Economics Bulletin

Volume 29, Issue 3

Specific Performance, Separability Condition and the Hold-Up Problem

M'hand Fares
INRA (AGIR) and University of Paris 1

Abstract

Edlin and Reichelstein (1996) claim that an efficient solution to the hold-up problem can be implemented with a specific performance contract if a separability condition is satisfied, i.e. if the effect of investments and the effect of the state of the world enter the parties valuation functions in an additively separable manner. This note shows that this separability condition generates the same solution than if the valuation functions are independent of the sate of nature (proposition 1). This implies that a simple menu of prices that does not specify the level of trade can solve the hold-up problem (proposition 2). That is, specifying the terms of trade by writing a specific performance contract is useless with the separability condition.

I am grateful to an anonymous referee, Szu-Wen Chou, Daniel Delalande, Tore Ellingsen, David Martimort, Marie-Odile Ogier, Anne Perrot, Patrick Rey, and seminar participants at University of Paris 1 for helpful comments and discussions.
Citation: M'hand Fares, (2009) "Specific Performance, Separability Condition and the Hold-Up Problem", *Economics Bulletin*, Vol. 29 no.3 pp. 2055-2062.
Submitted: Mar 17 2009. **Published:** August 20, 2009.

1 Introduction

Incomplete contract theory has addressed the issue of how to design the optimal contractual arrangement to achieve the efficient investments in the presence of contract incompleteness (Grossman and Hart 1986, Hart and Moore 1990, Hart 1995). In this theory, incompleteness comes from the parties inability to write complete contingent contract on all future states of nature and ex ante investments because they are too costly to describe (Dye 1985, Anderlini and Felli 1994 2001, Battigalli and Maggi 2002 2008) or unverifiable by court (Hart and Moore 1988, Anderlini et al. 2007). After the relevant state of nature is realized, the parties may renegotiate the initial contract. However, such ex post renegotiation may lead to inefficient investments. This is called under-investment result or the "hold-up problem" (Klein et al. 1978, Williamson 1985, Grossman and Hart 1986, Hart and Moore 1988).

In the contract solution literature (Rogerson 1992, MacLeod and Malcomson 1993, Aghion et al. 1994, Nöldeke and Schmidt 1995, Fares 2006), the hold-up problem can be solved by defining a renegotiation framework so that gains from renegotiation are never shared (renegotiation design). Aghion et al. (1994) show that such a renegotiation framework can be designed in a specific performance contract. Edlin and Reichelstein (1996) argue however that an efficient solution can be implemented with a specific performance contract without renegotiation design if a separability condition is satisfied, i.e. if the effect of investments and the effect of the state of the world enter the parties valuation functions in an additively separable manner.

This note shows that this separability condition generates the same solution than if the valuation functions are independent of the state of nature (proposition 1). This implies that a simple menu of prices that does not specify the level of trade can solve the hold-up problem (proposition 2). That is, specifying the terms of trade by writing a specific performance contract is useless with the separability condition. Therefore, we argue that a specific performance contract has some value to overcome the hold-up problem only with a renegotiation design.

The remaining sections are organized as follows. Section 2 presents the basic model. Section 3 analyzes the equivalence result between the separability condition and the no (direct) uncertainty assumption.

2 The Model

A seller (she) and a buyer (he) engage in a long-term relationship. They write a contract which specifies the delivery, at a precise date in the future, of a given quantity $q \in \mathbb{R}_+$ of the good produced by the seller, and the payment of a predetermined sum of money $p \in \mathbb{R}$. The production cost and the benefit to the parties are affected both by the realization of a random variable $\omega \in \Omega$ (distributed according to a known function $F(\omega)$). The cost of the seller is also affected by her investment $\sigma \in [0, \infty)$ and the benefit of the buyer by his investment $\beta \in [0, \infty)$. These investments are made before the value of ω is realized. They are relationship-specific, i.e. the parties cannot recoup the cost of the investment if they leave the specified relationship.

