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A note on testing parameter constancy in cointegrated vector autoregression: the case of near I(2) processes

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Abstract

This note investigates the behaviour of a parameter-constancy test statistic when near I(2) (integrated of order 2) variables are incorporated in a cointegrated vector autoregressive system. Simulation studies indicate that the presence of such variables has a significant impact on size properties of the constancy test.

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1 Introduction

This note investigates the behaviour of a parameter-constancy test when near $I(2)$ (integrated of order 2) variables exist in a cointegrated system. The introductory section briefly reviews the related literature and describes the most significant aspect of this note.

Economic time series data tend to exhibit non-stationary behaviour and should be treated as integrated processes rather than stationary. Cointegration introduced by Granger (1981) thus plays a crucial role in time series econometrics. A cointegrated vector autoregressive (VAR) model for $I(1)$ processes is introduced by Johansen (1988, 1996), and has become one of the most popular approaches to modelling economic time series. See Hendry and Mizon (1993), Juselius (2006), and Kurita (2007), *inter alia*, for econometric modelling using the cointegrated VAR analysis. The cointegrated VAR model requires parameter stability throughout the sample period of interest, and Hansen and Johansen (1999) explore various test statistics for constancy of the cointegrating vectors. Some of their tests are based on Nyblom (1989) and regarded as quasi Lagrange multiplier (LM) tests. Seo (1998) also investigates several likelihood-based tests for stability of the cointegrating parameters.

Macroeconomic time series data, in fact, can vary in terms of the degree of integration. Inflation data, for example, tend to be persistent and are often judged to be $I(1)$ or near $I(1)$ processes, so that time series data for price indices are deemed to be $I(2)$ or near $I(2)$. With the advances of $I(1)$ VAR analysis, econometric theories for $I(2)$ cointegrated VAR models are also developed in the literature: Johansen (1992, 1995, 1997, and 2006), Paruolo (1995, 2000), Haldrup (1998), Paruolo and Rahbek (1999), Rahbek, Kongsted and Jørgensen (1999), Boswijk (2000), Nielsen and Rahbek (2007), and Kurita (2008), *inter alia*. See also Juselius (2006, Part V) for empirical research using $I(2)$ VAR models. Whether a series is really $I(2)$ or not is a matter of debate. However, given the fact that smooth trending features are often found in time series data of stock variables such as price indices and aggregate money, such stock variables should be treated as near $I(2)$ series at least rather than conventional $I(1)$ or stationary series. Thus it is worthwhile to investigate testing parameter constancy when near $I(2)$ variables are involved in the underlying data generation process.

This note presents a limit theorem for a quasi LM parameter constancy test, which explicitly takes account of $I(2)$ roots in the cointegrated VAR system. The limiting distribution of the test statistic in this case consists of functionals of the standard Brownian motion and is thus different from that in a standard case where only $I(1)$ roots exist in the system. The asymptotic result suggests that the use of simulated quantiles based on a conventional $I(1)$ specification can lead to misleading statistical inference when the data in question are very close to $I(2)$. Monte Carlo experiments conducted in this note support the analytical reasoning. According to the experiments, the existence of near $I(2)$ series has a significant impact on size properties of a quasi LM test for parameter stability. An empirical illustration also gives weight to this argument. It is therefore important to check if the data are $I(1)$ or close to $I(2)$ processes when testing for parameter constancy. To the best of the author's knowledge, the present note is the first quantitative study that addresses the issue of testing stability of the cointegrating parameters in the presence of near $I(2)$ roots.

The organization of this note is as follows. Section 2 reviews a quasi LM test for constancy of the cointegrating parameters and presents a limit theorem allowing for $I(2)$ variables. Section 3 performs Monte Carlo experiments to inspect impacts of near $I(2)$ processes on the constancy test statistic. An empirical illustration of the quasi LM test is presented in Section 4. The overall summary and conclusion are provided in Section 5. All the numerical analyses and graphics in this note use *Ox* (Doornik, 2006) and *OxMetrics / PcGive* (Doornik and Hendry, 2006). This note uses the following notational conventions: For a certain matrix a with full column rank, $\overline{a} = a(a'a)^{-1}$ and so $a'\overline{a} = I$. An orthogonal complement $a_\perp$ is defined such that $a'_\perp a = 0$ with the matrix $(a, a_\perp)$ being of full rank. The symbol $\overset{w}{\to}$ signifies weak convergence.

