E C O N O M I C S B U L L E T I N

Mixed Motives of Simultaneous-move Games in a Mixed Duopoly

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Abstract

This paper investigates the simultaneous-move games in a mixed duopoly where firms are maximizers of either profits or relative profits. Contrary to previous results, if each firm has mixed motives about payoff in a simultaneous-move game, a private firm monopolizes whereas the public firm produces nothing.

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1 Introduction

There has recently been much interest in a mixed oligopoly that analyze the competitive relationship between public and private firms. In reality, many public firms are placed in a situation where they have to compete against private firms. In many countries, there are competitive relationships between the public and private firms. Analyses on typical public firms only are conducted essentially within the paradigm of a monopoly where the public firm and the private firm operate as a monopolist ignoring strategic interactions among themselves.

In pioneering work of the mixed oligopoply, De Fraja and Delbono (1989) made an important study on mixed oligopoly as a game theory. In particular, De Fraja and Delbono (1989) investigate the objective function and payoff patterns of the public firm as it is privatized in view of social welfare. In some cases a public firm should be privatized and maximize profits rather than social welfare. They also found that there exists a critical number of private firms such that privatization of the public firm improves social welfare. Thanks to the analysis by De Fraja and Delbono(1989), the analyses of the mixed oligopoly are actively tackled from various angles, such as private firms' endogenous and exogenous market entry, product differentiation between public and private firms, partial privatization, under capacity of public firms and excess capacity of private firms[1].

However, one of the most fundamental assumptions in mixed oligopoly is that all firms maximize absolute profits. From the viewpoint of mixed motives about profits, Schaffer (1989) and Vega-Redondo (1997) analyzed the relative payoff in terms of the firms' level as they examine Cournot equilibrium in the evolutionary game theory. According to Schaffer (1989) and Vega-Redondo(1997), a unilateral deviation from Cournot equilibrium that decreases the profit of deviator, but decrease the other firms' profit even more. Schaffer (1989) and Vega-Redondo (1997) argued that instability of Cournot equilibrium in the viewpoint of the evolutionary game theory as well as demonstrated that a Walrasian equilibrium forms as an evolutionary consequence resulting from the process where every firm tries to be ahead of every other one. According to Alos-Ferrer (2004), Riechmann (2002, 2006) and Schipper (2005), in case where firms aiming to maximize relative payoffs to win competitors and firms aiming to maximize absolute payoff coexist, the amount of production falls into the range between Cournot equilibrium and Walrasian equilibrium. That is, if firms do not care about their influence on the market price and behavior as price-taker, the equilibrium outputs result in Walrasian competitive market[2].

Although some theoretical works have already succeeded in explaining a mixed oligopoly,

[1]For pioneering work, see Beato and Mas-Collel (1984). For excellent surveys, see De Fraja and Delbono (1990), Nett (1993) and Bos (1991). For recent literature on mixed oligopoly, see Matsumura(1998), Matsumura and Matsushima (2003, 2004), Lu and Poddar (2005), Ogawa(2006), among others.

[2]In other words, if the relative payoff is the measure, it pays to hurt in terms of absolute payoff as long as by hurting yourself you hurt your opponent even more. This class of game called "spite effect" in Riechmann (2002).

this study adds mixed behavioral motives on the payoff for a public firm and a private firm in simultaneous-move game. We focus on the mixed situations in which each firm aims to maximize its absolute payoff and attempts to maximize relative payoff which defined as the difference between firm's absolute payoff and average absolute payoff of all firms.

2 The Model

Consider a mixed duopoly model with a public firm and a private firm, all producing a single homogenous product with inverse demand given by

$$p = 1 - Q \quad \text{where} \quad Q = q_0 + q_1$$

where p is a price for two firms ($i = 0, 1$), q_0 and q_1 denote the output of public firm and private firm, respectively. The price can not be equal to $1 - Q$ for all values of Q, for then it would be negative for $1 < Q$. Assume that total cost to firm i of producing quantity q_i is $C(q_i) = cq_i$. Assume that $1 > c > 0$, so that there is some value for total output Q for which market price is greater than the firms' common marginal cost c. That is, there is no fixed costs and common marginal cost is constant[3].

