E C O N O M I C S   B U L L E T I N

Marginal Utility of Income and value of time in urban transport

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Abstract

We relax the assumption of constancy of the marginal utility of income into a structural model of urban transportation with endogenous congestion. We examine the impact of unobserved heterogeneity in Marginal Utility (MU) of income on the determinants of travel by estimating the model using household survey data. We show that the value of time is no more statistically different across time slots and that the model is robust to all other results.

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1. Introduction

The objective of the present note is to test the constancy of the MU of income in the estimation of urban transportation demand with endogenous congestion. The decision of traveling is modeled as a Bayesian game, in which travelers impose externalities on one another and possess private information about their own aversion to traffic congestion. We assume that the traveler has a private knowledge of his MU of income and that it is proportional to his tolerance to traffic. In agreement with the literature stating that marginal utility of income decreases with income, we assume that higher income individuals are less constrained by traveling schedules than lower income individuals (See Frisch (1964), and Clark (1973) for empirical measures of MU of income in transportation).

Under the assumption of constant MU of income, Viauroux (2007) showed that the individual value of time in traffic is significantly different between peak and off-peak periods with an estimated valuation of .769 during peak period[1]against .7213 during off-peak period (and an t-test statistic of -22.18). We show that introducing unobserved heterogeneity in the MU of income results in a constant average aversion to traffic congestion across time slots. Consequently, the difference in aversion between periods is only explained by differences in the cost of transportation usage.

2. Framework

This section introduces the transportation demand model. The idea is that policies (such as tax or fare increases) affects individual decisions of traveling and these decision in aggregate change economic variables such as traffic congestion, which affect individual decisions again. Hence, the individual preference for traveling depends on the anticipated level of congestion, which in turn is determined by the travel decisions of all individuals.
We let θ_i index individual i value of traveling, and refer to θ_i as individual i's "type" (for $i = 1, \ldots, I$). Here $\theta_i \in \Theta_i = \Theta = [\underline{\theta}, \overline{\theta}]$, where $\underline{\theta}$ is a taste parameter indexing the least tolerance (greatest aversion) for congestion while $\overline{\theta}$ represents the greatest tolerance (lowest aversion) for the externalities. We denote by p_i the ex-ante probability that individual i's type is θ_i, and we assume it has a probability density function $f(\theta_i)$.

We also write

$$\theta = (\theta_i, \theta_{-i}) \in \Theta_i \times \Theta_{-i} \quad \text{with} \quad \Theta_{-i} := \underset{j \in I-i}{\times} \Theta_j.$$

Let $p_i(\theta_{-i}|\theta_i)$ denote the subjective probability that i would assign to the event that $\theta_{-i} \in \Theta_{-i}$ is the actual profile of types for the others, if his own type were θ_i. We assume that the probabilities p_i are independent, so that the density function of $p_i(\theta_{-i}|\theta_i)$ is given by $\prod_{j \neq i} f_j(\theta_j)$.

Let $q_i = (q_i^c, q_i^b) \in Q$ denote the number of trips made by individual i where q_i^c is the number of car trips creating congestion, while q_i^b is the number of bus trips that does not create externalities.

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[1] The peak period refers to individuals departing between 7:00 a.m. and 9:00 a.m. or between 4:00 a.m. and 7:00 p.m.

Travelers' utility functions are of the form

$$u_i(q,\theta,\nu_i) = \alpha q_i^c[1+\psi_i^c+\ln\theta_i-\ln s_{-i}-\ln q_i^c] \\ +(1-\alpha)q_i^b[1+\psi_i^b+\ln\theta_i-\ln s_{-i}-\ln q_i^b]+h(\theta_i)\nu_i$$

of Hanemann (1984), where s_{-i} is the average number of automobile trips made by individuals other than i, ψ_i^c (respectively ψ_i^b) denotes a measure of comfort of traveling by car (resp. by bus) for individual i, ν_i gives the amount of composite good consumed by i, α (respectively $1-\alpha$) represents the marginal utility of using the car (respectively the bus) and the marginal utility of income $h(\theta_i)$ is a function of θ_i.

