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Longevity and PAYG pension systems sustainability

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Abstract

In this paper we study the effects of an increasing longevity on the balanced pay-as-you-go pension budget in the basic overlapping generations model of growth (Diamond, 1965). It is shown that, when the capital's share in production is sufficiently high, the higher longevity the higher pension benefits. The policy implication is that there would be room for an increase, rather than the often threatened reduction, in future pension payments, by keeping unaltered the contribution rate paid by the young to finance pensions to retired people as well as a balanced PAYG pension budget.

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1 Introduction

As known, all developed countries will face a considerably aged population in the near future. According to authoritative estimates (United Nations, 1998) the expected population ageing will be very striking: for instance, while the dependency ratio age (over 65 in total population) was for most countries around 20 per-cent in 1995, it will be around 67 per-cent in Italy, around 57 per-cent in Japan, and around 49 per-cent in the Western Europe in 2040. Since in many developed countries, old-age pension benefits are financed on a PAYG-basis, it is commonly retained that this may cause serious concerns given the steadily rising share of old-aged people in total population. Governments are thus engaged in policies aiming to solve the expected PAYG budget problems, such as the compulsory reduction in the lengthening of the retirement period and/or a reduction in government pension benefits and/or an increase in the contribution rates.[1] All these policies are somewhat painful for either old-aged or young-aged people, but they are based on a widespread common wisdom between policymakers such as to motivate a recent literature aiming to document whether such a common belief is also shared by young and old people and, in any case, how to have success – for instance through appropriate informational campaigns – in popularising it. For instance, Boeri et al. (2001, 2002), drawing on surveys of European citizens, and Blinder and Krueger (2004) studying opinion polls in the US, noted that more informed individuals are more likely to support pension reforms and advise a more operative "advertising campaign". In this paper we address the following theoretical question: the negative relationship between increasing longevity and the balance of the PAYG pension system is really warranted? For doing this our model is simply the textbook Diamond (1965) style OLG framework extended with an exogenous longevity.[2] The analysis of our simple model yields the following result: when the capital share is sufficiently high, an increase in longevity may increase pensions.[3] Therefore, our results may constitute a policy warning suggesting that the striking population ageing could not be harmful for the PAYG system viability and, thus, some policy measures commonly invoked could be unnecessary.[4]

The paper is organised as follows. In Section 2 we develop the model and in Section 3 the main steady-state results are analysed and discussed. Section 4 winds up with some concluding remarks.

2 The model

2.1 Individuals

Young population N_t grows at a constant rate n and agents are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among two periods: youth and old-age (Diamond, 1965). Individuals belonging to generation t have a homothetic and separable utility function defined over young-aged and old-aged consumption, $c_{1,t}$ and $c_{2,t+1}$, respectively. Each young individual supplies inelastically one unit of labour in the labour market, and receives wage income at the competitive rate w_t. During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}. Moreover, we suppose old individuals survive to the second period with (constant) probability $0 < \pi < 1$. Therefore, the existence of a perfect annuity market

[1] For instance Pecchenino and Pollard (2005, p. 450) stated that "To maintain benefit levels, tax rates and/or productivity growth will have to rise."

[2] Although some papers (e.g., Pecchenino and Pollard, 2002, 2005; Zhang et al., 2001) addressed, in an endogenous growth context with unfunded social security, the issue of the relationship between longevity and growth, arguing that such a relation may be positive, the result of this paper that pension benefits may be increasing with population aging – obtained as a textbook result – has not been, to the best of our knowledge, pointed up.

[3] It must be emphasised that we have obtained the result using as parsimonious a model as possible, that is, the standard Diamond's (1965) OLG model of growth.

[4] Therefore, to the question posed by Boeri and Tabellini (2005, p. 2) "Why is it so difficult to reform the unsustainable and overly generous European pension systems?", our results could answer that fortunately, in some cases, it could not be a necessary measure.

implies old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased.[5] Furthermore, each old-age individual is entitled to a publicly provided pension benefit (p_{t+1}) financed at balanced budget by the government.
Thus, the representative individual born at time t is faced with the following program:

$$\max_{\{s_t\}} U_t = \ln(c_{1,t}) + \pi\gamma \ln(c_{2,t+1}), \tag{P}$$

subject to

$$c_{1,t} + s_t = w_t(1-\theta)$$

$$c_{2,t+1} = \frac{1+r_{t+1}}{\pi} s_t + p_{t+1},$$

where $0 < \theta < 1$ is the contribution rate and $0 < \gamma < 1$ is the subjective discount factor.
The maximisation of program (P) gives the following savings function:

$$s_t = \frac{\pi\gamma w_t(1-\theta)}{1+\pi\gamma} - \frac{\pi p_{t+1}}{(1+\pi\gamma)(1+r_{t+1})}. \tag{1}$$