Let $c(q, \omega, \sigma)$ denote the seller's production cost of delivering q units of output when ω and σ are the arguments of the cost function. Similarly, let $v(q, \omega, \beta)$ be the buyer's (gross) benefit of receiving q units of the good when the realized state of the world is ω, and when his level of investment is β. It is assumed that both $v(q, \omega, \beta)$ and $c(q, \omega, \sigma)$ are increasing in q and $v(q, \omega, \beta) - c(q, \omega, \sigma)$ is (strictly) increasing in (β, σ) at a (strictly) decreasing rate. The economically interesting case is one where incentives to invest are sensitive to expected levels of trade: $v_{q\beta} \geq 0$ and $c_{q\sigma} \leq 0$. Assume also that higher ω increases (decreases) marginal benefit (production cost) from investment, i.e. $v_{\beta\omega} > 0$ ($c_{\sigma\omega} < 0$). We next assume that for every ω, β, σ: $v(0, \omega, \beta) = c(0, \omega, \sigma) = 0$, i.e. the benefit and the cost of having no trade are both equal to zero. This assumption formalizes the idea that the cost of a specific investment cannot be recouped in the absence of trade.

Following the incomplete contract literature, we assume that ω, β, and σ as well as v and c are all observable by the two parties, but are nonverifiable by court. Therefore it is impossible to write a contract conditional upon any of them: the contract can only specify an allocation $(\widetilde{q}, \widetilde{p})$, where $\widetilde{q}$ is the level of output to be delivered by the seller and $\widetilde{p}$ the payment to be made by the buyer. Moreover, the court can impose this initial contract if either of the two parties demands it. In legal terms, the court uses the specific performance remedy with a possibility of renegotiation. Indeed, the court is assumed to be able to enforce the initial contract unless the parties voluntarily agree to replace this by another contract.

The sequence of events proceeds as follows. At date 0, the parties commit to a simple contract $(\widetilde{q}, \widetilde{p})$. At date 1, the parties decide simultaneously upon their respective investment levels. At date 2, seller and buyer learn their types after observing the state of nature. Trading the output and agreeing on the transfer payment, or perhaps breach of the contract, occurs at date 3, after which any outstanding claims are settled or go to court. At the stage between date 2 and 3, ex post renegotiation may occur if both parties agree to do so.

2.1 The benchmark

The first best, as a benchmark, is the outcome achieved when β, σ, ω are all observable by the two parties but also verifiable and hence contractible[1]. If both parties are risk neutral, given a realized value of ω and chosen levels of investment (β, σ), their preferences are described by the following utility function $u_B = v(q, \omega, \beta) - p$ and $u_S = p - c(q, \omega, \sigma)$. The social surplus is then defined as

$$\phi(q, \omega, \beta, \sigma) \equiv u_B + u_S = v(q, \omega, \beta) - c(q, \omega, \sigma)$$

For any given (β, σ), let

$$\{q^*(\omega, \beta, \sigma),\ p^*(\omega, \beta, \sigma) : \omega \in \Omega\}$$

be the complete contingent solution of this first best problem conditional on (β, σ), where

$$q^*(\omega, \beta, \sigma) = \underset{q}{\arg\max}\ \phi(q, \omega, \beta, \sigma)$$

[1] The benchmark cannot be interpreted as the solution under vertical integration. Indeed, in the incomplete contract theory (more precisely in the property rights approach) integration does not make the information verifiable (Hart 1995).

The efficient investments (β^*, σ^*) can be defined as follows

$$(\beta^*, \sigma^*) \equiv \underset{\beta,\sigma}{\arg\max} \int \phi(q^*(\omega, \beta, \sigma), \omega, \beta, \sigma) dF(\omega) - \beta - \sigma \tag{1}$$

The efficient investments maximize the difference between the total expected gain and the direct cost of investments. If both β and σ were verifiable, it would be possible to contract over investment levels and thus the parties would choose (β^*, σ^*) which maximizes the expected net gains from the relationship. Indeed, an optimum contract would stipulate that between dates 0 and 1 the buyer and the seller must undertake β^* and σ^* respectively; if a party failed to invest at the specified level, then he would be made to pay a large penalty to the other.

