E C O N O M I C S B U L L E T I N

A Reappraisal of the Irrelevance result in mixed duopoly: A note on RDcompetition

Vasileios Zikos
Department of Economics, Loughborough University

Abstract

We characterize the optimal policy-mix towards RDactivity and output production in the simultaneous moves mixed and private duopolies, as well as in the Stackelberg mixed duopoly. Our findings suggest that the government will opt for implementing jointly a tax on RDwith a subsidy on output to tackle the underlying market failures. Moreover, the optimal output subsidy, RDinvestment, output and welfare are identical irrespective of whether the public firm: (i) moves simultaneously with the private firm, (ii) is Stackelberg leader in RDand/or output, or (iii) is privatized and acts simultaneously with the private firm to maximize profits. Privatization reduces the optimal tax on Rbut leads to an increase in firms' profits. Finally, Stackelberg output leadership by the public firm induces an increase in RDtaxation, which is accompanied by a decrease in profits.

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1 Introduction

The study of mixed oligopolies has become increasingly popular over the last decade. While most of the existing work focuses on output, pricing and managerial incentives, empirical evidence suggests that public firms are present in highly innovative industries; for instance, in health-care. Indeed, they engage in R&D projects to enhance their organisational arrangements and introduce new processes of a lower cost (Aanestad et al. 2003). Further, the public sector has traditionally contributed to the development of national innovation systems in several countries (Katz 2001).

Among the noteworthy exceptions dealing with innovation in a mixed market is Delbono and Denicoló (1993). Their main conclusion is that a public firm may be used as a regulatory instrument for alleviating the overinvestment problem in the private duopoly. This result is derived under the assumption of perfect patent protection. Building on this observation, Poyago–Theotoky (1998) considered the more plausible case of easy imitation, finding that indeed almost all the main results of Delbono and Denicoló can be reversed. Relying on a non-tournament model for R&D, Nett (1994) studied the public firm's behaviour in choosing to produce at a higher than the private firm's marginal cost and showed that privatization may be welfare-improving. However, none of these papers investigated the potential role of public policy towards R&D. To this regard, it is worth stressing that in the real world, R&D subsidies have often promoted many technology discoveries with an example being biotechnology (see, for instance, Hart 1998).

An established result in the literature on mixed oligopoly—without consideration of innovation issues—is that the optimal output subsidy, as well as the first-best profits, output and welfare are identical whether the public firm: (i) moves simultaneously with the private firms, (ii) is a Stackelberg leader, or (iii) is privatized and acts simultaneously with the private firms to maximize profits (White 1996; Poyago–Theotoky 2001; Myles 2002).

However, we do not know to what extent the same outcome obtains when innovation is added into the frame of analysis, as well as whether a subsidy or tax will then be optimal. In order to address this issue, we propose a model that consists of an R&D market and a market for the (final) good. In this situation, the government faces distortions along both dimensions of production. Indeed, the combined use of 'corrective' policies towards R&D and output is considered (with a view to restoring the first-best optimum).

Our main findings can be summarised as follows. The government will opt for taxing R&D while subsidizing output production. Furthermore, the optimal output subsidy, R&D, output and welfare are the identical irrespective of whether the public firm: (i) moves simultaneously with the private firm, (ii) is a Stackelberg leader in R&D and/or output and (iii) is privatized and acts simultaneously with the private firm to maximize profits. Privatization reduces the optimal tax on R&D but increases firms' profits. Finally, Stackelberg output leadership by the public firm induces an increase in the R&D tax, whereas a reduction in firms' profits.

2 The model

We consider the following duopoly setting. The market consists of a public and a private firm, producing a homogeneous good. The inverse demand function is $P(Q) = a - Q$, $Q \leq a$, where $Q = q_0 + q_1$ denotes total output quantity that is made up, respectively, of a public and a private component. Both firms conduct process R&D that reduces their

marginal production cost by an amount x_i, $0 < x_i \leq c$, $i \in \{0,1\}$. We assume that such activity is perfectly protected against imitation and its cost is given by $\Gamma_i = x_i^2$, $i \in \{0,1\}$, which entails diminishing returns to the level of R&D expenditure. Further, firms receive a subsidy along both dimensions of their production. That is, $S_x = s_x x_i$, $S_q = s_q q_i$, denote the per-unit subsidy to R&D output and output quantity, respectively. We also allow for negative values of a subsidy, i.e., a tax can be optimal.