2.2 Government

The government balances the PAYG social security scheme in every period according to the following formula:

$$\pi p_t = \theta w_t(1+n), \tag{2}$$

where the left-hand side represents the social security expenditure and the right-hand side the tax receipts.[6]
Inserting (2) into (1) to eliminate p, the savings function chosen optimally by individuals modifies to become:

$$s_t = \frac{\pi\gamma w_t(1-\theta)}{1+\pi\gamma} - \frac{(1+n)\theta}{1+\pi\gamma}\frac{w_{t+1}}{1+r_{t+1}}. \tag{3}$$

2.3 Firms

As regards the production sector, we suppose firms are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas[7] technology of production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the time-t labour input respectively, $A > 0$ represents a scale parameter and $\alpha \in (0,1)$ is the capital's share on total output. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function may be written as $y_t = Ak_t^{\alpha}$. Assuming total depreciation of physical capital at the end of each period and knowing that final output is treated at unit price, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha Ak^{\alpha-1} - 1, \tag{4}$$

$$w_t = (1-\alpha)Ak_t^{\alpha}. \tag{5}$$

2.4 Equilibrium

[5] We note that also in the absence of a perfect annuity market, and therefore where adult mortality gives raise to accidental bequests, the qualitative results of this paper remain unchanged.
[6] We assume agents to have perfect foresight with respect to the level of the future public pension benefit.
[7] Notice that we have chosen this usual functional form for analytical tractability, but by using for instance a more general CES specification, the qualitative results of this paper, as numerical simulations not reported here reveal, are preserved as well.

Given the government budget (2) and knowing that population evolves according to $N_{t+1} = (1+n)N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the equality $(1+n)k_{t+1} = s_t$. Substituting out for s according to Eq. (3), exploiting (4) and (5), and assuming individuals are perfect foresighted, the dynamic equilibrium sequence of capital is determined by:

$$k_{t+1} = \frac{\pi\gamma(1-\theta)\alpha(1-\alpha)A}{(1+n)[\alpha(1+\pi\gamma)+\theta(1-\alpha)]}k_t^{\alpha}. \tag{6}$$

Steady-state implies $k_{t+1} = k_t = k^*$, so that:[8]

$$k^*(\pi) = \left\{\frac{\pi\gamma(1-\theta)\alpha(1-\alpha)A}{(1+n)[\alpha(1+\pi\gamma)+\theta(1-\alpha)]}\right\}^{\frac{1}{1-\alpha}}. \tag{7}$$

From Eq. (7) it can easily be seen that an increase in longevity always increases the long-run per-capita stock of capital, that is,

$$\frac{\partial k^*(\pi)}{\partial \pi} = \frac{k^*(\pi)[\alpha+\theta(1-\alpha)]}{\pi(1-\alpha)[\alpha(1+\pi\gamma)+\theta(1-\alpha)]} > 0. \tag{8}$$

3 Pensions and longevity in the long-run

A stylised fact emerged in developed countries is the population ageing problem. Which are the effects of an increasing longevity on pension payments? The common belief suggests that an increase in longevity, in order to keep balanced the PAYG pension budget, requires a reduction in pension benefits and/or an increase in the contribution rate paid by young-aged individuals to guarantee an adequate pension benefit to retired people. In what follows we give an answer to the simple question raised above finding that, rather surprisingly, in the basic OLG model of growth such a common belief may be reversed.

Given Eq. (2), the long-run pension benefit as a generic unction of the probability of surviving to the second period of life may be written as:

$$p^* = p^*\{\pi, w^*[k^*(\pi)]\}. \tag{9}$$

Therefore, the total derivative of Eq. (9) with respect to π gives:[9]

$$\frac{dp^*}{d\pi} = \overbrace{\frac{\partial p^*}{\partial \pi}}^{-} + \underbrace{\overbrace{\frac{\partial p^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \pi}}^{+}}_{+}. \tag{10}$$

Eq. (10) reveals that the final effect of an increase in longevity on the long-run pension payment depends on two counterbalancing forces, and it appears to be ambiguous: (1) a negative (direct) effect which tends to reduce pensions owing to the higher number of old-aged individuals, and (2) a positive (indirect) general equilibrium feedback effect which acts on pensions through an increased wage rate owing to an increased stock of capital per-capita. Given the positive relationship between PAYG pensions and wages, the higher the wage rate the higher the pension payment received by retired people.