Individual i faces the budget constraint

$$a_i^c+p_i^cq_i^c+a_i^b+p_i^bq_i^b+p_\nu\nu_i\le w_i,$$

where p_i^c (respectively p_i^b) denotes the car and bus unit price, a_i^c (respectively a_i^b) denotes the fixed charges associated to car and bus use, p_ν is the unit price associated with the composite good ν_i (normalized to 1) and w_i is the individual endowment. Assuming that the number I of individuals is sufficiently large, we have the following

Proposition 1. *The maximization of $u_i(\cdot)$ under the budget constraint gives the optimal allocations of trips for individual i:*

$$q_i^{c*}(\theta_i) = \frac{\theta_i}{s^*}e^{\psi_i^c-\frac{h(\theta_i)p_i^c}{\alpha}},$$
$$q_i^{b*}(\theta_i) = \frac{\theta_i}{s^*}e^{\psi_i^b-\frac{h(\theta_i)p_i^b}{1-\alpha}},$$

where we use the notation

$$s^* := \left(\frac{1}{I}\sum_{j\in I}\int_{\theta_j\in\Theta}\theta_j e^{\psi_j^c-\frac{h(\theta_j)p_j^c}{\alpha}}\ df_j(\theta_j)\right)^{1/2}.$$

Proof. : See Appendix 1. □

Note that when θ is common knowledge, $q_i^{c*}(\theta_i)$ and $q_i^{b*}(\theta_i)$ remain the same, but

$$s^* := \left(\frac{1}{I}\sum_{j\in I}\theta_j e^{\psi_j^c-\frac{h(\theta_j)p_j^c}{\alpha}}\right)^{1/2}.$$

Finally, one can write the indirect utility function of individual i as

$$V_i(w_i-a_i,p^c,p_i^b,\theta) = \alpha q_i^{c*}(\theta)+(1-\alpha)q_i^{b*}(\theta)+h\left(\theta_i\right)\left(w_i-a_i\right), \tag{1}$$

where $a_i = a_i^c+a_i^b$.

3. Data and Estimation

We use a household survey in the greater Montpellier area (south of France; 229,055 inhabitants) recording the transportation activity of 6341 individuals on a two days period. A trip is seen as a more-than-300-meters drive or run between two places on a public road. We focus on trips made for the purposes of work, school, shopping, leisure, returns home are not accounted for. We use a maximum likelihood estimation method. The observed number of trips is assumed to follow a

Poisson distribution, the expectation of which is the equilibrium conditional number of trips at the Nash equilibrium above (for a given mode of transportation). We use as average cost of the car, the price per kilometer times distance Origin-Destination. Bus fares vary by type and according to travelers'socioeconomic characteristics.

They include the unit ticket: FF7 (1.07 euros), a booklet of three tickets: FF20 (3.05 euros), a booklet of 10 tickets (discounted for handicapped or large families), a 30 days lump sum (discounted for students, non students-employed, unemployed, scholars depending on district subventions, retired with and without no "Carte Or" subscription) an annual lump sum (discounted for scholars and students, unemployed non students-scholars). Hence, each traveler i possesses the following mutually exclusive choices: he uses neither the car nor the bus (using another mode of transportation);
$V_i(\theta) = h(\theta)w_i$;
he uses at least once the car but never the bus;
$V_i^c(\theta) = \alpha\frac{\theta_i}{s^*_{-i}}e^{\psi_i^c - \frac{hp_i^c}{\alpha}} + h(\theta)(w_i - a_i^c)$;
he does not use the car but he uses at least once the bus; then he can choose among J payment option for the bus (J cases in total);
$V_i^{bj}(\theta) = (1-\alpha)\frac{\theta_i}{s^*_{-i}}e^{\psi_i^b - \frac{hp_i^b}{1-\alpha}} + h(\theta)(\theta)(w_i - a_i^{bj}), \quad j = 1, \ldots, J$;
he uses both the car and the bus at least once; then he can choose again among J payment option for the bus (J cases in total).
$V_i^{cbj}(\theta) = \alpha\frac{\theta_i}{s^*_{-i}}e^{\psi_i^c - \frac{hp_i^c}{\alpha}} + (1-\alpha)\frac{\theta_i}{s^*_{-i}}e^{\psi_i^b - \frac{hp_i^b}{1-\alpha}} + h(w_i - a_i^c - a_i^{bj}), \quad j = 1, \ldots, J.$

