E C O N O M I C S   B U L L E T I N

The Optimal Regulation of Product Quality under Monopoly

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Abstract

This paper characterizes the optimal quality regulation of a monopolist when quality is observable. In contrast to Sheshinski (1976) it is shown that a minimum quality standard may be desirable even if it induces the firm to reduce output.

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1 Introduction

It has long been known that the provision of product quality under monopoly is distorted away from the social optimum even if quality is observable. In particular, Spence (1975) and Sheshinski's (1976) by now classic papers have shown that a monopolist chooses the level of quality that suits the marginal consumer, while a social planner takes the valuations of all consumers into account. Therefore, to the extent that inframarginal consumers have a higher (lower) valuation for quality than marginal consumers, a monopolist provides too little (too much) quality, given the size of demand.[1]

In principle, this market failure can be corrected by imposing the welfare maximizing price and quality on the firm. In practice, however, it is often not feasible or desirable to regulate the price. One prominent reason is that the prospect of profits is what gives firms an incentive to provide desired products in the first place. Hence, Sheshinski (1976) analyzes the optimal quality regulation of a monopoly in the absence of price interventions.[2]

Two basic cases are distinguished. When quality and quantity are complements so that $p_{xq} > 0$, the optimal regulation can not generally be determined. However, when they are substitutes ($p_{xq} < 0$), there is a clear policy implication (Sheshinski 1976, p. 135):[3]

> "... when $p_{xq} < 0$ the regulator, starting from the monopoly equilibrium $(\hat{x}, \hat{q})$, should seek to raise quality levels if $m_q - c_{xq} > 0$ but to lower them if $m_q - c_{xq} < 0$ ..."

Alas, this result, which is the central proposition for the quality regulation of a monopolist, is wrong. In fact, the direction of quality regulation does not change with the sign of the effect of a quality increase on the monopolist's marginal profit ($m_q - c_{xq}$ in Sheshinski's model). Rather, the optimal policy can be described as follows: A minimum quality regulation improves welfare by providing a quality level that is closer to consumers' wants. However, if the price is unregulated, the monopolist responds to the regulation by increasing prices. If this price increase is so high that demand decreases despite higher quality, there is an additional monopoly distortion that counters the positive effect of the regulation. As a consequence, if the demand reduction is very strong, the regulator should reduce the level of quality the monopolist provides instead of increasing it. The aim of this note is to demonstrate this claim within Sheshinski's model.

2 The Model

I will use Sheshinski's original formulation to derive the optimal regulatory policy. There is a monopolist producing a good of quality $q \geq 0$ which he sells for price $p \geq 0$. Inverse demand is

[1] The argument extends to oligopolistic market structures.

[2] Sheshinski (1976) focusses on the case where quality is observable and the firm is a single-product monopolist. In alternative settings, different results may obtain (see for instance the survey by Sappington, 2005).

[3] Fortunately, the latter case seems to be more relevant in practice. $p_{xq} < 0$ means that consumers with a higher valuation for a product also have a higher valuation for quality improvements. This is routinely assumed in models of product differentiation and price discrimination (e.g., Mussa and Rosen, 1978, and Besanko, Donnenfeld, and White, 1987).

given by $p(x,q)$ where $x \geq 0$ denotes the quantity sold. It is assumed that $p_x(x,q) < 0$, $p_q(x,q) > 0$ and $p_{xq} < 0$.[4] Total costs are denoted by $c(x,q)$ with $c_x(x,q) > 0$ and $c_q(x,q) > 0$.

Firm and regulatory authority play a two-stage game. In stage one the regulator chooses q to maximize welfare taking into account that the monopolist chooses x to maximize profit in stage two. The firm's profit function is $\pi(x,q) = p(x,q)x - c(x,q)$. Solving the game by backward induction the second stage outcome is therefore given by the first order condition

$$\pi_x(x,q) = p_x(x,q)x + p(x,q) - c_x(x,q) = 0. \tag{1}$$

The appropriate second order condition $\pi_{xx}(x,q) < 0$ will be assumed to hold.

In stage one the regulator maximizes the welfare function

$$V(x(q),q) = \int_0^{x(q)} p(z,q)dz - c(x(q),q). \tag{2}$$

This yields the first order condition

$$\frac{dV}{dq} = (p - c_x)\frac{dx}{dq} + \int_0^x p_q(z,q)dz - c_q = 0 \tag{3}$$

where the appropriate second order condition $d^2V/dq^2 < 0$ is again assumed to be fulfilled. This gives us the (second best) regulatory equilibrium (x^*, q^*). We will have to compare this with an unregulated situation.