To analyse ultimately which of the two forces dominates, we now combine Eqs. (2), (5) and (7) to obtain the following steady-state pension benefit formula:

$$p^*(\pi) = \frac{\theta}{\pi}(1+n)(1-\alpha)A \cdot \left\{\frac{\pi\gamma(1-\theta)\alpha(1-\alpha)A}{(1+n)[\alpha(1+\pi\gamma)+\theta(1-\alpha)]}\right\}^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

[8] Using Eq. (7) it can be shown that the steady-state is always stable.

[9] Details are given in Appendix.

From Eq. (11) we have the following proposition:

Proposition 1. *1) Let $\alpha < 1/2$ hold. Then an increase in longevity always reduces pensions.*
2) Let $1/2 < \alpha < \overline{\alpha}$ hold. Then the pension benefit is an inverted U-shaped function of the rate of longevity and pensions are maximised at $\pi = \pi_p$.
3) Let $\overline{\alpha} < \alpha < 1$ hold. Then an increase in longevity always increases pensions.

Proof. The proof uses the following derivative:

$$\frac{\partial p^*(\pi)}{\partial \pi} = \frac{A \cdot [k^*(\pi)]^{\alpha} \theta(1+n) \cdot \{-\alpha(1-\alpha)\pi\gamma - (1-2\alpha)[\alpha + \theta(1-\alpha)]\}}{\pi^2[\alpha(1+\pi\gamma) + \theta(1-\alpha)]}.$$

Therefore, if $\alpha < 1/2$, then $\frac{\partial p^*(\pi)}{\partial \pi} < 0$ for any $0 < \pi < 1$ and the higher longevity the lower pension benefits. If $\alpha > 1/2$, then

$$\frac{\partial p^*(\pi)}{\partial \pi} \gtrless 0 \Leftrightarrow \pi \lessgtr \pi_p,$$

where

$$\pi_p \equiv \frac{(1-2\alpha)[\alpha + \theta(1-\alpha)]}{\alpha(\alpha-1)\gamma} > 0.$$

Therefore, $\pi_p < 1$ if and only if:

$$-H_1\alpha^2 + H_2\alpha + H_3 > 0, \tag{12}$$

where $H_1 \equiv 2(1-\theta) + \gamma > 0$, $H_2 \equiv 1 + \gamma - 3\theta$ and $H_3 \equiv \theta > 0$. Since $\Delta \equiv H_2{}^2 + 4H_1H_3 > 0$, then by applying the Descartes' rule of sign we find that, independently of the sing of H_2, there always exist two real roots $\underline{\alpha} \equiv \frac{1+\gamma-3\theta-\sqrt{(1+\gamma-\theta)^2+4\theta}}{2[2(1-\theta)+\gamma]} < 0$ and $\overline{\alpha} \equiv \frac{1+\gamma-3\theta+\sqrt{(1+\gamma-\theta)^2+4\theta}}{2[2(1-\theta)+\gamma]} > 0$ which solve Eq. (12).[10] Since $\underline{\alpha} < 0$ it is automatically ruled out. Thus, if $1/2 < \alpha < \overline{\alpha}$ then $\pi = \pi_p < 1$ represents the pension-maximising longevity rate. Finally, if $\overline{\alpha} < \alpha < 1$ then $\frac{\partial p^*(\pi)}{\partial \pi} > 0$ for any $0 < \pi < 1$ and thus the higher longevity the higher pension benefits. **Q.E.D.**

Proposition 1 says that the pension payment received by retired people is negatively linked with longevity if and only if the capital's share in production is low enough. On the contrary, when $\alpha > 1/2$, then, an increase in longevity may increase pensions and in particular, for moderate values of the capital share in production,[11] that is, $1/2 < \alpha < \overline{\alpha}$, there exists a pension-maximising longevity rate, that is

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[10] Notice that $1/2 < \overline{\alpha} < 1$. The first inequality in fact implies $\sqrt{(1+\gamma-\theta)^2+4\theta} > 1+\theta$. Therefore, we can write $(1+\gamma-\theta)^2 + 4\theta > (1+\theta)^2$. Rearranging terms, we get $\gamma[\gamma + 2(1-\theta)] > 0$. Therefore, $\overline{\alpha}$ is always higher than $1/2$. The second inequality yields $\sqrt{(1+\gamma-\theta)^2+4\theta} < 3+\gamma-\theta$. Therefore, we can write $(1+\gamma-\theta)^2 + 4\theta > (3+\gamma-\theta)^2$, which implies $2(1-\theta)+\gamma > 0$, so that $\overline{\alpha}$ is always smaller than unity.