Thus a firm's overall cost function (net of subsidies) is given by:

$$C_i(x_i, q_i) = (c - x_i)q_i + q_i^2 + \Gamma_i - S_x - S_q, \tag{1}$$

where x_i the i-th firm's R&D ouput and $a > c > 0$.[1]

Accordingly, the relevant profit functions are:

$$\pi_i = P(Q)q_i - C_i(x_i, q_i). \tag{2}$$

Social welfare, defined as the sum of consumer surplus, $CS = (1/2)Q^2$ and firms' profits, net of subsidies, is:

$$W = CS + \pi_i + \pi_j - s_x(x_i + x_j) - s_q(q_i + q_j),\ i \neq j,\ i, j \in \{0,1\}. \tag{3}$$

Note that the *direct effect* of a subsidy on welfare is zero, as it constitutes a transfer payment. However, the standard *indirect effect* amounts to saying that welfare will be affected via any change in the rate of subsidy, which will (directly) impact the private competitor's decision variables. Thus welfare rewrites:

$$W = CS + \sum_{\substack{i,j=0 \\ i \neq j}}^{1} [(a - q_i - q_j)q_i - (c - x_i)q_i - x_i^2]. \tag{4}$$

We investigate three Stackelberg and two Cournot games. In all cases considered, the government first commits to both an R&D and an output subsidy.[2] In the next stage, the public firm invests in R&D as the leader or simultaneously with the private firm. In the third stage, firms choose their output levels either simultaneously or with the public firm being the leader. We also examine the case where the public firm is privatized and acts simultaneously with the private firm in both R&D (second stage) and output (third stage) to maximize profits. Each game is solved by backward induction to obtain its Subgame Perfect Nash Equilibrium (SPNE henceforth).

[1] It is worth stressing that a firm's R&D investment shifts its marginal cost curve downwards, $mc_i = (c - x_i) + 2q_i$, while it does not alter its slope. This is the same effect that process R&D has on production costs in d' Aspremont and Jacquemin (1988) and subsequent authors, where production costs are assumed to be linear. The inclusion of a quadratic term in the (total) cost function is standard in the mixed oligopoly literature and serves the purpose of ruling out the uninteresting case of a public monopoly.

[2] We assume that commitment to both instruments takes place simultaneously. Alternatively, it can be shown that sequential commitment, preceding the decisions on R&D and output, would have no effect on the equilibrium outcome. Furthermore, as it will become evident from our analysis, a tax on R&D output will only emerge in the SPNE of the games considered, i.e., there is no ex ante commitment of the government to a negative subsidy.

2.1 *The simultaneous moves mixed duopoly*

At the last stage of the game, each firm chooses an output level to maximize its objective. The firms' output choices, for a given level of investment and subsidies, are:[3]

$$q_0^c(x_0, x_1) = \frac{3(a-c) + 4x_0 - x_1 - s_q}{11}, \;\; q_1^c(x_0, x_1) = \frac{2(a-c) + 3x_1 - x_0 + 3s_q}{11}, \qquad (5)$$

where 'c' stands for Cournot. At the preceding stage, firms make their R&D choices for given subsidies, anticipating how their decisions will affect competition at the output selection stage. The solution to this yields:

$$x_0^c(s_x, s_q) = \frac{275(a-c) - 77s_x - 51s_q}{1837}, \;\; x_1^c(s_x, s_q) = \frac{396(a-c) + 2167s_x + 648s_q}{3674}. \qquad (6)$$

Note the standard cost-redistribution effect of the policy instruments as in (5) and (6).

In stage one, the govenment maximizes welfare with respect to $\{s_x, s_q\}$, yielding the following subsidies:

$$s_q^c = \frac{2(a-c)}{7}, \; s_x^c = -\frac{2(a-c)}{77}. \qquad (7)$$

This result, new to the literature, shows that the optimal policy-mix consists in subsidizing output while taxing R&D activity. Although actual practice tends to provide tax concessions or subsidies rather than taxes to R&D, we may explain this somewhat surprising finding by referring to the objectives of public policy.

