E C O N O M I C S B U L L E T I N

A probabilistic analysis of a scheduling problem in the economics of tourism

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Abstract

The scheduling problem faced by a firm (or by a government agency) that is responsible for providing transportation to tourists who would like to visit a particular location has received scant theoretical attention in the tourism literature. Therefore, we conduct a probabilistic analysis of the scheduling problem in this paper. Specifically, we first delineate a generic model that accounts for the common features of visits to many locations such as fiords, game parks, lakes, and wildlife reserves. Next, we derive the transportation providing firm's long run expected profit per unit time function. Finally, we show that the optimal frequency with which transportation ought to be provided to tourists is the solution to our firm's long run expected profit maximization problem.

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1. Introduction

As the World Tourism Organization (UNWTO), a specialized agency of the United Nations, celebrates its sixtieth anniversary in 2006, there is no gainsaying the fact that tourism is one of the biggest growth industries in the world today. As noted by the UNWTO,[1] international tourism is the world's largest export earner and hence it is a significant factor in the balance of payments of most countries. In addition, tourism is also one of the world's most salient generators of employment. Many new tourism jobs and businesses are created in the non-urban parts of the world's developing nations and this provides a positive incentive to rural residents to stay in rural areas and not migrate to already overcrowded cities. Finally, it should not go unsaid that the intercultural awareness and personal friendships that are created through tourism can be a potent force in ameliorating international understanding and in contributing to peace between the various nations of the world.

Tourists from different parts of the world typically place a high premium on visiting naturally beautiful locations such as fiords, lakes, national parks, and wildlife reserves. Examples of such locations include the spectacular fiord known as Milford Sound in New Zealand's South Island, Lake Geneva bordering the city of Geneva in Switzerland, Grand Canyon National Park in the state of Arizona in the United States, and Krueger National Park in South Africa. Individuals wishing to tour these kinds of naturally beautiful locations typically do not arrange their own transport. Instead, private firms (or government agencies) generally provide the relevant transportation services. The reader should note that such firms also commonly provide transportation services to individuals interested in going on city tours, museum tours, canal tours (in Amsterdam), etc.

Whatever kind of tour one has in mind—lake, museum, national park, etc.—a key decision problem faced by a firm that is responsible for providing transportation services to tourists is a scheduling problem. This scheduling problem concerns the *frequency* with which a particular kind of transportation service ought to be provided. If this frequency is too high then it is quite likely that the buses, boats, or airplanes will be operating at less than full capacity and this excess capacity will have a negative impact on the transport providing firm's profits. In contrast, if the frequency is too low then this is likely to lead to long queues, congestion, loss of actual and potential customers and this negative publicity, as before, will also tend to have a negative impact on the transport providing firm's profits. Therefore, as far as the profit maximizing transport providing firm is concerned, it is of considerable importance to determine the *optimal frequency* with which transport service ought to be provided.

With regard to scheduling, Yan and Chen (2002) have developed a mixed integer multiple commodity network model to assist inter-city bus carriers in Taiwan with timetable and route maintenance. With an eye on the environmental impacts of fleet vehicle routing, Dessouky *et al.* (2003) have analyzed an optimization model and have shown that with only marginal increases in operating costs and service delays, it is possible to mitigate the environmental impacts of

[1] Go to www.unwto.org/aboutwto/eng/aboutwto.htm and see Chao *et al.* (2004), Mansfield and Winckler (2004), and Melisidou and Varvaressos (2004) for more details on international tourism and its economic impacts.

transportation substantially. Focusing on busy train stations, Carey and Carville (2003) have developed an algorithmic approach to what they call the scheduling and the "platforming" of trains. Miller *et al.* (2005) have used a random utility framework to study a tour based model of travel mode choice in which each individual selects the best combination of modes available to execute a tour subject to automobile availability constraints.

Focusing specifically on tourism, Kim and Ngo (2001) have attempted to model and forecast monthly airline passenger flows between three cities in Australia. Their analysis shows that univariate models generate more accurate forecasts than do multivariate models and that the time series for the Sydney/Melbourne route has a great impact on the other routes being analyzed. Finally, Nordstrom (2005) has modeled the demand for international tourism in Sweden and has shown that when studying this demand, it is important to use a utility function that has a dynamic and a stochastic part to it.

The studies that we have just discussed in the previous two paragraphs have certainly advanced our understanding of some aspects of scheduling in general and demand and forecast issues in the context of tourism. This notwithstanding, to the best of our knowledge, the existing literature has paid virtually no attention to the problem of *theoretically* determining the *optimal frequency* with which transportation ought to be provided to tourists by a profit maximizing firm. Therefore, we conduct a *probabilistic* analysis of this question in this paper. In particular, we first delineate a generic model that accounts for the common features of visits to many locations such as fiords, game parks, lakes, and wildlife reserves. Next, we derive a transportation providing firm's long run expected profit per unit time function ($LRE\Pi$). Finally, we show that the optimal frequency with which transportation ought to be provided to tourists is the solution to our firm's long run expected profit maximization problem.