2 Cointegrated VAR Model and Parameter Constancy

This section briefly reviews a cointegrated VAR model and parameter-constancy test statistic. Johansen (1996) is the main reference for details of the model. Let us consider an unrestricted VAR(k) model for a p-dimensional time series X_t as follows:

$$\Delta X_t = (\Pi, \Pi_c) \begin{pmatrix} X_{t-1} \\ 1 \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \quad \text{for} \quad t = 1, ..., T, \tag{1}$$

where the innovations $\varepsilon_1, ..., \varepsilon_T$ have independent and identical normal $N(0, \Omega)$ distributions conditional on the starting values $X_{-k+1}, ..., X_0$, and Π, $\Gamma_i \in \mathbf{R}^{p\times p}$ and $\Pi_c \in \mathbf{R}^p$ all vary freely. In order to justify a standard $I(1)$ cointegration analysis, three regularity conditions need to be introduced. These are given in Assumption 2.1.

Assumption 2.1 *(cf. Theorem 4.2 in Johansen, 1996)*

1. *The characteristic roots obey the equation* $|A(z)| = 0$, *where*

$$A(z) = (1 - z) I_p - \Pi z - \sum_{i=1}^{k-1} \Gamma_j (1 - z) z^i,$$

and the roots satisfy $|z| > 1$ *or* $z = 1$.

2. $(\Pi, \Pi_c) = \alpha (\beta', \gamma')$, *where* $\alpha, \beta \in \mathbf{R}^{p\times r}$ *and* $\gamma' \in \mathbf{R}^r$ *for* $r < p$.

3. *rank* $(\alpha'_\perp \Gamma \beta_\perp) = p - r$, *where* $\Gamma = I_p - \sum_{i=1}^{k-1} \Gamma_i$.

The first condition ensures that the process is neither explosive nor seasonally cointegrated, and the second is a reduced rank condition, implying that there are at least $p - r$ common stochastic trends and cointegration arises when $r \geq 1$. The third condition is of particular importance in the present note, in that it prevents the process from being $I(2)$ or of higher order. This paper is concerned with a case where the third condition is satisfied only marginally. Let $\beta^{*\prime} = (\beta', \gamma')$ and $X^*_{t-1} = \left(X'_{t-1}, 1\right)'$ for future reference. A set of vectors α are called adjustment vectors, while β^* are referred to as cointegrating vectors or cointegrating parameters.

Parameter constancy is usually required in order to demonstrate the validity of empirical models, and various testing procedures have been explored in the literature on non-stationary time series. Hansen and Johansen (1999), among all, investigate various test statistics for parameter-constancy in the cointegrated VAR framework. Brűggeman, Donati and Warne (2003) then find that some of the Hansen-Johansen tests, if constructed directly from a score function, tend to perform better in finite sample than the original formulations suggested by Hansen and Johansen (1999). The present note adopts a modification by Brűggeman *et al.* to construct the Hansen-Johansen average test for constancy of β^*. For this purpose, define $Z_t = \left(\Delta X'_{t-1}, ..., \Delta X'_{t-k+1}\right)'$ and introduce notational conventions as follows: R_{0t} denotes residuals from regression of ΔX_t on Z_t, while R_{1t} represents residuals from regression of X^*_{t-1} on Z_t. A sequence of sample product moments of R_{0t} and R_{1t} is then given by

$$\begin{pmatrix} S_{00}^{(t)} & S_{01}^{(t)} \\ S_{10}^{(t)} & S_{11}^{(t)} \end{pmatrix} = \frac{1}{t}\sum_{i=1}^{t} \begin{pmatrix} R_{0i} \\ R_{1i} \end{pmatrix} \begin{pmatrix} R_{0i} \\ R_{1i} \end{pmatrix}', \quad \text{for} \quad t = 1, ..., T. \tag{2}$$