Before proceeding, examples of public funds towards R&D in mixed markets may be useful in opening the discussion of the issue at hand. The Norwegian oil industry consists of the state-owned company Statoil that competes with two private firms, Norske Shell (or Shell Technology) and Exxon (Pal 1998). Emerging technologies, such as fuel cells and hydrogen technologies are priority on their agenda and are related to Norway's participation in energy oriented R&D projects within the EU, ie., the 5th and 6th Framework Programmes (see Godø et al. 2003). The development of these technologies is part of national innovation policies, as such potentially radical innovations are often subject to important market failures (Godoe and Nygaard 2006).[4] To this regard, established in 2001 is Enova, a public organization with main aim to subsidize enviromentally clean (and efficient) energy technologies.[5] Moreover, the Research Council of Norway is responsible for directing public funds towards R&D, which form part of the budgets of the Ministry of Petroleum and Energy.[6]

To proceed with our intuition, consider the case prior to any public intervention. In this situation, the public firm produces an output for which marginal cost equals price, as in models with an output focus alone (De Fraja and Delbono 1989). In contrast, the private firm chooses a lower output such that marginal cost exceeds price. Furthermore, total production is below the social optimum level (see Appendix A).[7] Thus, the govern-

[3] The second order conditions, which are available upon request, are all fulfilled.

[4] This is illustrated in what follows.

[5] Public funding to R&D in promoting innovations in fuel cells and related hydrogen technologies was approximately US$ 18 millions in 2001 (Godø et. al., 2003).

[6] In Greece, the telecommunications industry is another example of mixed oligopoly. Currently, there are debates on denationalizing some part of the National Telecommunications company (OTE), which has traditionally received subsidies from the state.

[7] Note that in the absence of any subsidies towards R&D or output, the public firm overproduces (both in terms of R&D and output) relative to the social optimum, whereas the private firm underproduces. The

ment is faced with the two standard failures of a mixed market: underproduction and inefficiency in the allocation of production costs across firms. The fact that the distortions on the side of R&D reinforce the ones on the side of output is unique to the current model and is in fact taken into account by the government when designing the optimal policies (i.e., the first-best allocation can be restored as we shall see).[8]

Turning to the optimal intervention, assume first that the government has only an output subsidy at her disposal. This implies that a second-best optimum can be reached, since there is only one instrument available and two targets to be controlled for (Leahy and Neary 1997). Moreover, the second-best allocation does reflect *overinvestment* in R&D (relative to the case of the first-best).[9] The reason is that even though the public firm underinvests in R&D, the private firm overinvests. As a result, it is possible, and turns out to be the case that the private firm can redress the balance and indeed can yield an amount of total investment beyond the social optimum. These results are presented in Appendix B.

An immediate implication of this observation is that it now becomes socially beneficial for the government to tax R&D. This result is also reminiscent of the finding by Leahy and Neary (1997), who showed (in the context of a purely private market) that R&D should be taxed, when there are no spillovers or even if spillovers are low enough, and actions are strategic substitutes (the latter is supported by the current model). More importantly, the optimal tax will be accompanied by an increase in the amount of output subsidy[10] in order to counteract the associated reduction in private output due to the introduction of R&D taxation (and hence eliminate the gap between price and marginal cost). Thus, it is important to mention that the result of imposing a tax on R&D is *combined* in the sense that the introduction of a policy scheme aimed at 'correcting' the failures along the side of innovation takes into account how this may influence welfare along the side of output (with also the reverse to hold).[11]

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public firm's behaviour, however, cannot redress the balance, implying that the total level of production remains suboptimal.

[8] It is worth noting that while the market failures related to output production (underproduction and inefficiently distributed production costs) have received some attention in the respective literature, the ones related to R&D seem much less recognised. More precisely, the private firm does not take into account consumer surplus in its objective function and therefore carries out less investment compared to the social optimum (so-called under-valuation effect). However, the public firm's behaviour, being consistent with welfare maximization, partially alleviates the ensuing under-investment problem. Therefore, the task of the government still remains to address underproduction in addition to the inefficiency in the distribution of post-innovation cost.

[9] The first-best optimum is defined as a public duopoly due to diminishing returns in production (both for R&D and output).