The rest of this paper is organized as follows. Section 2.1 describes the renewal-reward theorem that will form an essential part of our transport providing firm's $LRE\Pi$ determination problem. Section 2.2 delineates the probabilistic features of a general model that captures the typical features of visits to places such as fiords, game parks, lakes, and wildlife reserves. Section 2.3 derives our transport providing firm's $LRE\Pi$ from the probabilistic model features described in section 2.2. Section 2.4 solves this transport providing firm's $LRE\Pi$ maximization problem and, in the process, determines the optimal frequency of transport provision. Section 3 concludes and discusses plausible extensions of the research described in this paper.

2. The Theoretical Framework

2.1. Preliminaries

The textbook by Ross (2003, pp. 416-425) tells us that a stochastic process $\{Z(t):t\geq 0\}$ is a counting process if $Z(t)$ represents the total number of counts that have taken place by time t. Obviously, since $Z(t-1)$, $Z(t)$, $Z(t+1)$, etc. are stochastic, the time between any two counts $Z(t)$ and $Z(t-1)$ is also stochastic. This time between any two counts is called the interarrival time. A counting process for which the interarrival times have a general cumulative probability distribution function is said to be a renewal process.

Consider a renewal process $\{Z(t){:}t\geq 0\}$ with interarrival times X_z, $z\geq 1$, which have a cumulative probability distribution function $F(\cdot)$. In addition, assume that a monetary reward R_z is earned when the zth renewal is completed. Let $R(t)$, the total reward earned by time t, be $\Sigma_{z=1}^{Z(t)} R_z$, and let $E[R_z]=E[R]$, and $E[X_z]=E[X]$. The renewal-reward theorem—see Ross (2003, p. 417) or Tijms (2003, p. 41)—tells us that if $E[R]$ and $E[X]$ are finite, then with probability one,

$$\lim_{t\to\infty}\frac{E[R(t)]}{t}=\frac{E[R]}{E[X]}. \tag{1}$$

In words, equation (1) is telling us that if we think of a cycle being completed every time a renewal occurs, then the long run average reward—the left-hand-side (LHS) of equation (1)—is the average reward in a cycle or $E[\textit{reward per cycle}]=E[R]$ divided by the average amount of time it takes to complete that cycle or $E[\textit{length of cycle}]=E[X]$. We now proceed to the probabilistic features of a generic model that captures the typical features of visits to places such as fiords, game parks, lakes, and wildlife reserves.

2.2. A generic model

Consider a profit maximizing private firm that provides transportation to tourists wishing to visit a particular location. For concreteness, we shall think of this location as a lake but the reader should note that, without loss of generality, our analysis holds for other locations—such as fiords, national parks, and wildlife reserves—as well. Our transport providing firm's boat departs for a tour of the lake under study every T time periods where T is deterministic.

We suppose that visitors wishing to go on a sightseeing tour of the lake arrive in accordance with a Poisson process with rate λ.[2] Now, it is clear that not every tourist in the vicinity of the lake will want to take the boat trip. For a variety of reasons, some tourists may prefer to simply stroll along the lake shore. To model this feature of the underlying problem, we suppose that a potential tourist who sees a boat leaving t time periods from now will join the boat with probability $e^{-\delta t}$ where $t\in[0,T]$.

Revenues and costs accrue to our firm from the provision of transport (boat rides) to tourists. There are various ways to model these revenues and costs but one straightforward way is as follows: The firm in question incurs a fixed cost of $C>0$ for every boat round trip. In addition, this firm collects ticket or marginal revenue of $R>0$ from each tourist who takes the boat ride. With this specification of revenues and costs, our task now is to derive the firm's long run expected profit per unit time function or $LRE\Pi$.

[2] See Ross (2003, chapter 5) or Tijms (2003, chapter 1) for textbook accounts of the Poisson process.

2.3. Long run expected profit

We want to use the renewal-reward theorem (see equation (1)) to undertake the necessary derivation. However, before we do this, we must first resolve three matters. To this end, we now note that the renewal stochastic process that we want to work with is the continuous time stochastic process describing the number of tourists waiting for the departure of a boat. Further, the renewal epochs or time points for this renewal stochastic process are the epochs or time points at which a boat departs for a lake trip.