The Hansen-Johansen average test consists of an element denoted by $Q_T^{(t)}$, which is defined as

$$Q_T^{(t)} = \left(\frac{t}{T}\right)^2 tr\left\{\left(V^{(T)}\right)^{-1} S^{(t)\prime} \left(M^{(T)}\right)^{-1} S^{(t)}\right\}, \quad \text{for} \quad t = 1, ..., T,$$

where

$$\begin{aligned} V^{(T)} &= \widehat{\alpha}' \left(\widehat{\Omega}\right)^{-1} \widehat{\alpha}, \\ S^{(t)} &= \overline{c}'_\perp \left(S_{01}^{(t)} - \widehat{\alpha}\widehat{\beta}^{*\prime} S_{11}^{(t)}\right)' \left(\widehat{\Omega}\right)^{-1} \widehat{\alpha}, \\ M^{(T)} &= T^{-1} c'_\perp S_{11}^{(T)} c_\perp, \end{aligned}$$

for

$$c_\perp = \begin{pmatrix} \widehat{\beta}_\perp & 0 \\ 0 & 1 \end{pmatrix}.$$

where $\widehat{\theta}$ denotes the maximum likelihood estimator of a certain parameter θ. Note that $S^{(t)}$ corresponds to a score function based on the Gaussian distribution. The average test statistic for constancy of β^* is then defined as

$$QLM_T = T^{-1} \sum_{t=1}^{T} Q_T^{(t)},$$

which can be regarded as a quasi LM test statistic in the context of Nyblom (1989). Hansen and Johansen (1999) show that, under Assumption 2.1, QLM_T has the following asymptotic distribution:

$$QLM_T \xrightarrow{w} \int_0^1 tr\left\{S^*(s)' J(1) S^*(s)\right\} ds, \tag{3}$$

where

$$S^*(s) = S(s) - J(s) J(1)^{-1} S(1),$$

for

$$J(s)=\int_0^s \begin{pmatrix} B_{1,u} \\ 1 \end{pmatrix} \begin{pmatrix} B_{1,u} \\ 1 \end{pmatrix}' du \quad \text{and} \quad S(s)=\int_0^s \begin{pmatrix} B_{1,u} \\ 1 \end{pmatrix} (dB_{2,u})', \tag{4}$$

and $s \in [0,1]$. The processes $B_{1,u}$ and $B_{2,u}$ are independent standard Brownian motions of dimension $(p-r)$ and r, respectively.

The limiting distribution given in (3) assumes the processes to be $I(1)$. Given the $I(2)$-type features of price indices and monetary aggregates observed in a number of empirical studies, it is also useful to investigate the asymptotic distribution of QLM_T when $I(1)$ roots are all replaced by $I(2)$ roots. The limit result is presented in the next theorem.

Theorem 2.2 *Suppose that both 1 and 2 in Assumption 2.1 are fulfilled, whereas 3 in Assumption 2.1 is violated with the number of $I(2)$ stochastic trends given by $p-r$. Then, the limiting distribution of QLM_T given in (3) is modified such that $B_{1,u}$ in (4) is replaced by $B_{1,u}^{**}$ defined as*

$$B_{1,u}^{**}=B_{1,u}^{*}-\int_0^u B_{1,v}^{*}B_{1,v}'dv\left(\int_0^u B_{1,v}B_{1,v}'dv\right)^{-1}B_{1,u}, \tag{5}$$

for

$$B_{1,u}^{*}=\int_0^u B_{1,v}dv,$$

and the restricted intercept is also corrected for $B_{1,u}$.

Proof. See the Appendix. ■

That is, the limiting distribution of QLM_T varies according to $I(1)$ and $I(2)$ cases. This result suggests the possibility that the use of simulated $I(1)$ quantiles based on (3) can lead to misleading statistical inference when the data in question are very close to $I(2)$. The next section, using Monte Carlo experiments, inspects the performance of QLM_T in the presence of near $I(2)$ roots.

3 Monte Carlo Experiments

A data generation process (DGP) for Monte Carlo experiments is given by the following bivariate system:

$$\begin{aligned}\begin{pmatrix} \Delta X_{1,t} \\ \Delta X_{2,t} \end{pmatrix} &= \begin{pmatrix} -0.1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & -1 & -0.5 \end{pmatrix} \begin{pmatrix} X_{1,t-1} \\ X_{2,t-1} \\ 1 \end{pmatrix} \\ &\quad + \begin{pmatrix} 0.2 & 0 \\ 0 & \psi \end{pmatrix} \begin{pmatrix} \Delta X_{1,t-1} \\ \Delta X_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix},\end{aligned}$$

where $(\varepsilon_{1,t},\varepsilon_{2,t})' \sim IN(0,\Omega)$ for

$$\Omega = 0.02 \times \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}.$$