Without regulation the producer chooses quality directly to maximize profits. The market outcome $(\hat{x}, \hat{q})$ is characterized by (1) and the first order condition with respect to q,

$$\pi_q(x,q) = p_q(x,q)x - c_q(x,q) = 0. \tag{4}$$

Additional second order conditions are $\pi_{qq} < 0$ and $\pi_{xx}\pi_{qq} - \pi_{xq}^2 > 0$ which are again assumed to hold.

The most convenient way of comparing q^* and $\hat{q}$ is by representing the two solutions graphically in an (x,q) plane. $(\hat{x}, \hat{q})$ is determined by the intersection of the curves $\pi_x = 0$ and $\pi_q = 0$, given by equations (1) and (4). From (1) we can infer the slope of the curve $\pi_x = 0$ as

$$\left.\frac{dq}{dx}\right|_{\pi_x=0} = -\frac{\pi_{xx}}{\pi_{xq}} = -\frac{p_{xx}x + 2p_x - c_{xx}}{p_{xq}x + p_q - c_{xq}}. \tag{5}$$

By the second order condition this has the same sign as π_{xq}, the marginal impact of q on (1). Figure 1 (a) displays the case where $\pi_{xq} > 0$ (with an upward sloping curve $\pi_x = 0$) while the case $\pi_{xq} < 0$ is represented in Figure 1 (b) (with a downward sloping curve $\pi_x = 0$). Using (4), the slope of the curve $\pi_q = 0$ is found to be

$$\left.\frac{dq}{dx}\right|_{\pi_q=0} = -\frac{\pi_{xq}}{\pi_{qq}} = -\frac{p_{xq}x + p_q - c_{xq}}{p_{qq}x - c_{qq}} \tag{6}$$

[4] Subscripts denote partial derivatives.

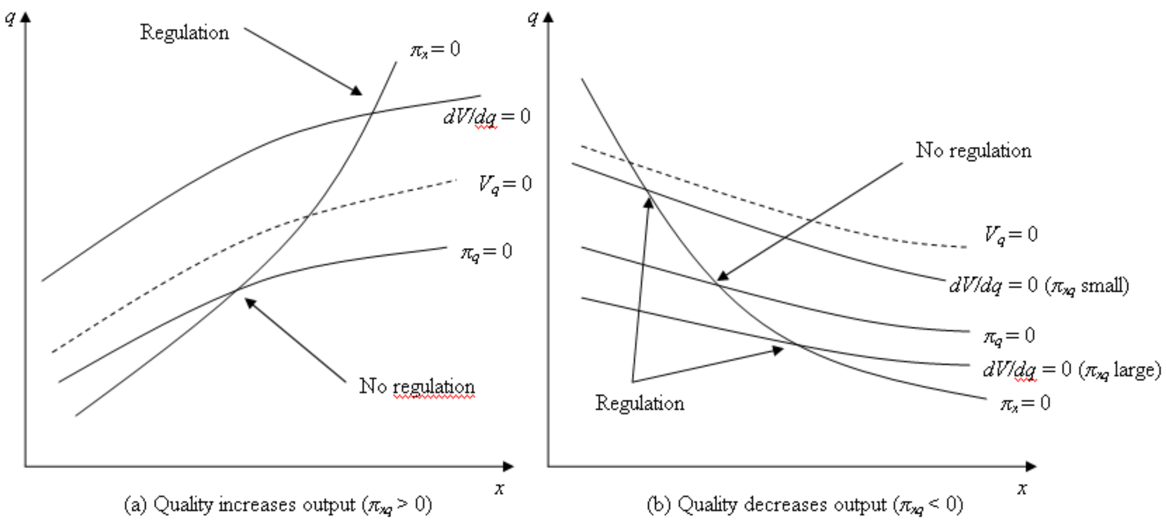

Figure 1: The regulatory solution

which (again by the second order condition) also has the same sign as π_{xq}. Simple algebra shows that $dq/dx|_{\pi_x=0} > dq/dx|_{\pi_q=0}$ is equivalent to $\pi_{xx}\pi_{qq} > \pi_{xq}^2$ (when $\pi_{xq} > 0$), respectively $\pi_{xx}\pi_{qq} < \pi_{xq}^2$ (when $\pi_{xq} < 0$). As we know $\pi_{xx}\pi_{qq} > \pi_{xq}^2$ to be true from the second order conditions, the curve $\pi_x = 0$ is steeper than the curve $\pi_q = 0$ around $(\hat{x}, \hat{q})$ in both cases. Figure 1 presents the unregulated solution graphically.