3 Characterization result

When β, σ, ω are unverifiable and hence uncontractible, parties can write only an incomplete (contingent) initial contract $(\widetilde{q}, \widetilde{p})$. We assume that the renegotiation of $(\widetilde{q}, \widetilde{p})$ between dates 2 and 3 results in an efficient quantity of trade $q^*(\omega, \beta, \sigma)$ and that both parties appropriate the entire net surplus (i.e. budget balancing). This net surplus, denoted $S \equiv [\phi(q^*(.), \omega, \beta, \sigma) - \phi(\widetilde{q}, \omega, \beta, \sigma)]$, is the difference between the joint surplus $\phi(q^*(.), \omega, \beta, \sigma)$ that would result from renegotiation and the joint surplus under the original contract terms, i.e. $\phi(\widetilde{q}, \omega, \beta, \sigma) \equiv (v(\widetilde{q}, \omega, \beta) - c(\widetilde{q}, \omega, \sigma))$. The bargaining positions of the parties are supposed to be exogenously determined and we simply assume that the parties share the surplus from bargaining, with the seller receiving a fraction $\alpha \in [0, 1]$. Unlike renegotiation, we do not model the negotiations that result in the initial contract. Rather we simply assume that these negotiations yield an efficient contract. This can be modeled as the buyer choosing a contract $(\widetilde{q}, \widetilde{p})$ subject to the constraint that the seller achieves, in expectation, a reservation utility. As a normalization of utility, we set this reservation level of utility to zero. Given the initial contract and the subsequent renegotiation, the seller's expected payoff from choosing σ at date 1 is

$$\begin{aligned} U_S(\beta, \sigma; \widetilde{q}) &= \int [\widetilde{p} - c(\widetilde{q}, \omega, \sigma)] + \alpha S \; dF(\omega) - \sigma \\ &= \int [\widetilde{p} - c(\widetilde{q}, \omega, \sigma)] + \alpha[\phi(q^*(.), \omega, \beta, \sigma) - \phi(\widetilde{q}, \omega, \beta, \sigma)] dF(\omega) - \sigma \end{aligned} \tag{2}$$

where the first term in the integral $[\widetilde{p} - c(\widetilde{q}, \omega, \sigma)]$ represents the seller's status quo under the initial contract. The next term represents the seller's share of the net surplus arising from renegotiation. Similarly, the buyer's expected payoff from choosing β at date 1 is

$$\begin{aligned} U_B(\beta, \sigma; \widetilde{q}) &= \int [v(\widetilde{q}, \omega, \beta) - \widetilde{p}] + (1 - \alpha) S \; dF(\omega) - \beta \\ &= \int [v(\widetilde{q}, \omega, \beta) - \widetilde{p}] + (1 - \alpha)[\phi(q^*(.), \omega, \beta, \sigma) - \phi(\widetilde{q}, \omega, \beta, \sigma)] dF(\omega) - \beta \end{aligned} \tag{3}$$

The seller chooses non cooperatively his investment σ to maximize $U_S(\beta, \sigma; \widetilde{q})$. The equilibrium investment, if positive, satisfies the first-order conditions

$$\frac{\partial U_S(\beta, \sigma; \widetilde{q})}{\partial \sigma} = -\int \alpha c_\sigma(q^*(.), \omega, \sigma) + (1 - \alpha) c_\sigma(\widetilde{q}, \omega, \sigma) dF(\omega) - 1 = 0 \tag{4}$$

The buyer also chooses non cooperatively her investment β to maximize $U_B(\beta, \sigma; \widetilde{q})$. The equilibrium investment, if positive, satisfies the first-order conditions

$$\frac{\partial U_B(\beta, \sigma; \widetilde{q})}{\partial \beta} = \int (1-\alpha) v_\beta(q^*(.), \omega, \beta) + \alpha v_\beta(\widetilde{q}, \omega, \beta) dF(\omega) - 1 = 0 \tag{5}$$

Since the investments are chosen simultaneously, given equation (2), σ^* is the best reply to β^* if there is an initial quantity $\widetilde{q}_S$ such that

$$\frac{\partial U_S(\beta^*, \sigma^*; \widetilde{q}_S)}{\partial \sigma} = -\int c_\sigma(q^*(.), \omega, \sigma^*) dF(\omega) = 1 \tag{6}$$

