consecutive number of times, formally:

**Definition 1** A set of parameters  $(\delta, p_0, \widehat{\alpha}^m, z^L, z^R) \in ]0, 1[\times [0, 1]^4$  exhibits **Political cycles** if and only if W ( $W(t) \in \{L, R\}\}$ ) does not converge when t goes to infinity.

A first important remark, is that in situations where the median voter is indifferent between the two policies, the dynamic strongly depends on the randomized winner. Indeed, if the median voter is indifferent between the two parties in election t, he will not be indifferent in election t + 1. Let  $v_t^m$  be the utility of voter m over the set of programs at date t:

$$v_t^m(z) = u^i((1-\delta)z + \delta p_{t-1}),$$

The following result is straightforward at the light of the previous remark. Let  $\tilde{p}^m$  be the effective policy such that the median voter is indifferent between both parties, it is defined by

$$u^{m}\left(\left(1-\delta\right)z^{L}+\delta\widetilde{p}^{m}\right)=u^{m}\left(\left(1-\delta\right)z^{R}+\delta\widetilde{p}^{m}\right).$$

Remark that this equation implies that  $z^L < \tilde{p}^m < z^R$ .

Now suppose that in election t-1,  $p_{t-1} = \tilde{p}^m$ . In this situation, the median voter is indifferent between the two parties and the result of the election t is undetermined. Suppose that L is the winner, then the policy

implemented is  $z^L$  and  $p_t = (1 - \delta) z^L + \delta \widetilde{p}^m < \widetilde{p}^m$ . This implies that the median voter (and then a majority of voters) will vote for party R. With the same argument, if R wins at time t, then L wins at election t+1. We can conclude that:

**Lemma 2** If  $p_{t-1}^m = \widetilde{p}^m$  then  $W(t-1) \neq W(t)$ .

Hence, the situation where the median voter is indifferent between both parties is not stable. Now we characterize the set of parameters such that political cycles appear:

**Proposition 3** The set of parameters  $(\delta, p_0, \widehat{\alpha}^m, z^L, z^R)$  exhibits Political cycles if and only if  $\frac{1-\delta}{1-\delta} \leq \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} \leq \frac{1+\delta}{1-\delta}$ . , W converges to L if and only if  $\frac{1+\delta}{1-\delta} < \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L}$ , and W converges to R if and only if  $\frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} < \frac{1-\delta}{1+\delta}$ .

**Proof.** Suppose  $\frac{1+\delta}{1-\delta} \geq \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} \geq \frac{1-\delta}{1+\delta}$ . **step 1:** Suppose that W converge to L. Then, there exists an election k such that  $\forall t \geq k, \ W\left(t\right) = L$ . This is implies  $\forall t \geq k, \left|p_{t+1}^{m} - z^{L}\right| \leq \left|z^{R} - p_{t+1}^{m}\right|$ . Furthermore,

$$p_{t+1} = (1 - \delta) z^L + \delta p_t,$$

Then,

$$p_{t+1} - z^L = \delta \left( p_t - z^L \right),\,$$

Since  $\delta \in ]0,1[$ ,  $\lim_{t\to+\infty} (p_{t+1}-z^L)=0$ . Finally,  $\lim_{t\to+\infty} p_{t+1}=z^L$ . Now, we must have that the median voter (strictly, because of the Lemma) prefers L to R:

$$u^{i}\left(z^{L}\right)-u^{i}\left(z^{R}\right)=-\left|\frac{\widehat{\alpha}^{m}-z^{L}}{1-\delta}\right|+\left|z^{R}-\widehat{\alpha}^{m}-\delta\frac{\widehat{\alpha}^{m}-z^{L}}{1-\delta}\right|>0,$$

Since  $\widehat{\alpha}^m > z^L$ , this is equivalent to:

$$\left| z^R - \widehat{\alpha}^m - \delta \frac{\widehat{\alpha}^m - z^L}{1 - \delta} \right| > \frac{\widehat{\alpha}^m - z^L}{1 - \delta}, \tag{1}$$

This inequality is equivalent to:

$$\begin{split} \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} &> & \frac{1 + \delta}{1 - \delta} \text{ if } \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} > \frac{\delta}{1 - \delta}, \\ \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} &< & -1 \text{ if } \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} \leq \frac{\delta}{1 - \delta}, \end{split}$$

The second case cannot hold because  $\frac{z^R - \hat{\alpha}^m}{\hat{\alpha}^m - z^L} > 0$ , then (1) is equivalent to:

$$\frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} > \frac{1 + \delta}{1 - \delta},$$

Contradiction.