The second matter to resolve is the stochastic process describing the arrival of tourists who *actually take* a boat trip. Note that because all arriving tourists at the lake under study do not necessarily take a boat trip, even though the arrival process of tourists in general is Poisson, the arrival process of tourists who actually take a boat trip is *not* a Poisson process but instead a non-homogeneous Poisson process with intensity function $\lambda(t)$[3] where

$$\lambda(t)=\begin{cases}\lambda e^{-\delta(T-t)}, & t\in[0,T)\\ \lambda(t-T), & t\geq T\end{cases}. \tag{2}$$

Finally, we note for subsequent use that because the arrival of tourists who actually take a boat trip follows a non-homogeneous Poisson process with intensity function described by equation (2), we can use Theorem 5.10 in Kulkarni (1995, p. 224) and conclude that the number of tourists who actually take a boat trip is Poisson distributed with mean

$$\int_0^T \lambda(t)dt=\frac{\lambda}{\delta}(1-e^{-\delta T}). \tag{3}$$

With equation (3) in place, we can now derive our transport providing firm's $LRE\Pi$. Specifically, because the renewal epochs of the renewal stochastic process we are working with are the epochs at which a boat departs for a lake trip, it is clear that the expected length of a cycle or $E[length\ of\ cycle]=T$. Further, to compute the mean profit per cycle note that the mean revenue per cycle equals $R\times(\lambda/\delta)(1-e^{-\delta T})$. Further, the mean cost is simply C. Therefore, the average profit per cycle or $E[reward\ per\ cycle]=R\times(\lambda/\delta)(1-e^{-\delta T})-C$. Now, using the two previous expectations and the renewal-reward theorem (equation (1)), we can tell that our transport providing firm's long run expected profit per unit time function is

$$LRE\Pi=\frac{R\times(\lambda/\delta)(1-e^{-\delta T})-C}{T}. \tag{4}$$

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The non-homogeneous Poisson process is sometimes also referred to as the non-stationary Poisson process. For textbook accounts of this process, see Ross (2003, pp. 316-321) or Tijms (2003, pp. 22-24).

Equation (4) conforms well with our intuition regarding the expected impact of changes in the marginal revenue term R and the cost term C on our firm's long run expected profit. Specifically and as expected, we see that an increase in R raises the firm's $LRE\Pi$. In contrast, an increase in C lowers the firm's $LRE\Pi$. The two parameters λ and δ are related to the stochastic arrival process of the tourists and hence it is reasonable to suppose that they are not controllable by our transport providing firm. In contrast, the trip frequency variable or T is the key control variable for our firm and hence this firm can certainly maximize its profits from the provision of transport services to tourists by choosing T optimally. Therefore, we now turn to this long run expected profit maximization problem.

2.4. The profit maximization problem

Our firm's objective function is given by equation (4). Therefore, this firm's optimization problem is to solve

$$\max_{\{T\}}\left[\frac{R\times(\lambda/\delta)(1-e^{-\delta T})-C}{T}\right]. \tag{5}$$

The first order necessary condition for an optimum is

$$e^{-\delta T}\left(\lambda RT+\frac{\lambda R}{\delta}\right)=\frac{\lambda R}{\delta}-C. \tag{6}$$

Now, to keep the transport providing firm's maximization problem meaningful, let us suppose that the right-hand-side (RHS) of equation (6) is positive, i.e., $(\lambda R)/\delta>C$. Then, we can say that this firm's profits are maximal for the unique value of T, say T^*, which satisfies equation (6). Put differently, the optimal frequency with which our firm ought to provide transport service to tourists is given by T^* and T^* is the unique solution to the optimality equation (6). This completes our discussion of the transport providing firm's optimization problem.

3. Conclusions

In this paper, we conducted a probabilistic analysis of the optimal frequency with which transport ought to be provided by a private firm (or by a government agency) to tourists who would like to visit a particular location. Specifically, we first delineated a generic model that accounted for the common features of visits to many locations such as fiords, game parks, lakes, and wildlife reserves. Next, we derived the transportation providing firm's long run expected profit per unit time function. Finally, we demonstrated that the optimal frequency with which transportation ought to be provided to tourists is the solution to our firm's long run expected profit maximization problem.

The analysis in this paper can be extended in a number of directions. Here are two suggestions for extending the research described in this paper. First, it would be useful to generalize the analysis in this paper by analyzing a scenario in which tourists arrive at the location of interest such as a lake in accordance not with a Poisson process but instead with a more general renewal process. Second, it would be interesting to investigate a model in which the cost per trip incurred by the transport providing firm is not fixed but variable and a function of the expected number of tourists who actually make use of the transport service being offered. Studies of scheduling in general and optimal trip frequency determination in particular that incorporate these features of the problem into the analysis will provide additional insights into a demand management problem in tourism that has salient economic implications.

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