The model is estimated by maximum likelihood where the likelihood function is the joint probability of doing a number of trips and choosing a mode of payment. It is decomposed into a probability of making a certain number of trips conditional on making trips with that mode of transportation and payment times the marginal probability of choosing that mode of transportation and mode of payment. The first probability is chosen to be a normalized Poisson distribution while the second is a multinomial logit. The likelihood functions are reported in Appendix 2. We assume that $h(\theta_i) = h\theta_i$, that is, the more tolerant to traffic congestion, the more an individual uses transportation despite traffic conditions and the higher the marginal utility of income. Vectors of comfort of traveling are specified as $\psi_i^b = \beta^b X_i^b$, and $\psi_i^c = \beta^c X_i^c$ where X_i^b and $X_i^c$are vectors characterizing the trip such as the time between the Origin and the Destination (O-D) as well as socioeconomic characteristics of individual i. The respective vectors of parameters to be estimated are denoted $\beta^c = \left\{\beta_j^c\right\}_{j=1,..J}$, $\beta^b = \left\{\beta_{j'}^b\right\}_{j'=1,..J'}$ where J and J' are the numbers of variables introduced to respectively characterize car and bus trips.

Estimation results are presented in Appendix 3. They show that the estimated average aversion to traffic congestion ($\frac{1}{1.1980} = 0.8347$) is low and no more different across time slots as this difference is accounted for by the MU of income function. Under the assumption of a constant MU of income, Viauroux's (2007) estimation results showed a significant difference of the average aversion to congestion between peak and off-peak periods. Allowing the MU of income to depend on aversion to congestion results in a constant average aversion. Consequently, the difference in aversion between periods is only explained by differences in the cost of transportation usage. The other results remain robust. The higher the bus frequency, the more the individual travels by bus and the effect is significantly stronger during off-peak time. As the distance from the Origin to the Destination increases, individuals travel more by bus and less by car, while both modes are used during off-peak times. Scholars and unemployed travel more by bus for all times.

4. Conclusion

We show that the Bayesian approach used to model the endogeneity of the congestion process in urban areas is robust to the relaxation of the assumption on the constancy of the marginal utility of income. The only difference is in the difference in aversion to traffic congestion found similar across periods.

References

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Appendix 1. Proof of Proposition 1

Proof. Taking into account the budget constraints, the utility functions u_i are given by the formula

$$\begin{aligned} u_i(q_i, q^*_{-i}, \theta) = \alpha q_i^c \left[1 + \psi_i^c + \ln\theta_i - \ln s^*_{-i} - \ln q_i^c\right] \\ + (1-\alpha)\left[1 + \psi_i^b + \ln\theta_i - \ln s^*_{-i} - \ln q_i^b\right] \\ + h(\theta_i)\,(w_i - a_i^c - p_i^c q_i^c - a_i^b - p_i^b q_i^b). \end{aligned}$$

This definition leads to a pure multistrategy equilibrium corresponding to the value of $q_i(\theta)$ which maximizes $u_i(q_i, q^*_{-i}, \theta)$. In order to simplify the computation, let us admit that the variable $q_i(\theta)$ can be changed continuously, and let us write down the first order conditions associated to the above maximization. Both partial derivatives with respect to q_i^c and q_i^b must vanish at the equilibrium point, namely,

$$\alpha\left[\psi_i^c + \ln\theta_i - \ln s^*_{-i} - \ln q_i^{c*}\right] - h(\theta_i)\,p_i^c = 0, \tag{2}$$

$$(1-\alpha)\left[\psi_i^b + \ln\theta_i - \ln s^*_{-i} - \ln q_i^{b*}\right] - h(\theta_i)\,p_i^b = 0. \tag{3}$$