[10] That is, $s^{sq} - s^* = \frac{67749(a-c)}{250025} - \frac{2(a-c)}{7} = -\frac{25807(a-c)}{1750175} < 0$, where the asterisc denotes the socially optimal output subsidy and 'sq' denotes the case of the subsidised mixed duopoly with an output subsidy alone.

[11] By way of contrast, recent work by Gil Molto et al. (2006) characterizing the optimal intervention with a subsidy to R&D alone, showed that the existence of market failures justify positive rather than negative subsidies (i.e., taxes) at the case of the second-best optimum.

Substituting (7) into (5) and (6), we obtain the SPNE solutions for the entire game:

$$\begin{aligned} q_0^c &= q_1^c = \frac{2(a-c)}{7}, \;\; x_0^c = x_1^c = \frac{a-c}{7}, \\ \pi_0^c &= \pi_1^c = \frac{75(a-c)^2}{539}, \;\; P = (c - x_i) + 2q_i = \frac{3a+4c}{7} = mc_i, \\ CS^c &= \frac{8(a-c)^2}{49}, \;\; W^c = \frac{2(a-c)^2}{7}. \end{aligned} \tag{8}$$

Notice that the SPNE obtained is characterized by some undesirable *redistributional* effects. Although output quantity, R&D spending, welfare and consumer surplus are at the first-best, it turns out that the interaction between a tax on R&D with a subsidy on output entails a *net* positive effect on profits, thus overshooting the social optimum (i.e., $\pi_i^c > \pi_i^* = (3/49)(a-c)^2$, $i \in \{0,1\}$).

The intuition behind the latter is related to the way that the cost redistribution effect of the optimal policies operates. More precisely, the driving forces behind the equilibrium outcome are the direct effect of the subsidy (and tax) on one side, and their subsequent indirect effect on the other side. That is, a change in the rate of an instrument will directly affect the private firm's behaviour, inducing then an indirect change in the public firm's decision variables (see eq. (4), (2) and (1)). Conversely, there is a cost redistribution effect, implying a reallocation of production from the public to the private competitor. It turns out that, due to the optimality of the policy instruments, the reduction in the public firm's R&D and output and the countervailing increase in the private firm's production, will not only balance one another but also promote an increase in aggregate investment and in the output production levels (i.e., the policy tools can tackle both market failures). In turn, this corresponds to the social optimum, where the firms' production is indeed equalized (i.e., productive efficiency is attained and underproduction resolved). Nevertheless, the public firm and its private rival may still make a larger profit than the level dictated by the optimal allocation.

2.2 *The simultaneous moves private duopoly*

We proceed to solve for the SPNE in the private duopoly. The solutions for output, price, output subsidy, consumer surplus and welfare coincide with the ones in the simultaneous moves mixed duopoly. The same does not hold true for the remainder equilibrium values of profits and R&D subsidy presented below:

$$\pi_0^p = \pi_1^p = \frac{103(a-c)^2}{735}, \;\; s_x^p = -\frac{2(a-c)}{105}, \tag{9}$$

where 'p' denotes the case of the private duopoly.

Now, we are in position to make the following comparisons:

$$(i) \;\; \pi_i^c - \pi_i^p = -\frac{8(a-c)^2}{8085}, \;\; i \in \{0,1\}, \quad (ii) \;\; |s_x^c| - |s_x^p| = \frac{8(a-c)}{1155}. \tag{10}$$

According to part (i), privatization increases firms' profits (for instance, De Fraja and Delbono 1989). The intuition for part (ii), being less obvious, can be exposed as follows. In both the mixed and the private duopolies, the task of the government is to 'correct' distortions along both dimensions of production (R&D and output), while taking into

account the potential feedback of the optimal policies on one another. In particular, the mixed duopoly suffers from both underproduction and inefficiency in the distribution of equilibrium costs. In the private duopoly, however, the latter distortion vanishes as firms conduct the same level of R&D (and produce the same amount of output). Hence, naturally, a tax on R&D will be less effective, and thus lower, in the post-privatization setting.[12] Further, notice that the standard irrelevance result for the optimal output subsidy obtains (i.e., privatization has no consequences on the optimal output subsidy).

