Irrelevance of productivity difference: A case with labor union

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Abstract

Common wisdom suggests that firms with higher productivities earn higher profits and the higher productivities of the firms benefit consumers by increasing outputs. We show that productivity difference may not matter for outputs and profits in presence of wage bill maximizing labor unions. Our results hold under decentralized (i.e., firm specific) and centralized union-firm bargaining, for any degree of product differentiation and for any bargaining power of the firms. Hence, our results have important implications for firms' incentives for innovation.

1. Introduction

Common wisdom suggests that firms with higher productivities earn higher profits and higher productivities of the firms benefit consumers by increasing outputs. However, we show that these results may not hold in presence of labor unions. We show that if there are wage bill maximizing labor unions, the outputs and profits of the firms' may be same irrespective of their productivity differences. Hence, in presence of wage bill maximizing labor unions, the higher productivities of the firms do not benefit consumers and the firms do not have incentives for improving their productivities. However, productivity difference matters for outputs and profit if there are rent maximizing unions. Our results hold under decentralized (i.e., firm specific) and centralized union-firm bargaining,[1] for any degree of product differentiation and for any bargaining power of the firms.[2] We also show that, in the case of wage bill maximizing labor unions, the productivity difference does not matter for the outputs and profits of the firms even if the marginal costs of production are increasing.

There is a controversy about the appropriate modeling strategy for union behavior and the debate can be traced back to Dunlop (1944) and Ross (1948). As pointed out in Farber (1986), "… Dunlop was right in that it is fruitful to analyze labor union as maximizing a well-defined objective function but that the internal structure of the union and its political process, emphasized by Ross, are important determinants of the objective function". In a seminal work, Dunlop (1944) argues that wage bill maximization subject to the constraint imposed by the labor demand function is the standard case, while the subsequent works (see, e.g., Pencavel, 1984) consider the alternatives to wage bill maximization such as rent maximization. However, what the appropriate maximand for labor union is an empirical question, which is yet to be settled. We show that productivity difference can have significantly different implications for different union behavior.

The remainder of the paper is organized as follows. With linear marginal costs of production, sections 2 and 3 show the effects of the labor productivity difference on the outputs and profits under decentralized and centralized unionization structures. Section 4 discusses the case of increasing marginal costs of production. Section 5 concludes.

2. Decentralized bargaining

Consider an economy with two firms: firm 1 and firm 2. Assume that the firms produce horizontally differentiated products and production requires only labor. Assume that firm 1 requires 1 laborer to produce one unit of output, while firm 2 requires $\lambda > 1$ laborers to produce one unit of output. Hence, the firms differ in labor productivities.

In this section, we assume that there are firm-specific labor unions, who bargain with the respective firms. We consider a right-to-manage model of labor union where the unions and firms bargain for the wage rates but only the firms decide

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[1] Under decentralized bargaining, union-firm bargaining takes place at the firm level, while under centralized bargaining, union-firm bargaining involves all unions and firms (see, e.g., Vannini and Bughin, 2000).

[2] If the firms have full bargaining power, the productivity difference does not matter for outputs and profits, since the equilibrium wage rates are equal to the reservation wage rates irrespective of the unions' objective functions. We ignore this case by assigning positive bargaining power to the unions.

on the employment levels.[3] Assume that the reservation wage rate for each laborer is c.

We consider the following game. At stage 1, the firms bargain with the respective labor unions simultaneously and the wage rates are determined. At stage 2, the firms produce their outputs simultaneously by hiring laborers according to their need, and the profits are realized. We solve the game through backward induction.

Assume that the inverse demand function faced by the ith firm is

$$P_i = a - q_i - \gamma q_j, \qquad i = 1,2, \ i \neq j, \tag{1}$$

where $a > 0$, q_i and q_j are the outputs of firms i and j, and $\gamma \in [0,1]$ is the degree of product differentiation. $\gamma = 0$ implies that the products are isolated, while $\gamma = 1$ implies that the products are perfect substitutes.