[11] In order to better clarify the meaning of the coefficient α (the capital's share in production), it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including thus physical and human components, and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992, p. 417), if we take a broad view of capital, then the physical capital's share of income is expected to be about 1/3 and the human capital's share of income should be between 1/3 and one half. In sum, the coefficient α may be fairly about 0.6 and 0.8. Indeed, for instance, Barro and Sala-i-Martin (2003, p. 110) used $\alpha = 0.75$ saying that: "Values in the neighbourhood

$\pi = \pi_p$ (which is, it must be note, independent of the population growth rate). Finally, if the capital's share in technology is sufficiently high, then the higher longevity the higher pension payments independently of the value of the contribution rate, that is the indirect general equilibrium effect of an increasing longevity which acts positively on wages through an increased capital accumulation always dominates over the negative direct effects which tends to reduce pensions owing the lower number of old-aged people deceased. This result suggests that the idea that population ageing is always detrimental for pension benefits paid to retired people is not warranted from a theoretical point of view. In fact, the policymaker may even reduce the contribution rate by keeping the PAYG pension budget balanced in every period of time as the number of old-aged individuals increase.

3.1 A numerical illustration

An example, chosen only for illustrative purposes, of the results stated in Proposition 1 is summarised in the following Tables 1 and 2. We take the following parameter values: $A = 10$ (simply a scale parameter in the Cobb-Douglas production function), $\gamma = 0.30$ (as in De La Croix and Michel, 2002, p. 50), $n = 1$ (the constant population growth rate) and $\theta = 0.40$ (the contribution rate). This parameter set generates $\bar{\alpha} = 0.5508$. As regards the value of the capital's share in production, it must be noted that, as observed by Jones (2003, p. 8), countries such as Italy and Spain, which are strongly plagued by population ageing problems, show a value of α without self-employment correction between 0.5 and 0.6. Therefore, in the following Tables 1 and 2 we used $\alpha = 0.54$ and $\alpha = 0.60$, respectively, to illustrate the effects of an increasing longevity on pension payments as stated in points 1) and 2) in Proposition 1. In particular, in Table 1 we show there exists a pension-maximising value of the rate of longevity, whereas in Table 2 we illustrate the case in which the relationship between longevity and pension payments is monotonically positive.

Table 1. Effects of an increasing longevity on pension payments. Case $1/2 < \alpha < \bar{\alpha}$ ($\alpha = 0.54$ and $\pi_p = 0.7772$).

π	$p^*(\pi)$
0.10	0.6047
0.20	0.6650
0.30	0.6961
0.40	0.7142
0.50	0.7249
0.60	0.7310
0.70	0.7338
0.7772	0.73445
0.80	0.73441
0.90	0.7332
0.99	0.7310

Table 2. Effects of an increasing longevity on pension payments. Case $\bar{\alpha} < \alpha < 1$ ($\alpha = 0.60$ and $\pi_p = 2.1111$).

π	$p^*(\pi)$
0.10	0.1480
0.20	0.2022
0.30	0.2395

of 0.75 accord better with the empirical evidence, and these high values of α are reasonable if we take a broad view of capital to include human components".

0.40	0.2677
0.50	0.2898
0.60	0.3076
0.70	0.3222
0.80	0.3342
0.90	0.3442
0.99	0.3517

4 Conclusions

We investigated, by using the textbook Diamond (1965) style OLG model, if the common belief that the increased longevity poses a threat to the PAYG pension system viability is really warranted, obtaining the following result: when the capital's share in production is sufficiently high, an increase in longevity may increase pensions. The policy implication is that in developed countries plagued by a strong population aging problem the compulsory reduction in the lengthening of the retirement period and/or a reduction in pension benefits received by old-aged individuals and/or an increase in the contribution rates paid by the young might be unnecessary policies to keep balanced the PAYG pension budget.

Appendix

In this appendix we presets details to clarify the role of the rate of longevity on the long-run pension benefits. In particular, we have that:

$$\frac{\partial p^*}{\partial \pi} = \frac{-\theta(1+n)w^*}{\pi^2} < 0, \tag{A1}$$

$$\frac{\partial p^*}{\partial w^*} = \frac{\theta}{\pi}(1+n) > 0, \tag{A2}$$

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A(k^*)^{\alpha-1} > 0. \tag{A3}$$

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