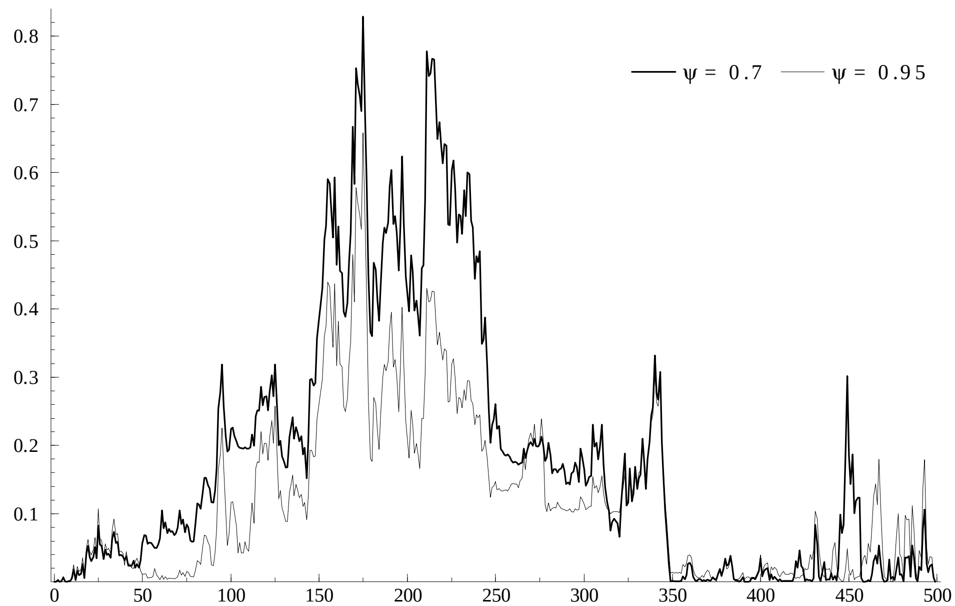

Figure 1: Sample Paths of $Q_T^{(t)}$

In the DGP, ψ is allowed to take four different values: 0.7, 0.8, 0.9, and 0.95. Note that the third condition in Assumption 2.1 is satisfied but only marginally when ψ approaches to 1, corresponding to near $I(2)$. If $\psi = 1$, then $X_{2,t}$ is a complete $I(2)$ process, which carries over to $X_{1,t}$ by way of the cointegrating linkage. The complete $I(2)$ case is precluded in the DGP considered here. The number of replications is set to be 10,000, although the sample size T varies according to experiments.

Figure 1 displays a sample path of $Q_T^{(t)}$ with $T = 500$ for two cases, $\psi = 0.7$ and $\psi = 0.95$. The former case should fall in the category of $I(1)$, while the latter case clearly corresponds to a near $I(2)$ process. According to the example in Figure 1, the overall path for $\psi = 0.95$ seems to be located lower than that for $\psi = 0.7$, suggesting that the magnitude of the QLM_T statistic for $\psi = 0.95$ can be quite different from that for $\psi = 0.7$, in line with the theoretical argument presented in the previous section.

Figure 2 plots estimated quantiles of QLM_T for $T = 500$ against the corresponding simulated asymptotic quantiles. The methodology in Johansen (1996, Ch.15) enables the author to tabulate the asymptotic quantiles of (3); the tabulation is performed using $2,000$ observations with $10,000$ replications. The vertical axis corresponds to the estimated quantiles, while the horizontal axis to the asymptotic quantiles. The baseline asymptotic quantile-quantile (QQ) plots are given by the straight thick line of 45 degree, whereas the estimated QQ plots are represented by various other lines. Figure 2 shows that all the estimated QQ plots lie below the baseline asymptotic case. In addition, the figure indicates that the test statistic tends to be under-sized as ψ gets closer to 1, even though the number of observations is very large. In other words, the test statistic, calculated from the above specific DGP even using a large number of observations, tends to be very conservative as the degree of a near $I(2)$ root becomes larger. This result is difficult to