Solving for q_i^{c*} and q_i^{b*} we obtain the first two inequalities of the proposition :

$$q_i^{c*}(\theta_i) = \frac{\theta_i}{s^*_{-i}} e^{\psi_i^c - \frac{h(\theta_i)p_i^c}{\alpha}}, \tag{4}$$

$$q_i^{b*}(\theta_i) = \frac{\theta_i}{s^*_{-i}} e^{\psi_i^b - \frac{h(\theta_i)p_i^b}{1-\alpha}}. \tag{5}$$

In order to determine the value of s^*_{-i}, integrate both parts of (4) with respect to θ_i of density $f(\theta_i)$; we obtain

$$\begin{aligned} s_{-i}(q^{c*}_{-i}) &= \frac{1}{I-1}\sum_{j\in I-i}\int_{\theta_j\in\Theta} q_j^{c*}(\theta_j) f(\theta_j)d\theta_j \\ &= \frac{1}{I-1}\sum_{j\in I-i}\int_{\theta_j\in\Theta} \frac{\theta_j}{s^*_{-j}} e^{\psi_j^c - \frac{h(\theta_j)p_j^c}{\alpha}} f(\theta_j)d\theta_j. \end{aligned}$$

Assuming that one individual is negligible in the continuum of individuals so that $s^*_{-i} = s^*_{-j}$, we may denote this common value by s^*. It follows that

$$(s^*)^2 \approx \frac{1}{I}\sum_{j\in I-i} \theta_j e^{\psi_j^c - \frac{h(\theta_j)p_j^c}{\alpha}} f(\theta_j)d\theta_j.$$

In the case of complete information the proof remains the same, except the determination of s^* where we do not have to integrate over Θ. Then we obtain

$$(s^*)^2 \approx \frac{1}{I}\sum_{j\in I} \theta_j e^{\psi_j^c - \frac{h(\theta_j)p_j^c}{\alpha}}.$$

To obtain the expression of the indirect utility function, let us rewrite the equalities (2) and (3) in the form

$$1+\psi_i^c+\ln\theta_i-\ln s_{-i}^*-\ln q_i^{c*}=1+\frac{h\left(\theta_i\right)p_i^c}{\alpha},$$
$$1+\psi_i^b+\ln\theta_i-\ln s_{-i}^*-\ln q_i^{b*}=1+\frac{h\left(\theta_i\right)p_i^b}{1-\alpha}.$$

Then we obtain

$$\begin{aligned}
&V_i(w_i-a_i^c-a_i^b,p^c,p_i^b,\theta)\\
&\quad=[\alpha+h\left(\theta_i\right)p_i^c]q_i^{c*}(\theta)+[1-\alpha+h\left(\theta_i\right)p_i^b]q_i^{b*}(\theta)\\
&\qquad+h\left(\theta_i\right)[w_i-a_i^c-p_i^cq_i^{c*}(\theta)-a_i^b-p_i^bq_i^{b*}(\theta)]\\
&\quad=\alpha q_i^{c*}(\theta)+(1-\alpha)q_i^{b*}(\theta)+h\left(\theta_i\right)\left(w_i-a_i^c-a_i^b\right)
\end{aligned}$$

as stated. □

Appendix 2. Expression of the likelihood function

Let l_{ikn}^{cbj} denote the contribution to the likelihood in case of k car trips and n bus trips by using the mode of payment j to take the bus if $n>0$. It is the product of the probability to observe q_i^{c*} car trips and/or q_i^{b*} bus trips by the logistic probability to choose the car and/or bus modes of transportation. The probability of making q_i^{b*} or q_i^{b*} trips follows a Poisson distribution. Then the unconditional likelihood function is given by the product