The absolute payoff of each firm i is given by

$$\pi_i^a = p(q_i - c). \tag{1}$$

Relative payoff to firm i is defined in the evolutionary game theory (Samuelson(1997, pp. 66) and Weibull(1995, pp. 71-74)) as the difference between i's absolute payoff and the average absolute payoff of all firms. The average absolute payoff is given by $(1/2)\pi_j^a$ in our framework[4]. Thus, the relative payoff to firm i is as follows:

$$\pi_i^r = \left(\frac{2-1}{2}\right)\pi_i^a - \frac{1}{2}\pi_j^a = \frac{1}{2}[p(q_i - c) - p(q_j - c)]. \tag{2}$$

To distinguish notations, the superscript a and r are defined as the absolute-payoff-maximizer and relative-payoff-maximizer, respectively. Thus, the public firm 0 maximizes the social welfare W, given by

$$W^{lm} = \frac{Q^2}{2} + \pi_0^l + \pi_1^m \quad \text{where} \quad l = a, r, m = a, r \tag{3}$$

where $Q^2/2$ is the consumer surplus and π_1^m is the profit of private firm.

[3]To simplify the analysis, cost function is the same for both private and public firm in our paper. Thus, we rule out the possibility that privatization of the public firm reduces production cost. It turns out that it is impossible for political or economics reasons to nationalize the public firm. There are theoretical and empirical works for the equality of cost efficiency between the public firm and private firm, see Matsumura and Kanda (2005) for appropriate references. In the literature, the public firm is assumed to be less efficient than the private firm, e.g. Pal (1998).

[4]In the case of n firms, the relative payoff to firm i will be defined $\pi_i^r = [(n-1)/n]\pi_i^a - (1/n)\pi_j^a$.

3 Results

We consider firms' choice of quantity in the following four cases.

[**Absolute payoff in mixed duopoly**]: Following standard equilibrium concept, the public firm's absolute payoff objective is to maximize welfare defined as the sum of consumer surplus and profit all firms, and the private firm's absolute payoff objective is to maximize its own proftit as follows:

$$\max_{q_0} W^{aa} = \frac{Q^2}{2} + \pi_0^a + \pi_1^a,$$

and

$$\max_{q_1} \pi_1^a = (1 - q_0 - q_1)(q_1 - c).$$

The maximization problem of each firm yields

$$q_0 = \frac{1-c}{2}, \tag{4}$$

$$q_1 = \frac{1-c-q_0}{2}. \tag{5}$$

By solving (4) and (5), we obtain the equilibrium output levels in simultaneous-move game as

$$q_0^* = \frac{1-c}{2}, \tag{6}$$

$$q_1^* = \frac{1-c}{4}. \tag{7}$$

[**Relative payoff in mixed duopoly**]: Similarly, the maximization problems for the relative-payoff-maximizer of public firm and private firm, and private firm's objective function are given by

$$\max_{q_1} \pi_1^r = \frac{1}{2}[p(q_1 - c) - p(q_0 - c)], \tag{8}$$

and

$$\max_{q_0} W^{rr} = \frac{Q^2}{2} + \pi_0^r + \pi_1^r, \tag{9}$$

respectively. Given the relative-payoff-maximizer, the private firm maximizes (8) with respect to q_1. The maximization problem yields

$$q_1 = \frac{1-c}{2}.$$

However, the relative-payoff-maximizer of public firm maximizes (9) with respect to q_0, which yields

$$\frac{\partial W^{rr}}{\partial q_0} < 0 \Leftrightarrow q_0 = -q_1.$$

Hence we obtain the equilibrium output levels as

$$q_0^* = 0, \tag{10}$$

$$q_1^* = \frac{1-c}{2}. \tag{11}$$

Notice that if both private and public firm are relative-payoff-maximizer in simultaneous-move game where welfare-maximizer public firm competes against the private firm, the private firm who is relative-payoff-maximizer can achieve the monopolistic position because the public firm's output becomes zero. Thus, in the point of public firm's view, welfare-maximizer public firm prefers absolute payoff function to relative payoff function. Thus, we will examine a case where mixed payoff motives exist for this reason.