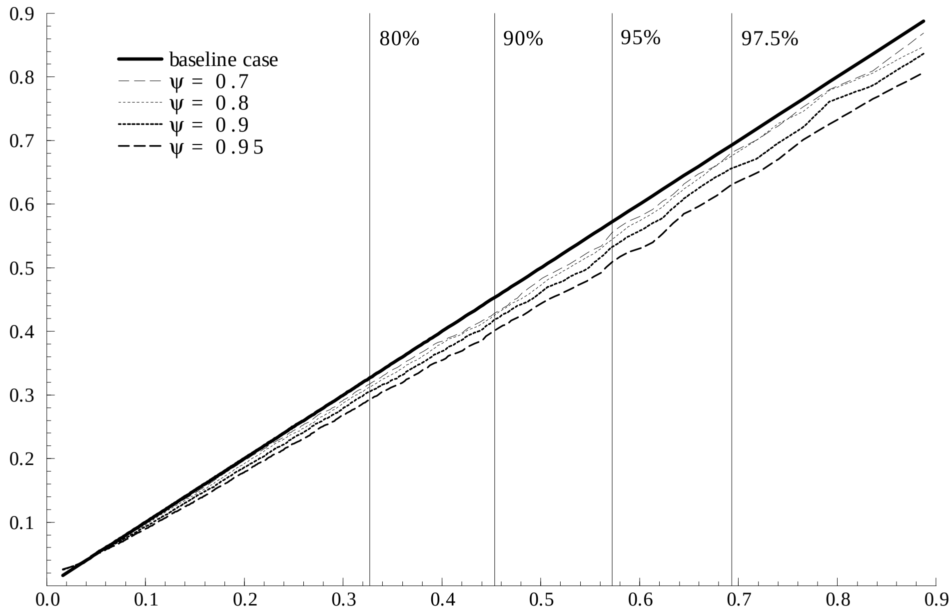

Figure 2: QQ Plots of QLM_T

generalise, but is consistent with the fact that the limiting distribution of QLM_T varies according to cases where $I(1)$ or $I(2)$ roots are involved in the system.

Furthermore, Figure 3 presents recursive rejection frequencies of QLM_T. The 95% tabulated quantile is used as a critical value, so that a nominal significance level coincides with 5%, and the figure also provides the 95% confidence band. The sample size starts at $T = 50$ and increases by 50 observations until it reaches $T = 500$. According to Figure 3(a) for $\psi = 0.7$, the rejection frequencies tend to approach to the nominal level as the sample size increases. However, such tendency becomes weaker when ψ gets close to 1, as shown in Figures 3 (b), (c) and (d). According to Figure 3(d), the rejection rate is only around 3.5% for $T = 500$. The finding indicates that the existence of a near $I(2)$ root in the above DGP renders the test statistic conservative, in line with Figure 2. The analytical argument in the previous section is again supported by the experiment.

The experiments thus far suggest that near $I(2)$ variables in the specified DGP cause the problem of under-size distortions when using the standard $I(1)$ quantile *i.e.* the test is prone to be very conservative in the presence of near $I(2)$ variables. Size control is a fundamental requirement in classical statistical inferences, so this problem needs to be addressed. The limiting distribution given in Theorem 2.2, which allows for the presence of $I(2)$ roots, could provide a better approximation in a situation where the data are very close to $I(2)$. Figure 4 similarly displays recursive rejection frequencies of QLM_T, but in this case the critical value is given by a simulated 95% quantile based on the limiting distribution in Theorem 2.2. The tabulation of the new limiting distribution is again conducted based on Johansen (1996, Ch.15). It turns out that the rejection rates for $\psi = 0.7$ in Figure 4(a) are much larger than the nominal level. The rejection rates, however, tend to approach the nominal level as ψ becomes larger, according to Figures 4