**step 2:** Suppose that W converge to R. Then, there exists an election k such that  $\forall t \geq k$ , W(t) = R. This implies  $\forall t \geq k$ ,  $\left|z^{R} - p_{t+1}^{m}\right| \leq \left|p_{t+1}^{m} - z^{L}\right|$ . Furthermore,

$$p_{t+1} = (1 - \delta) z^R + \delta p_t,$$

Then,

$$p_{t+1} - z^R = \delta \left( p_t - z^R \right),\,$$

Since  $\delta \in ]0,1[$ ,  $\lim_{t\to +\infty} (p_{t+1}-z^R)=0$ . Finally,  $\lim_{t\to +\infty} p_{t+1}=z^R$ . Now, we must have that the median voter prefers R to L (strictly, because of the Lemma).

$$u^{i}\left(z^{R}\right)-u^{i}\left(z^{L}\right)=-\left|\frac{z^{R}-\widehat{\alpha}^{m}}{1-\delta}\right|+\left|\widehat{\alpha}^{m}-z^{L}-\delta\frac{z^{R}-\widehat{\alpha}^{m}}{1-\delta}\right|>0,$$

Since  $z^R > \widehat{\alpha}^m$  is equivalent to:

$$\frac{z^R - \widehat{\alpha}^m}{1 - \delta} < \left| \widehat{\alpha}^m - z^L - \delta \frac{z^R - \widehat{\alpha}^m}{1 - \delta} \right|,$$

Equivalent to:

$$\frac{z^{R} - \widehat{\alpha}^{m}}{\widehat{\alpha}^{m} - z^{L}} < \frac{1 - \delta}{1 + \delta} \text{ if } \frac{z^{R} - \widehat{\alpha}^{m}}{\widehat{\alpha}^{m} - z^{L}} < \frac{1 - \delta}{\delta},$$

$$\frac{z^{R} - \widehat{\alpha}^{m}}{\widehat{\alpha}^{m} - z^{L}} < -1 \text{ if } \frac{z^{R} - \widehat{\alpha}^{m}}{\widehat{\alpha}^{m} - z^{L}} \ge \frac{1 - \delta}{\delta},$$

The second case cannot hold because  $\frac{z^R - \hat{\alpha}^m}{\hat{\alpha}^m - z^L} > 0$ , then this is equivalent to:

$$\frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} < \frac{1 - \delta}{1 + \delta},$$

Contradiction.

Now, suppose  $\frac{1+\delta}{1-\delta} < \frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L}$ , this is equivalent to  $\widehat{\alpha}^m < \frac{z^R (1-\delta) + z^L (1+\delta)}{2}$ . Furthermore,

$$p_t = (1 - \delta) \sum_{k=1}^{t} \delta^{t-k} z^{W(k)} + \delta^t p_0,$$

Then,

$$p_{t+1}^m \underset{t \to +\infty}{\sim} \frac{\widehat{\alpha}^m}{1-\delta} - \sum_{k=1}^t \delta^{t+1-k} z^{W(k)},$$

Thus,

$$p_{t+1}^{m} \underset{t \to +\infty}{<} \frac{z^{R} + z^{L} \frac{(1+\delta)}{1-\delta}}{2} - \sum_{k=1}^{t} \delta^{t+1-k} z^{W(k)} < \frac{z^{R} + z^{L}}{2},$$

Then W(t) converges to L.

Finally, suppose  $\frac{z^R - \widehat{\alpha}^m}{\widehat{\alpha}^m - z^L} < \frac{1 - \delta}{1 + \delta}$ , this is equivalent to  $\widehat{\alpha}^m > \frac{z^R (1 + \delta) + z^L (1 - \delta)}{2}$ . Then,

$$p_{t+1}^{m} > \frac{z^{R} + z^{L}}{2} + \frac{\delta}{1 - \delta} z^{R} - \sum_{k=1}^{t} \delta^{t+1-k} z^{W(k)},$$

Furthermore,  $\forall k, z^{W(k)} \leq z^R$ , then

$$\sum_{k=1}^{t} \delta^{t+1-k} z^{W(k)} \leq \frac{\delta}{1-\delta} z^{R},$$

Thus,

$$p_{t+1}^m > \frac{z^R + z^L}{2},$$

Then W(t) converges to R.

Finally, when the policy inertia degree is high enough  $\left(\frac{|z^R+z^L-2\widehat{\alpha}^m|}{z^R-z^L} \le \delta\right)$ , parties alternate in power. Indeed, if party L is the incumbent, he will necessary loose one future election. Then R wins the power and he will necessary loose one future election, and so on...

We have claimed that the dynamic can take two different paths when the median voter is indifferent between L and R. Indeed, suppose the median voter is indifferent in election t, formally,  $\left|p_t^m-z^L\right|=\left|p_t^m-z^R\right|$ . Since  $z^R\neq z^L$ , we have  $p_t^m=\frac{z^L+z^R}{2}$ . Now we compare the case where L wins the election to the case where R wins the election.

If L wins the election,  $p_t = \widehat{\alpha}^m + (1 - \delta) \frac{z^L - z^R}{2}$ . If R wins the election  $p_t = \widehat{\alpha}^m + (1 - \delta) \frac{z^R - z^L}{2}$ . Then the dynamic can change dramatically.

### 4 Conclusion

We have proposed a simple infinite horizon dynamic model of elections where candidates have fixed programs and policies present some degree of inertia. We have shown that inertia can generate political cycles.

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