Similarly, given equation (3), β^* is the best reply to σ^* if there is an initial quantity $\widetilde{q}_B$ such that

$$\frac{\partial U_B(\beta^*, \sigma^*; \widetilde{q}_B)}{\partial \beta} = \int v_\beta(q^*(.), \omega, \sigma^*) dF(\omega) = 1 \tag{7}$$

Aghion et al. (1994) show that if the initial contract allocates the whole bargaining power to the seller ($\alpha = 1$) in the renegotiation subgame (renegotiation design), a specific performance contract with a single quantity $\widetilde{q}_B = \widetilde{q}$ can solve the system of two independent equations (4)-(5). (see Proposition 3.1, p. 267). Edlin and Reichelstein (1996) show that the same efficient solution can be implemented without renegotiation design if the parties valuation functions satisfy the following separability condition: $v(q, \omega, \beta) \equiv v_1(\beta).q + v_2(q, \omega) + v_3(\omega, \beta)$ and $c(q, \omega, \sigma) \equiv c_1(\sigma).q + c_2(q, \omega) + c_3(\omega, \sigma)$. Indeed, with such a condition they prove that a single quantity $\widetilde{q} = \widetilde{q}_S = \widetilde{q}_B = \int q^*(.) dF(\omega)$ can solve the system (4)-(5) (see proposition 6, p. 493).

The following proposition 1 shows that the Edlin-Reichelstein (1996)'s efficient result is due to the equivalence between the separability condition and a no (direct) uncertainty assumption on the valuation functions, i.e. the valuation functions are supposed to be independent of ω.

Proposition 1 *The separability condition is equivalent to a no (direct) uncertainty assumption since both generate the same solution, i.e.* $\widetilde{q}_B = \widetilde{q}_S = \int q^*(.) dF(\omega)$.

Proof We prove this in two steps. First, we show that when parties face a (direct) uncertainty, i.e. their valuation functions are dependent of ω, the optimal quantities $\widetilde{q}_B$ and $\widetilde{q}_S$ on average exceed the actual quantity level ($q^*(.)$) and therefore we cannot get the unique solution generated by the separability condition *(i)*. Second, we show that when parties face no (direct) uncertainty we get the same unique solution than when the valuations functions are separable, i.e. $\widetilde{q}_B = \widetilde{q}_S = \int q^*(.) dF(\omega)$ *(ii)*.

(i) Suppose that parties face a (direct) uncertainty and their valuation functions are linear in q, i.e. $v(q, \omega, \beta) \equiv v(\omega, \beta) q$ and $c(q, \omega, \sigma) \equiv c(\omega, \sigma) q$. Rewriting equation (5), we get

$$\begin{aligned} \frac{\partial U_B(\beta, \sigma; \widetilde{q}_B)}{\partial \beta} &= \int (1-\alpha) v_\beta(\omega, \beta) q^*(\omega, \beta^*, \sigma^*) + \alpha v_\beta(\omega, \beta^*) \widetilde{q}_B dF(\omega) \\ &= \int v_\beta(\omega, \beta) q^*(\omega, \beta^*, \sigma^*) dF(\omega) \end{aligned} \tag{8}$$

that is,

$$\widetilde{q}_B \int v_\beta(\omega,\beta^*)dF(\omega) = \int v_\beta(\omega,\beta^*)q^*(\omega,\beta^*,\sigma^*)dF(\omega) \tag{9}$$