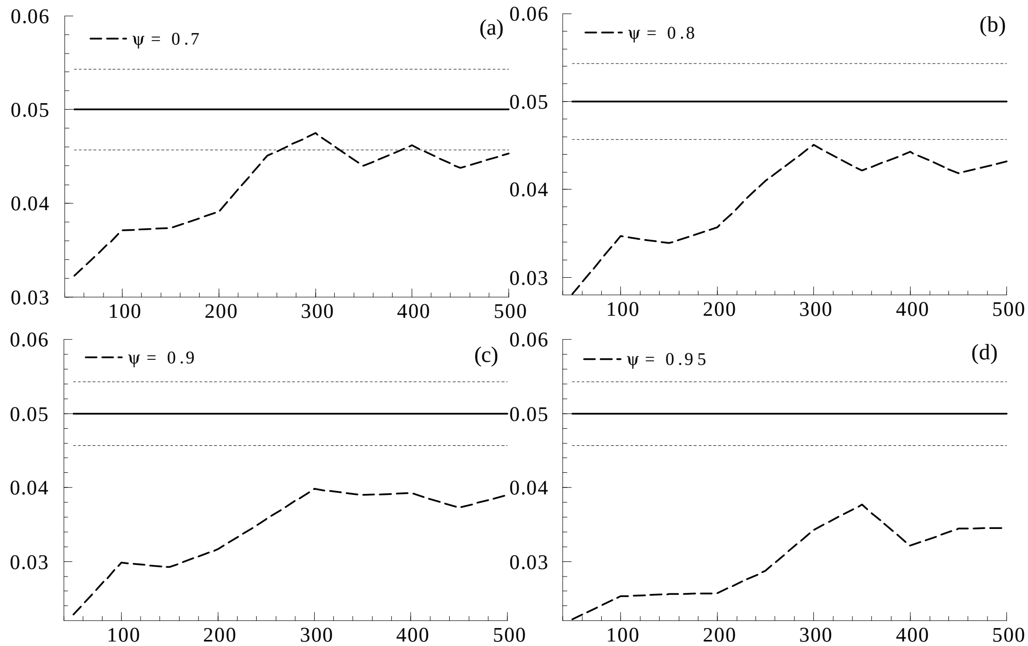

Figure 3: Recursive Rejection Frequencies: Case of the $I(1)$ Limiting Quantile

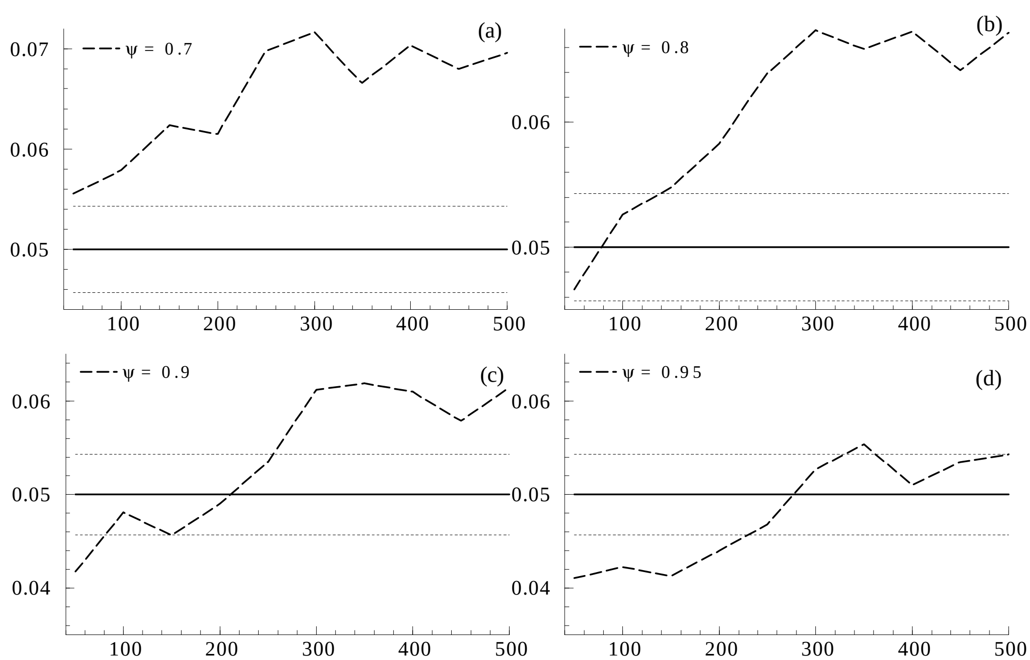

Figure 4: Recursive Rejection Frequencies: Case of the $I(2)$ Limiting Quantile

(b), (c) and (d). The rejection frequencies in Figure 4(d) tend to lie inside the confidence band with an increase in the sample size. According to Figures 3 and 4, it is presumably important to distinguish between $I(1)$ and near $I(2)$ variables in terms of size control of the test for parameter stability; the limiting distribution allowing for the presence of $I(2)$ roots is likely to be a better approximation when the data are very close to $I(2)$. The results of the experiments are all in line with the analytical argument given in the previous section.

4 Empirical Illustration

This section provides an empirical illustration of QLM_T when the data are judged to be near $I(2)$ processes. A vector autoregression consists of p_t and w_t, which are the logs of price and wage indices in Japan, respectively. See the Appendix for details of the data. An overview of quarterly data for p_t and w_t is provided in Figure 5, and the sample period runs from the fourth quarter in 1982 to the second quarter in 1999. The scale of the figure is normalised for p_t.