[**Mixed motives of payoff in mixed duopoly**]: In this case, given the private firm aims to maximize absolute(resp. relative) payoff and the public firm aims to maximize relative(resp. absolute) payoff, each objective function of public firm is as follows:

$$\max_{q_0} W^{ra} = \frac{Q^2}{2} + \pi_0^r + \pi_1^a, \tag{12}$$

and

$$\max_{q_0} W^{ar} = \frac{Q^2}{2} + \pi_0^a + \pi_1^r. \tag{13}$$

By maximizing (12) or (13) with respect to q_0, each first-order condition becomes

$$\frac{\partial W^{ar}}{\partial q_0} = \frac{\partial W^{ra}}{\partial q_0} < 0 \Leftrightarrow c < 1, \tag{14}$$

respectively. Given the assumption, $1 > c$ in basic model, we can verify that both $\partial W^{ar}/\partial q_0$ and $\partial W^{ra}/\partial q_0$ are negative. Consequently, at any given pair of relative-payoff-maximizers and absolute-payoff-maximizer of public firm, the public firm does not produce the output. There would be no output for the public firm at which the public firm could make any profit, because the market price never exceeds 1.

On the other hand, we obtain the optimal output level of the private firm as

$$q_1^* = \frac{1-c}{2}.$$

The private firm wants to produce monopoly output whether there is absolute payoff function or relative payoff function in simultaneous-move game.

3.1 Implication for Simultaneous-move Game in Mixed Duopoly

We have analyzed four cases in a simultaneous-move game. The result shows that if the public firm aims to maximize the absolute payoff, the private firm will produce less than when it aims to maximize the absolute payoff (see Equation (6) and (7)). Therefore, if the public firm attempts to maximize the absolute payoff, the private firm would not have an incentive to maximize the absolute payoff. In other cases, the public firm's production becomes zero, allowing the private firm's monopoly payoff and price set. In the case where each firm's payoff motives is mixed, the resulting equilibrium turns out to be an inefficient level with the monopoly of private firm even if the public firm participates in the productive activity. Therefore, the market price equals to the marginal cost of private firm, and public firm does not produce any positive output where the mixed payoff motives exist in the simultaneous-move game of a mixed oligopoly. The existence of the pubic firm never affects the equilibrium output in the simultaneous-move games discussed in mixed motives.

Alos-Ferrer (2004), Reichmann (2002, 2006) and Schipper (2005) focused on only the private firms in the simultaneous-move game. They argued that a private firm that values relative payoff would not have an incentive to maximize the absolute payoff, whereas a private firm that values the absolute payoff would not pursue the maximization of relative payoff. Thus, they concluded that the evolutionary equilibrium is stable. In the case when the firms that pursue to maximize relative payoff and those that pursue to maximize absolute payoff to surpass competitors, Alos-Ferrer (2004), Reichmann (2002, 2006) and Schipper (2005) argued, the industry's production is determined between the range of Cournot equilibrium and Walrasian equilibrium output. If the public and private firms coexist in a simultaneous-move game, however, Alos-Ferrer (2004) and Reichmann (2002, 2006)'s equilibrium would not be obtained.

Our result derives from the fact that in the mixed oligopoly, the relative-payoff-maximizer of the private firm has a dominant strategy. In this regard, whether the public firm is payoff maximizer or not, the best payoff maximizer for the private firm is the absolute-payoff-maximizer. Therefore, investigating the sequential-move game in the mixed oligopoly is needed for future research[5].

4 Concluding Remarks

In this paper, we investigate the simultaneous-move game in a mixed duopoly where firms are maximizers of either profits or relative profits. Contrary to previous results, if each firm has mixed motives about payoff in a simultaneous-move game, a private firm monopolizes whereas the public firm's amount of production becomes zero. However, our result might depend on the assumption that common marginal cost is constant regardless of who

[5]For the sequential-move games in the mixed duopoly, it is available from author upon request.

produces the output in mixed duopoly. Thus, extending our formulation to encompass a different marginal cost and endogenous the number of entry firms are further subjects for future research.

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