Given the wage rates w_1 and w_2 charged by the respective labor unions to firms 1 and 2, the equilibrium outputs of firms 1 and 2 are respectively

$$q_1 = \frac{a(2-\gamma) - 2w_1 + \gamma\lambda w_2}{4 - \gamma^2} \quad \text{and} \quad q_2 = \frac{a(2-\gamma) - 2\lambda w_2 + \gamma w_1}{4 - \gamma^2}. \tag{2}$$

Given the wage rates, the profits of firms 1 and 2 respectively

$$\pi_1 = \left(\frac{a(2-\gamma) - 2w_1 + \gamma\lambda w_2}{4 - \gamma^2} \right)^2 \quad \text{and} \quad \pi_2 = \left(\frac{a(2-\gamma) - 2\lambda w_2 + \gamma w_1}{4 - \gamma^2} \right)^2. \tag{3}$$

The wage rates are determined by the generalized Nash bargaining between the firms and the firm-specific labor unions. Assume that the bargaining power of each labor union (and the respective firm) is α (and respectively by $1-\alpha$), where $\alpha \in (0,1]$. Note that if $\alpha = 0$, i.e., if the unions have no bargaining power, it is trivial that the equilibrium wage rates are equal to the reservation wage rates of the laborers and therefore, do not depend on the productivity difference of the firms. Hence, to make our analysis non-trivial, we assume that $\alpha \in (0,1]$.

The wage rates are determined by maximizing the following expressions:

$$\underset{w_1}{Max} \left(\frac{(w_1 - c)(a(2-\gamma) - 2w_1 + \gamma\lambda w_2)}{4 - \gamma^2} \right)^\alpha \left(\frac{a(2-\gamma) - 2w_1 + \gamma\lambda w_2}{4 - \gamma^2} \right)^{2(1-\alpha)} \tag{4}$$

$$\underset{w_2}{Max} \left(\frac{(w_2 - c)\lambda(a(2-\gamma) - 2\lambda w_2 + \gamma w_1)}{4 - \gamma^2} \right)^\alpha \left(\frac{a(2-\gamma) - 2\lambda w_2 + \gamma w_1}{4 - \gamma^2} \right)^{2(1-\alpha)}. \tag{5}$$

We assume that, under disagreements with the unions, the firms' profits are zero. The maximization problems in (4) and (5) are for the rent maximizing labor unions. The wage bill maximizing labor unions will maximize (4) and (5) with $c = 0$.

The equilibrium wage rates for the rent maximizing labor unions are

$$w_1 = \frac{a\alpha(2-\gamma)(4+\alpha\gamma) + 2c(2-\alpha)(4+\alpha\gamma\lambda)}{16 - \alpha^2\gamma^2} \tag{6}$$

and $\quad w_2 = \frac{a\alpha(2-\gamma)(4+\alpha\gamma) + 2c(2-\alpha)(4\lambda+\alpha\gamma)}{\lambda(16 - \alpha^2\gamma^2)}. \tag{7}$

The second order conditions for maximization are satisfied.

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[3] The 'efficient bargaining' model, which stipulates that the firms and unions bargain over the wages and employments, is an alternative to the right-to-manage model. See, Layard et al. (1991) for arguments in favor of right-to-manage models.