$$L=\prod_{i=1}^{N}l_{ikn}^{cbj}$$

with

$$\begin{aligned}
l_{i00}^{cbj}&=\int_{\Theta_i}\frac{e^{V_i}}{S}\,dF(\theta_i);\\
l_{ik0}^{cbj}&=\int_{\Theta_i}\frac{\exp\left(-q_i^{c*}\right)\left(q_i^{c*}\right)^k}{k!\left(1-\exp\left(-q_i^{c*}\right)\right)}\frac{e^{V_i^c}}{S}\,dF(\theta_i),\quad k=1,2,\ldots;\\
l_{i0n}^{cbj}&=\int_{\Theta_i}\frac{\exp\left(-q_i^{b*}\right)\left(q_i^{b*}\right)^n}{n!\left(1-\exp\left(-q_i^{b*}\right)\right)}\frac{e^{V_i^{bj}}}{S}\,dF(\theta_i),\quad n=1,2,\ldots,\quad j=1,\ldots,J;\\
l_{ikn}^{cbj}&=\int_{\Theta_i}\frac{\exp\left(-q_i^{c*}\right)\exp\left(-q_i^{b*}\right)\left(q_i^{c*}\right)^k\left(q_i^{b*}\right)^n}{k!n!\left(1-\exp\left(-q_i^{c*}\right)\right)\left(1-\exp\left(-q_i^{b*}\right)\right)}\frac{e^{V_i^{cbj}}}{S}\,dF(\theta_i),\\
&\qquad k,n=1,2,\ldots,\quad j=1,\ldots,J,
\end{aligned}$$

where

$$S:=e^{V_i}+e^{V_i^c}+\sum_{j=1}^{J}\left(e^{V_i^{bj}}+e^{V_i^{cbj}}\right).$$

Note that the structure of the likelihood is highly nonlinear in $h(\theta_i)$. Our expression of the likelihood is sensibly different from constant MU of income case.

Appendix 3. Estimation results

Table 1: Estimation Results

	Variable	Parameter	$\theta \rightarrow$Beta(1,μ)		Comparison
			Peak	Off-Peak	t-test
	Marginal Utility Transport	α	0.7726 (0.0097)	0.8355 (0.0069)	-5.2840354
	Private information	μ	0.1980 (0.0008)	0.1981 (0.0009)	-0.083045480
	Slope of MU income	h	3.4928 (0.1367)	3.8937 (0.1630)	-1.8845102
Bus	1	β_1^b	1.8035 (0.1356)	0.1363 (0.1753)	7.5226316
	Bus Frequency	β_2^b	0.0719 (0.0143)	0.1659 (0.0219)	-3.5939171
	Distance O-D	β_3^b	-0.0031 (0.0128)	0.1030 (0.0180)	-4.8037089
	Time O-D	β_4^b	0.0042 (0.0007)	-0.005 (0.0018)	4.7635794
	Student-Scholar	β_5^b	0.0824 (0.0769)	0.1623 (0.1313)	-0.52509796
Car	1	β_1^c	1.3895 (0.1795)	0.4428 (0.2221)	3.3151559
	Distance O-D	β_2^c	0.1137 (0.0084)	0.1394 (0.0127)	-1.6878334
	Time O-D	β_3^c	0.0008 (0.0004)	-0.005 (0.0018)	3.1454916
	Power of the car	β_4^c	0.0231 (0.0066)	0.0449 (0.0104)	-1.7698444
	Unemployed	β_5^c	-0.2294 (0.0963)	-0.2373 (0.0765)	0.064234139
	Student-Scholar	β_6^c	-0.4005 (0.0957)	-0.4261 (0.1280)	0.16018002
	Age	β_7^c	0.0528 (0.0087)	0.0313 (0.0096)	1.6595020
	Age*age	β_8^c	-0.0006 (0.0001)	-0.0004 (0.0001)	-1.4142136
	Adj. (Mean Log L.)		-6.98321	-6.07535	

Standard Deviations in parentheses.