2.3 *The Stackelberg mixed duopoly*

As expected, we obtain the same SPNE outcome as in the simultaneous moves mixed duopoly. This amounts to saying that when the government has a sufficient number of instruments at her discretion—to 'correct' for the underlying failures of a mixed market—she may then restore the first-best allocation, at least partially. Consequently, the order of the firms' moves is not relevant given that the social optimum is unique (see Poyago–Theotoky 2001).

2.4 *Irrelevance of the optimal output subsidy*

As discussed in the introduction, the so-called irrelevance result shows that the optimal output subsidy (as well as profits, output, welfare) coincide independently of whether the public firm: (i) moves simultaneously with the private firms, (ii) is a Stackelberg leader, or (iii) is privatized and moves simultaneously with the private firms to maximize profits. In this paper we have studied so far the existence of the irrelevance result in terms of the R&D activity. As a next step, it would be interesting to examine whether the same outcome applies to the optimal output subsidy. To this regard, following the seminal contributions by White (1996) and Poyago–Theotoky (2001) as in (i)–(iii), one additional case needs to be taken into account; the public firm moving as the leader in output, whereas investing in R&D simultaneously with the private firm. (It can be readily verified that when the public firm retains Stackelberg leadership in both R&D and output the same SPNE obtains.)

Solving for the SPNE of this game by backward induction, we are able to confirm the irrelevance of the optimal output subsidy. This adds to a series of related results in the literature by treating within a unified framework a number of issues: R&D activity, output production and the optimal public intervention. Hence, the optimal output subsidy does not depend on whether the public firm acts as a Stackelberg leader in R&D and/or output; in addition to this, it remains independent of the government's decision to privatize the public firm. This is because provided that the government has a sufficient number of policy tools at her disposal, then she may restore the first-best allocation.[13] In this case, privatization as well as the order of firms' moves bear no consequences on the output subsidy, R&D, output and welfare, since the social optimum is unique.

Interestingly, the optimal R&D subsidy does depend on the timing of moves—and as was shown on privatization; in fact, it is higher compared to the case of the simultaneous

[12] Fjell and Heywood (2004) provide a similar intuitive argument for a model without R&D spending. Recall also that in the present setting the result of taxing R&D activity should be viewed as combined with that of subsidizing the production of output.

[13] Recall that in the 'form' of social optimum obtained, R&D, output and welfare are at the first-best except for the firms' profits.

moves mixed duopoly. That is,

$$|s_x^{**}| - |s_x^c| = \frac{10(a-c)}{301} - \frac{2(a-c)}{77} = \frac{24(a-c)}{3311}. \tag{11}$$

Recall that when the government subsidizes output alone, the second-best requires that the total amount of R&D investment is beyond the social optimum. In this case, it turns out that total investment under Stackelberg output leadership by the public firm is higher than under the Cournot conjecture. As a result, it is optimal for the government to tax R&D more heavily in the former case with a view to restoring the optimal allocation (together with the use of an output subsidy). Finally, higher R&D taxation will naturally reduce firms' profits compared to the simultaneous moves mixed market, as output subsidies are equal in both cases; i.e.,

$$\pi_i^{**} - \pi_i^c = \frac{291(a-c)^2}{2107} - \frac{75(a-c)^2}{539} = -\frac{24(a-c)^2}{23177}, \ i \in \{0,1\}. \tag{12}$$

3 Conclusion

Using a simple model, we have characterized the optimal policy intervention both for R&D and output. Our results show that the government will optimally tax the amount of R&D invested, whereas it will subsidize the production of output. The reason is that an optimal output subsidy alone tends to increase R&D above the welfare-maximizing level and so mandates an R&D tax. Moreover, what matters here is the inter-relation between the innovation and the output markets and indeed the potential feedback effects of the employed policies on one another. Therefore, the introduction of R&D taxation calls for an increase in the amount of output subsidy at the first-best allocation (relative to the second-best allocation). This also suggests that the result of taxing R&D activity should be viewed as combined with that of subsidizing the production of output.

Our findings confirm the "irrelevance result" for the optimal output subsidy, R&D investment, output and welfare. Since the government has available two instruments to control for the market failures at work (underproduction and inefficient allocation of production costs), she may then restore the first-best allocation. Indeed, whether the public firm moves as a leader in R&D and/or output is not relevant, since the optimal allocation is unique; in addition to this, the government's decision to privatize the public firm is without consequences. The only exception is the tax on R&D, which is conditional on the order of firms' moves and the possibility for privatization.