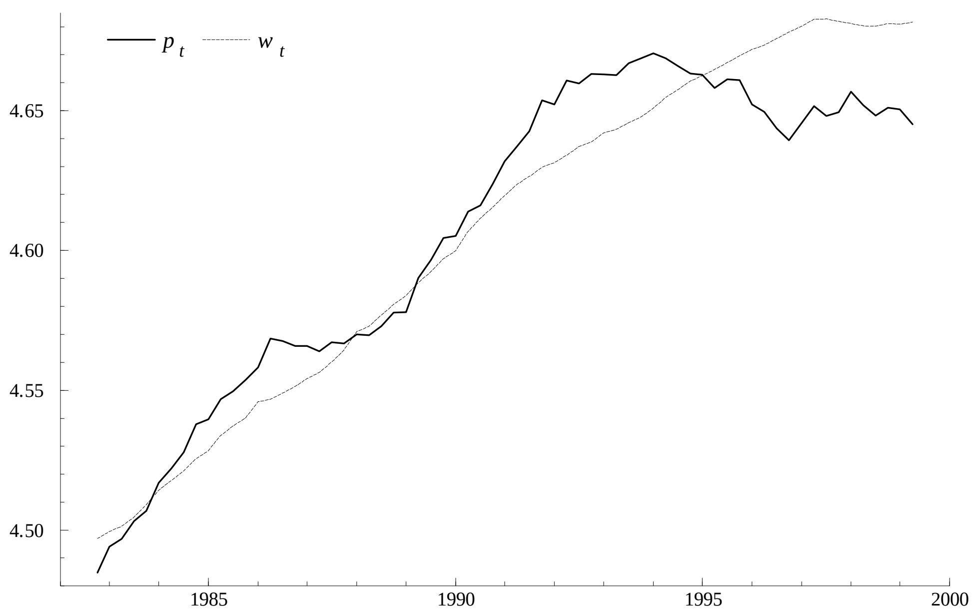

Figure 5: An Overview of Japan's Time Series Data

In line with the discussion given in the introduction, smooth trending features are found in both p_t and w_t, indicating that both of them may be interpreted as near $I(2)$ variables and share a common stochastic trend. However, according to the figure, one may conjecture that a relationship between p_t and w_t becomes unstable from 1995 onwards. It is well known that Japan suffered from a long-lasting deflation since the mid-1990s.

A VAR(2) model is estimated to conduct Johansen's $I(1)$ testing procedure (see Johansen, Ch.6, 1996) for the determination of the cointegrating rank r. The test suggests the choice of $r = 1$, which implies the validity of a restriction of a single unit root on the VAR system. Imposing the restriction on the system, it is then found that modulus of eigenvalues of a companion matrix for the system, corresponding to reciprocal values of the roots of $A(z)$ in Section 2, are given by

$$1.000, \quad 0.984, \quad 0.291, \quad 0.188.$$

The first largest eigenvalue is unity due to the restriction of a single unit root, and the second largest one is also nearly unity. This finding, as anticipated from the overview of the data, suggests that the data may be very close to $I(2)$ series.

The parameter constancy test statistic QLM_T is then constructed using the price-wage data, and its value is given by $QLM_T = 0.503$. The 95% simulating quantile based on (3) for $I(1)$ roots is 0.572, while the quantile using the $I(2)$ modification in Theorem 2.2 is 0.496. Thus, the hypothesis of time-invariant parameters is not rejected at the 5% level based on the former $I(1)$ quantile, whereas the hypothesis is marginally rejected at the same level based on the latter $I(2)$ quantile. The possibility of parameter instability, suggested by the use of the latter quantile taking account of $I(2)$ roots, seems to be in line with the conjecture based on Figure 5. This empirical illustration, together with Monte Carlo experiments given above, indicates the importance of allowing for $I(2)$ roots when testing parameter constancy in a VAR system for stock variables, such as price indices and monetary aggregates.

5 Summary and Conclusion

This note investigates the behaviour of a parameter-constancy test when near $I(2)$ variables exist in a cointegrated system. A cointegrated VAR analysis requires parameter stability throughout the sample period of interest. Hansen and Johansen (1999) explore various test statistics for this purpose, and this note focuses on a quasi LM test among them. This note presents a limit theorem for a quasi LM parameter constancy test, which explicitly allows for $I(2)$ roots in the cointegrated VAR system. The limiting distribution of the test statistic comprises functionals of the standard Brownian motion. The distribution is therefore different from that in a standard case where only $I(1)$ roots are involved in the system. The limit theorem indicates that the use of simulated quantiles based on a conventional $I(1)$ specification can lead to misleading statistical inference when the data are very close to $I(2)$. According to the Monte Carlo experiments conducted in this note, the existence of near $I(2)$ series, in fact, has a significant impact on size properties of a quasi LM test for parameter stability. An empirical illustration also lends weight to this argument. It is therefore important to check if the data are $I(1)$ or close to $I(2)$ processes when testing stability of the cointegrating parameters.