Then because $\int v_\beta(\omega,\beta^*)dF(\omega) > 0$ we get

$$\widetilde{q}_B \geq \int q^*(\omega,.)dF(\omega) \tag{10}$$

if the following covariance or Chebyshev's inequality (Wagener, 2006)

$$\int v_\beta(\omega,\beta^*)q^*(\omega,.)dF(\omega) \geq \int v_\beta(\omega,\beta^*)dF(\omega)\int q^*(\omega,.)dF(\omega) \tag{11}$$

is satisfied. As $v_\beta(\omega,.)$ and $q^*(\omega,.)$ are non-negative and increasing variables in ω, this condition (8) holds. Consider now the seller' side. After rewriting equation (4) and rearranging we get

$$\widetilde{q}_S \int -c_\sigma(\omega,\sigma)dF(\omega) = \int -c_\sigma(\omega,\sigma)q^*(\omega,\beta,\sigma)dF(\omega) \tag{12}$$

since $\int -c_\sigma(\omega,\sigma)dF(\omega) > 0$ then

$$\widetilde{q}_S \geq \int q^*(\omega,\beta^*,\sigma^*)dF(\omega) \tag{13}$$

if the following condition

$$\int -c_\sigma(\omega,\sigma^*)q^*(\omega,.)dF(\omega) \geq \int -c_\sigma(\omega,\sigma^*)dF(\omega)\int q^*(\omega,.)dF(\omega) \tag{14}$$

holds. Which is true for the same reasons as above. Then in a general setting $\widetilde{q}_S$ and $\widetilde{q}_B$ are no more equal, which implies that a single quantity cannot implement the efficient solution.

(ii) If parties do not face any (direct) uncertainty, i.e. $v(\omega,q,\beta)$ and $c(\omega,q,\sigma)$ are independent of ω, then (9) and (12) can be rewritten as follows

$$\begin{aligned} \widetilde{q}_B v_\beta(\beta^*) &= v_\beta(\beta^*)\int q^*(\omega,\beta^*,\sigma^*)dF(\omega) \\ -\widetilde{q}_S c_\sigma(\sigma) &= -c_\sigma(\sigma)\int q^*(\omega,\beta,\sigma)dF(\omega) \end{aligned}$$

which implies the equality $\widetilde{q}_B = \widetilde{q}_S = \int q^*(.)dF(\omega)$. □

Note that with renegotiation design the initial quantity $\widetilde{q}_B$ has to be upward biased to ensure efficiency. Indeed, if $\alpha = 1$ in (8), we also get $\widetilde{q}_B \geq \int q^*(\omega,.)dF(\omega)$. The question is then: why can we define a unique quantity ($\widetilde{q} = \widetilde{q}_B = \widetilde{q}_S$) and set this quantity at an "average" trade level ($\widetilde{q} = \int q^*(.)dF(\omega)$) under the separability condition? This can be explained as follows. What really matter in the separable valuation functions are the first terms $v_1(\beta).q$ and $c_1(\sigma).q$ independent of ω, and the other terms play no role in the efficiency result. That is why the separability condition is equivalent to a no (direct) uncertainty assumption.

Suppose now that the buyer offers at date 1 a contract that specifies a menu of two prices $p(q)$, with $p(q) = p_1$ if $q > 0$ and $p(q) = p_0$ if $q = 0$, to be paid by the buyer to the seller when they trade q. That is, in contrast to a specific performance, no trade level like a minimum quantity $\widetilde{q}$ is specified. The menu price contract just defines a trade price of p_1 and a no-trade price of p_0[2]. The proposition 2 below shows that the equivalence between the separability condition and the no (direct) uncertainty assumption has a strong consequence since it implies that a simple renegotiable menu of prices (p_1, p_0) can implement the efficient solution.

Proposition 2 *With the no (direct) uncertainty assumption, the efficient solution can be achieved with a menu of prices* (p_1, p_0).