We get the equilibrium wage rates for the wage bill maximizing labor unions by putting $c = 0$ in (6) and (7). Therefore, the equilibrium wage rates for the wage bill maximizing labor unions are

$$w_1 = \frac{a\alpha(2-\gamma)}{4-\alpha\gamma} \qquad \text{and} \qquad w_2 = \frac{a\alpha(2-\gamma)}{\lambda(4-\alpha\gamma)}. \tag{8}$$

Proposition 1: *In case of decentralized union-firm bargaining, if the labor unions maximize wage bills, the equilibrium outputs and profits of firms 1 and 2 are the same and do not depend on the productivity difference parameter λ.*

Proof: If the labor unions maximize wage bills, it is immediate from (8) that $w_1 = \lambda w_2 = \frac{a\alpha(2-\gamma)}{4-\alpha\gamma}$. Since the marginal costs of firms 1 and 2 are w_1 and λw_2 respectively, we get that the marginal costs of the firms evaluated at the equilibrium wage rates are the same. Hence, it follows from (2) and (3) that the equilibrium outputs and profits of firms 1 and 2 are the same irrespective of the productivity parameter λ.   Q.E.D.

Note that the above result holds irrespective of the bargaining power of the firms and the degree of product differentiation.

However, if the labor unions behave like rent maximizing unions, it is immediate from (6) and (7) that $w_1 \neq \lambda w_2$, and therefore, the productivity difference matters for the equilibrium outputs and profits of the firms.

The reasons for the above findings are as follows. The labor unions will choose the productivity adjusted wage rates to maximize their objective functions. Hence, the labor union specific to firm 1 chooses w_1 to maximize its objective function, while the labor union specific to firm 2 chooses λw_2 to maximize its objective function. Notice that if we consider that the strategic variables of the labor unions are the productivity adjusted wage rates, the objective functions of both labor unions are the same when we have wage bill maximizing labor unions, i.e., when $c = 0$. As a result, in equilibrium, the productivity adjusted wage rates charged by the unions are the same. However, if the labor unions are rent maximizing unions, this symmetry about the unions' objective functions breaks down, and creates different equilibrium productivity adjusted wage rates.

Interesting implications follow immediately from Proposition 1. Since the productivity difference does not matter for the profits in presence of wage bill maximizing labor unions, the firms may not have the incentives for doing innovation in an economy with wage bill maximizing labor unions. Further, since the outputs are also independent of the productivity difference if the labor unions maximize wage bills, productivity difference between the firms do not affect the consumers. It is also expected to observe that the incentives for innovation are higher under rent maximizing labor unions than under wage bill maximizing labor unions.

3. Centralized bargaining

Using decentralized unionization structure, we have shown in section 2 that productivity difference does not matter for the outputs and profits of the firms if there are wage bill maximizing unions. In this section, we consider a game similar to section 2 with the exception that the bargaining between the firms and labor union is

centralized. In other words, we assume that, at stage 1, all the unions and firms bargain to set the wage rates w_1 and w_2, and, at stage 2, the firms produce their outputs simultaneously by hiring laborers according to their need, and the profits are realized. We solve the game through backward induction.

It is important to note that, under a centralized union, we allow the union to charge different wage rates to the firms. Hence, centralization eliminates competition between the labor unions by forming a single labor union, though it keeps the flexibility of charging different wage rates to different firms. Therefore, following the terminology of Haucap and Wey (2004), centralized labor union of this section can be termed as "coordination" between the labor unions.

The reason for considering wage discrimination by the centralized labor union can be found in Yoshida (2000), which shows that an upstream agent prefers price discrimination over uniform pricing if there are differences in the downstream agents.

Therefore, it is immediate that, given the wage rates, the equilibrium outputs and profits of the firms are given by (2) and (3). The equilibrium wage rates are determined by maximizing the following expression:

$$\underset{w_1, w_2}{Max}\left((w_1 - c)q_1 + (w_2 - c)\lambda q_2\right)^\alpha \left(\sum_{i=1}^{2}\pi_i\right)^{1-\alpha}. \tag{9}$$