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A Appendix

A.1 Proof of Theorem 2.2

Without loss off generality, the VAR model (1) under the fulfillment of both 1 and 2 in Assumption 2.1 is expressed as

$$\Delta^2 X_t = \alpha\beta^* X_{t-1}^* - \Gamma\Delta X_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 X_{t-i} + \varepsilon_t,$$

where $\Psi_i = -\sum_{j=i+1}^{k-1} \Gamma_j$. Define $Z_t^* = \left(\Delta^2 X_{t-1}', ..., \Delta^2 X_{t-k+2}'\right)'$ and introduce notational conventions as follows: R_{0t}^* denotes residuals from regression of $\Delta^2 X_t$ on ΔX_{t-1} and Z_t^*, while

$$R_{1t}^* = X_{t-1}^* - \sum_{i=1}^{t} X_{i-1}^* \Delta X_{i-1}' \left(\sum_{i=1}^{t} \Delta X_{i-1} \Delta X_{i-1}'\right)^{-1} \Delta X_{t-1} + O_p(T),$$

which also represents residuals from regression of X_{t-1}^* on ΔX_{t-1} and Z_t^*. The final term on the right hand side of R_{1t}^* arises because Z_t^* is a vector of $I(0)$ processes. The statistic $Q_T^{(t)}$ is then constructed using (2), in which R_{0t} and R_{1t} are replaced by R_{0t}^* and R_{1t}^*, respectively. Let R_{1t}^* be expressed in two elements such that $R_{1t}^* = \left(R_{1t}^{*A}, R_{1t}^{*B}\right)'$, in which R_{1t}^{*A} corresponds to X_{t-1} corrected for ΔX_{t-1} and Z_t^*, while R_{1t}^{*B} corresponds to a constant corrected for ΔX_{t-1} and Z_t^*. As the number of $I(2)$ trends is $p-r$ and the trends can be captured by expansion into the direction $\beta_\perp'$, the sequence R_{1t}^{*A} is normalised by $T^{-\frac{3}{2}}\beta_\perp'$, which leads to a weak limit

$$T^{-\frac{3}{2}}\beta_\perp' R_{1[Tu]}^{*A} \xrightarrow{w} V_u^{**} = V_u^* - \int_0^u V_\upsilon^* V_\upsilon' d\upsilon \left(\int_0^u V_\upsilon V_\upsilon' d\upsilon\right)^{-1} V_u,$$

where

$$V_u^* = \int_0^u V_\upsilon d\upsilon \quad \text{and} \quad V_\upsilon = \beta_\perp' C_2 W_\upsilon,$$

and W_υ denotes a p dimensional Brownian motion with variance Ω, and a $p \times p$ matrix C_2 is defined as $C_2 = \beta_\perp(\alpha_\perp'\Theta\beta_\perp)^{-1}\alpha_\perp'$ for $\Theta = \Gamma\overline{\beta}\overline{\alpha}'\Gamma + I_p - \sum_{i=1}^{k-2}\Psi_i$ (see Johansen, 1995, 1997). Thus, the $p-r$ dimensional functional $B_{1,u}^{**}$ defined in (5) is obtained from

$$B_{1,u}^{**} = \left(\beta_\perp' C_2 \Omega C_2' \beta_\perp\right)^{-\frac{1}{2}} V_u^{**},$$

and the analogous argument as in Hansen and Johansen (1999) is used to find the limiting distribution consisting of $B_{1,u}^{**}$.

A.2 Details of the Data

(Data Definitions)

p_t = the log of the implicit deflator for Gross Domestic Products (GDP) in Japan, <1>
w_t = the log of monthly earning index in Japan, <2>

(Sources and Notes)

<1> *National Account Statistics*, Cabinet Office web page (http://www.esri.cao.go.jp/).

<2> *International Financial Statistics*, International Monetary Fund.

The implicit deflator is constructed from the division of the nominal GDP by the real GDP. The nominal and real GDP seasonally-adjusted series, and the monthly earning index is also seasonally adjusted.

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