(x^*, q^*) are jointly determined by the intersection of the curves $\pi_x = 0$ and $dV/dq = 0$ given by (1) and (3). Characterizing the latter turns out to be a bit more tedious. What complicates matters is that $dV/dq = 0$ takes the impact of q on x into account. In order to determine its position, it is helpful to first derive the curve $V_q = 0$ which leaves aside this indirect effect. From (2) we find

$$V_q = \int_0^x p_q(z, q)dz - c_q = 0. \tag{7}$$

Since by assumption $p_{xq} < 0$, it follows that $p_q(z, q) > p_q(x, q)$ for all q and $z < x$. Therefore, $V_q = \int_0^x p_q(z, q)dz - c_q(x, q) > p_q(x, q)x - c_q(x, q) = \pi_q$. That is, whenever $V_q = 0$, we have $\pi_q < 0$. As $\pi_{qq} < 0$, q must be decreased to reach $\pi_q = 0$ given x. In other words, the curve $V_q = 0$ lies everywhere above the curve $\pi_q = 0$. This is shown in Figure 1 (a) and (b).

Next we will determine the position of $dV/dq = 0$. Plugging (1) and (7) into (3) we find that $dV/dq = -p_x x \cdot dx/dq + V_q = 0$. Using (5) and the fact that $p_x < 0$, we have $sign[-p_x x \cdot dx/dq] = sign[dx/dq] = sign[\pi_{xq}]$. Assume first that $\pi_{xq} > 0$ (Figure 1 (a)). Then from an arbitrary (x, q) that satisfies $V_q = 0$, q must be increased given x to reach $dV/dq = 0$ if and only if $V_{qq} < 0$.[5] Note that $V_{qq} < 0$ is a second order condition for the first best solution (x^{**}, q^{**}) where both quantity and quality are regulated. Hence, it is a mild requirement and I follow Sheshinski (1976) in assuming it. From the previous arguments it can be concluded that $dV/dq = 0$ lies above $V_q = 0$. As Figure 1 (a) shows, we therefore have $q^* > \hat{q}$ and $x^* > \hat{x}$ at the intersection of $dV/dq = 0$ and $\pi_x = 0$.

[5] At (x, q) satisfying $V_q = 0$ we must decrease V_q to reach $dV/dq = 0$ as $dV/dq > V_q = 0$. When $V_{qq} < 0$ such a decrease implies an increase in q.

Next assume $\pi_{xq} < 0$ (Figure 1 (b)). By a similar argument we find that $dV/dq = 0$ lies below $V_q = 0$. Clearly, *how much* it lies below depends on the magnitude of π_{xq}. Figure 1 (b) presents two possibilities, the curve $dV/dq = 0$ (π_{xq} small) and the curve $dV/dq = 0$ (π_{xq} large), demonstrating that we may either have $q^* > \hat{q}$ and $x^* < \hat{x}$ or $q^* < \hat{q}$ and $x^* > \hat{x}$, contradicting the mistaken result in Sheshinski (1976) who claims that one can always conclude $q^* < \hat{q}$ and $x^* > \hat{x}$.

The difference is seen most clearly when $\pi_{xq} = 0$. In this case a regulatory intervention in q does not alter the monopolist's quantity choice. Hence, an infinitesimal increase in q has the welfare improving impact of increased quality without side-effects.[6] There is no effect on welfare regarding the monopoly quantity distortion, so the overall effect is clearly positive while Sheshinski's result suggests that there is no impact on welfare. When $\pi_{xq} > 0$, a minimum quality requirement even has a positive effect on the monopoly distortion through increased output. Finally, if $\pi_{xq} < 0$, then a small quality increase unfortunately triggers a larger monopoly distortion as monopoly prices go up too much to sustain the demand level. As long as the positive welfare effect of a quality increase is strong enough, however, there is scope for a minimum quality regulation even in this case.

References

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[6] As noted earlier, Spence (1975) and Sheshinski (1976) show that a monopolist underprovides quality given output when $p_{xq} < 0$.