The first order conditions for maximization of (9) with respect to w_1 and w_2 are respectively

$$\frac{\alpha(a(2-\gamma) - 4w_1 + 2\gamma\lambda w_2 + c(2-\gamma\lambda))}{(w_1-c)(a(2-\gamma) - 2w_1 + \gamma\lambda w_2) + (\lambda w_2 - \lambda c)(a(2-\gamma) - 2\lambda w_2 + \gamma w_1)}$$
$$+ \frac{(1-\alpha)(-2a(2-\gamma)^2 + 2w_1(4+\gamma^2) - 8\gamma\lambda w_2)}{(a(2-\gamma) - 2w_1 + \gamma\lambda w_2)^2 + (a(2-\gamma) - 2\lambda w_2 + \gamma w_1)^2} = 0 \tag{10}$$

and

$$\frac{\alpha(a(2-\gamma) - 4\lambda w_2 + 2\gamma w_1 + c(2\lambda - \gamma))}{(w_1-c)(a(2-\gamma) - 2w_1 + \gamma\lambda w_2) + (\lambda w_2 - \lambda c)(a(2-\gamma) - 2\lambda w_2 + \gamma w_1)}$$
$$+ \frac{(1-\alpha)(-2a(2-\gamma)^2 + 2\lambda w_2(4+\gamma^2) - 8\gamma w_1)}{(a(2-\gamma) - 2w_1 + \gamma\lambda w_2)^2 + (a(2-\gamma) - 2\lambda w_2 + \gamma w_1)^2} = 0. \tag{11}$$

The second order conditions for maximization are satisfied.

Subtracting (11) from (10), we get

$$\frac{-2\alpha(2+\gamma)(w_1 - \lambda w_2) - \alpha c(2+\gamma)(\lambda-1)}{(w_1-c)(a(2-\gamma) - 2w_1 + \gamma\lambda w_2) + (\lambda w_2 - \lambda c)(a(2-\gamma) - 2\lambda w_2 + \gamma w_1)}$$
$$+ \frac{2(1-\alpha)(2+\gamma)^2(w_1 - \lambda w_2)}{(a(2-\gamma) - 2w_1 + \gamma\lambda w_2)^2 + (a(2-\gamma) - 2\lambda w_2 + \gamma w_1)^2} = 0, \tag{12}$$

which implies that, if $c > 0$, $w_1 \neq \lambda w_2$. So, if the centralized labor union maximizes rent, productivity difference creates the difference in outputs and profits of the firms by creating different marginal costs of production for these firms.

However, if $c = 0$, (12) reduces to

$$(w_1 - \lambda w_2)G = 0, \tag{13}$$

where $G = \left[\frac{-\alpha}{w_1(a(2-\gamma) - 2w_1 + \gamma\lambda w_2) + \lambda w_2(a(2-\gamma) - 2\lambda w_2 + \gamma w_1)} + \frac{(1-\alpha)(2+\gamma)}{(a(2-\gamma) - 2w_1 + \gamma\lambda w_2)^2 + (a(2-\gamma) - 2\lambda w_2 + \gamma w_1)^2} \right].$

We get that (13) holds if $w_1 = \lambda w_2$.[4] Hence, if the union is maximizing wage bills, in equilibrium, both firms face the same marginal cost of production, and therefore, productivity difference does not create the difference in outputs and profits of the firms.

To make things clearer, consider an example with $\alpha = 1$, i.e., with the full bargaining power of the union. Then, we get the equilibrium wage rates as $w_1 = \dfrac{a+c}{2}$ and $w_2 = \dfrac{a+\lambda c}{2\lambda}$. Hence, if $c = 0$ (i.e., the union maximizes wage bills), the equilibrium outputs and profits of the firms are independent of the productivity differences.

Hence, the following proposition is immediate from the above discussion.

Proposition 2: *The result of Proposition 1 holds under centralized union-firm bargaining with wage discrimination.*

Note that the above proposition allows wage discrimination between the firms. However, it is trivial that if the centralized labor union requires to charge a uniform wage rate to the asymmetric producers (may be because of the institutional reasons as mentioned in Haucap et al., 2000 and 2001), the labor productivity matters for outputs and profits, irrespective of the union's objective. Given that the firms differ in labor productivities, the marginal costs of the firms differ if the centralized union sets a uniform wage rate to the firms, thus affecting the outputs and profits of the firms.