Finally, it is worth noting that when the public firm remains as a Stackelberg leader in the post-privatization regime (a case not considered here), this will naturally induce a reduction in the rate of both policy instruments. The rationale is that the effectiveness of both instruments will be lower after privatization, with the reason being that the optimal subsidy (combined with a tax) cannot restore cost efficiency. Indeed, as the privatized leader always produces more than the follower, this will imply a higher marginal cost for the leader (see also Fjell and Heywood 2004).

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Appendix

A. Unsubsidized mixed duopoly

Solving this game by the same procedure (as in case 2.1) for $s_x = 0$ and $s_q = 0$, we obtain the SPNE solutions for R&D, output, profits, social welfare, price and marginal costs:

$$\begin{aligned} x_0(0) &= \frac{25(a-c)}{167},\ x_1(0) = \frac{18(a-c)}{167},\ q_0(0) = \frac{53(a-c)}{167},\ q_1(0) = \frac{33(a-c)}{167}, \\ \pi_0(0) &= \frac{2184(a-c)^2}{27889},\ \pi_1(0) = \frac{1854(a-c)^2}{27889},\ SW = \frac{7736(a-c)^2}{27889}, \\ P(0) &= \frac{81a+86c}{167} = mc_0(0),\ mc_1(0) = \frac{48a+119c}{167},\ P(0) - mc_1(0) = \frac{33(a-c)}{167}. \end{aligned}$$

The social optimum, defined as the case of two public firms due to diminishing returns in production, is presented in (8) apart from the expression for profit, which in the optimum corresponds to:

$$\pi_i^* = \frac{3(a-c)^2}{49},\quad i \in \{0,1\}.$$

We proceed to show that the (unsubsidized) mixed duopoly is characterized by underproduction in both R&D and output (relative to the social optimum):

$$\begin{aligned} x_0(0) + x_1(0) - 2x_i^* &= -\frac{33(a-c)}{1169}, \\ q_0(0) + q_1(0) - 2q_i^* &= -\frac{66(a-c)}{1169}. \end{aligned}$$

B. Mixed duopoly with output subsidies

When an output subsidy is available, the SPNE outcomes of the entire game (denoted by the superscript 'sq') are:

$$\begin{aligned} s_q^{sq} &= \frac{67749(a-c)}{250025},\ x_0^{sq} = \frac{35548(a-c)}{250025},\ x_1^{sq} = \frac{38898(a-c)}{250025},\ q_0^{sq} = \frac{14284(a-c)}{50005}, \\ q_1^{sq} &= \frac{71313(a-c)}{250025},\ \pi_0^{sq} = \frac{8675789676(a-c)^2}{62512500625},\ \pi_1^{sq} = \frac{8658033534(a-c)^2}{62512500625}, \\ CS^{sq} &= \frac{20372709289(a-c)^2}{125025001250},\ SW^{sq} = \frac{142787(a-c)^2}{500050}. \end{aligned}$$

Comparing the R&D investment with the social optimum both for the private and the public firm, we obtain that the private firm overinvests, whereas the public firm

underinvests; total investment exceeds the optimal level:

$$\begin{aligned} x_0^{sq} - x_i^* &= -\frac{1189(a-c)}{1750175}, \\ x_1^{sq} - x_i^* &= \frac{22261(a-c)}{1750175}, \\ x_0^{sq} + x_1^{sq} - 2x_i^* &= \frac{21072(a-c)}{1750175}. \end{aligned}$$

Notice further that the public firm chooses an output such that price equals marginal cost. The gap between price and marginal cost of the private firm is smaller compared to the case of the unsubsidized mixed duopoly:

$$P^{sq} = \frac{107292a + 142733c}{250025} = mc_0^s,$$

$$mc_1^{sq} = \frac{103728a + 146297c}{250025}, \quad P^{sq} - mc_1^{sq} = \frac{3564(a-c)}{250025},$$

$$[P(0) - mc_1(0)] - [P^{sq} - mc_1^{sq}] = \frac{7655637(a-c)}{41754175}.$$