Proof When there is no (direct) uncertainty, the parties can either trade the efficient quantity $q^*(.)$ or choose not to trade. The initial contract has then just to share the surplus so that parties are willing to trade. This can be done with a menu of prices (p_1 if $q^*(.)$ is exchanged and p_0 otherwise). These prices can be renegotiated at date 2. Choose (p_1, p_0) such that at date 2, $q^*(\omega, \beta, \sigma)v(\beta) - p_1 < p_0$, i.e. the buyer would prefer no trade to the efficient trade $q^*(.)$ if there were no renegotiation. After renegotiation, the outcome is the trade of $q^*(.)$ at a price $\widetilde{p}_1$ which makes the buyer indifferent between trade and no trade, i.e. $q^*(.)v(\beta) - \widetilde{p}_1 = p_0$. Since the buyer's ex ante utility is $U_B(\beta, p_0) = p_0 - \beta$, he invests efficiently if p_0^* is such that $p_0^* = \beta^*$. Now, choose $p_1 = p_1^*$ such that at date 2 $q^*(\omega, \beta, \sigma)v(\beta) - p_1^* < p_0^*$. Given the subsequent renegotiation outcome, the seller's ex ante utility is $U_S(\sigma, p_0^*) = \widetilde{p}_1 - c(\sigma) \int q^*(\omega, \beta^*, \sigma)dF(\omega) = [v(\beta^*) - c(\sigma)] \int q^*(\omega, \beta^*, \sigma)dF(\omega) - \beta^* - \sigma$. That is, the seller is residual claimant at the margin and thus will invest efficiently ($\sigma = \sigma^*$). $\square$

This result casts doubt on the relevance of the specific performance contract when a separability condition is satisfied. Indeed, since a menu of prices can implement the efficient solution, there is no need to write a specific performance contract where specifying the level of trade can be costly (Battigali and Maggi 2008).

References

[1] Aghion P., Dewatripont, M. and P. Rey (1994) “Renegotiation Design with Unverifiable Information ” *Econometrica* **62**, 257-282.

[2] Anderlini, L. and L. Felli (1994) ”Incomplete Written Contracts : Undescribable States of Nature” *Quarterly Journal of Economics* **109**, 1085-1124.

[3] Anderlini, L. and L. Felli (2001) ”Costly Bargaining and Renegotiation” *Econometrica* **69**, 377-411.

[4] Anderlini L., Felli, L. and A. Postlewaite (2007) ”Courts of Law and Unforseen Contingencies” *Journal of Law, Economics and Organization* **23**, 662-684.

[2] This contract is very similar to the voluntary contract considered by Hart and Moore (1988). Indeed, enforcement of such a contract requires the court to verify only whether the parties trade or not, which is clearly feasible since q is verifiable.

[5] Battigalli P., and G. Maggi (2002) ”Rigidity, Discretion and the cost of writing contracts” *American Economic Review* **92**, 798-817.

[6] Battigalli P., and G. Maggi (2008) "Costly contracting in Long-Term Relationship" *Rand Journal of Economics* **39**, 352-377.

[7] Dye, R. (1985) ”Costly Contract Contingencies” *International Economic Review* **26**, 233-250.

[8] Edlin A., and S. Reichelstein (1996) ”Holdups, Standard Breach Remedies, and Optimal Investment *American Economic Review* **86**, 478-501.

[9] Fares, M. (2006) "Renegotiation Design and Contractual Solutions to the Hold-Up Problem" *Journal of Economic Surveys* **20**, 731-756.

[10] Grossman S., and O. Hart (1986) ”The Costs and Benefits of Ownership : A Theory of Vertical and Lateral Integration *Journal of Political Economy* **94**, 691-719.

[11] Hart, O. (1995) *Firms, Contracts, and Financial Structure*, Oxford University Press : Oxford.

[12] Hart, O. and J. Moore (1988), ”Incomplete Contracts and Renegotiation” *Econometrica* **56**, 755-785.

[13] Hart, O. and J. Moore (1990), ”Property Rights and the Nature of the Firm” *Journal of Political Economy* **98**, 1119-1158.

[14] Klein B., Crawford, R.G and A.A. Alchian (1978) ”Vertical Integration, Appropriable Rents, and the Competitive Contracting Process” *Journal of Law and Economics* **28**, 297-326.

[15] MacLeod, B. and J. Malcomson (1993) ”Investments, Hold-up, and the Form of Market Contracts” *American Economics Review* **83**, 811-837.

[16] Rogerson, W. (1992) ”Contractual Solutions to the Hold-up Problem” *Review of Economic Studies* **59**, 777-793.

[17] Wagener, A. (2006) "Chebyshev’s Algebraic Inequality and comparative statics under uncertainty" *Mathematical Social Sciences* **52**, 217-221.

[18] Williamson, O.E. (1985) *The Economic Institutions of Capitalism* Free Press : New-York.