4. Increasing marginal cost of production

So far, we have considered the case where the marginal costs of production of the firms are constant. The purpose of this section is to show that, if the labor unions maximize wage bills, the labor productivity difference may not affect the outputs and profits of the firms even if the marginal costs of production are increasing. To show this we assume that, besides labor cost, the firms need to incur other costs such as managerial costs. We assume that the managerial costs of the ith firm, $i = 1, 2$, are given by $\dfrac{q_i^2}{2}$. Hence, the firms are symmetric in terms of the managerial expertise, and the marginal cost of the ith firm attributable to the managerial cost is increasing in output. Therefore, the total costs of firms 1 and 2 arte respectively $w_1 q_1 + \dfrac{q_1^2}{2}$ and $\lambda w_2 q_2 + \dfrac{q_2^2}{2}$. For simplicity, here we assume that $\alpha = 1$, i.e., the labor unions have full bargaining power.

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[4] We get that G falls with higher α. Further, G is negative at $\alpha = 1$, while it is positive at $\alpha = 0$. Hence, there is a critical α, say $\overline{\alpha}$, at which G is zero irrespective of $w_1 = \lambda w_2$ or $w_1 \neq \lambda w_2$. However, except $\overline{\alpha}$, it is necessary to have $w_1 = \lambda w_2$ for (13) to hold.

To avoid repetition, we do not show the mathematical details here. However, it is easy to find that, given the wage rates, the equilibrium outputs of firms 1 and 2 are respectively $q_1 = \dfrac{2a - 3w_1 + \lambda w_2}{8}$ and $q_2 = \dfrac{2a - 3\lambda w_2 + w_1}{8}$.

If the labor unions are decentralized, the wage rates charged by the firm-specific unions to firms 1 and 2 are respectively $w_1 = \dfrac{2a}{5}$ and $w_2 = \dfrac{2a}{5\lambda}$. Hence, it is immediate that the difference in labor productivities does not affect the outputs and profits of the firms.

If there is a centralized union, the wage rates charged to firms 1 and 2 are respectively $w_1 = \dfrac{a}{2}$ and $w_2 = \dfrac{a}{2\lambda}$, which shows that the productivity difference does not matter for the outputs and profits of the firms.

5. Conclusion

Common wisdom suggests that firms with higher productivities produce more and earn higher profits compared to the firms with lower productivities. We show that if there are wage bill maximizing labor unions, the productivity difference between the firms do not matter for the equilibrium outputs and profits. Hence, in this situation, higher productivities of the firms do not benefit consumers, and also the firms do not have incentives for improving their productivities. However, productivity difference matters for outputs and profits if there are rent maximizing unions. Our results hold under decentralized (i.e., firm specific) and centralized union-firm bargaining, for any degree of product differentiation and for any bargaining power of the firms. We also show that, if there are wage bill maximizing labor unions, the productivity difference does not affect the outputs and profits of the firms even if the marginal costs of production are increasing. Therefore, the innovations incentives of the firms are expected to be higher under rent maximizing unions than under wage bill maximizing unions.

We have proved our results with linear demand functions. However, it must be clear that our qualitative results hold even for non-linear demand functions. We have seen that productivity difference does not matter if the objective functions of the unions are symmetric when we consider the productivity adjusted wage rates. Given the wage rates w_1 and w_2, the firms' equilibrium outputs and labor demands depend on the marginal costs of production w_1 and λw_2 irrespective of the type of market demand function. Hence, the labor demand functions are the same in terms of the productivity adjusted wage rates. So, if there is an asymmetry in the unions' objective functions, it is due to the existence of c in the unions' price-cost margin. Hence, irrespective of the type of market demand function, the unions' objective functions are symmetric if $c = 0$.

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