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Growth and Distributional Effects of Inflation with Progressive Taxation

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Abstract

This paper examines the growth and income distribution effects of inflation in a growing economy with heterogeneous households and progressive income taxation. Assuming that the cash-in-advance constraint applies to investment as well as to consumption spending, we show that a higher growth of monetary supply yields a negative impact on growth and an ambiguous effect on income distribution. Numerical example with plausible parameter values, however, demonstrate that those long-run effects of money growth are rather small. In contrast, fiscal distortion caused by progressive taxation yield significant impacts on growth and distribution.

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1 Introduction

The purpose of this paper is to examine interactions between money growth and real income tax in a simple model of monetary endogenous growth with heterogeneous agents. We construct a cash-in-advance model in which there are two types of households, each of which has different time discount rate. In our setting, the long-run level of relative income and the balanced growth rate of real income are uniquely determined unless the elasticity of intertemporal substitution in consumption is sufficiently high. Provided that the cash-in-advanced constraint applies to both consumption and to investment spending, we inspect how a change in the growth rate of nominal money supply affects growth and income distribution in the long-run equilibrium. We show that a monetary expansion has a negative impact on growth and an ambiguous effect on income distribution.[1] Numerical example with plausible parameter values, however, demonstrate that the quantitative effects of money growth are rather small. In contrast, the fiscal distortion caused by progressive taxation may yield considerable impacts on growth and distribution.

2 The Model

Consider a competitive, growing economy with an Ak technology. The aggregate production function is given by

$$y = Ak, \tag{1}$$

where y is output and k is capital stock. Since the production employs capital alone, the competitive gross rate of return to capital is determined by $r = A$. As for the consumers' side, we assume that there are two types of households. Those type of agents differ in the time discount rates and initial

[1] Several authors examine the growth effect of inflation in the context of representative-agent models of endogenous growth: see, for example, De Gregorio (1993), Jha, Wang and Yip (2002), Jones and Manuelli (1995), Marquis and Reffett (1995) and Mino (1997). In general, the foregoing studies find a negative relation between growth and inflation. The present paper reexamines the same issue in a prototype model of endogenous model with heterogeneous agents. It is to be noted that the steady-state impact on inflation in exogenous growth models have produced more diverse results. For example, Chen et al. (2008) reveal that a rise in money growth may or may not increase the steady-state level of capital depending on whether or not the marginal impatience increases with consumption.

holdings of wealth. We assume that type 1 household is more impatient than those of type 2. There is a continuum of households and the total number is normalized to unity. It is assumed that population share of type 1 is $\theta \in (0,1)$ and type 2 is $1-\theta$.

Except for the presence of heterogeneous households, the rest of the setting is standard. We use a cash-in-advance model in which households face a liquidity constraint for their investment as well as for consumption expenditure. The objective of type i household maximizes its discounted sum of utilities

$$U_i = \int_0^\infty \frac{c_i^{1-\sigma}-1}{1-\sigma} e^{-\rho_i t} dt, \ \sigma > 0, \quad \rho_i > 0, \quad i = 1,2,$$

where c_i denotes consumption of type i household. By our assumption, the time discount rate ρ_i satisfy that $\rho_1 > \rho_2$.

The households hold capital and money. The real money balances held by type i household changes according to

$$\dot{m}_i = \left[1 - \xi \left(\frac{y_i}{y}\right)^{\varepsilon}\right] y_i - c_i - v_i - \pi m_i + z, \quad \xi > 0, \quad \varepsilon > 0, \tag{2}$$

where y_i, m_i, and v_i are respectively denote income, real money holding and investment for physical capital. Additionally, π stands for the rate of inflation and z denotes the lump-sum transfer from the government. We assume that the government levies progressive income tax and the rate of tax is specified as $\xi\left(\frac{y_i}{y}\right)^{\varepsilon}$, where ε (> 1) represents the degree of progressiveness of taxation. We have assumed that the total population is one, implying that y also represents the average per-capita output so that $y = \theta y_1 + (1-\theta) y_2$. Since we deal with a growing economy with persistent expansion of individual income, we assume that the rate of tax depends on the relative income rather than the absolute level of income. This formulation follows Guo and Lansing (1998) and Li and Sarte (2004).[2] The holding of capital stock changes in the following manner:

$$\dot{k}_i = v_i - \delta k_i, \quad 0 < \delta < 1, \tag{3}$$

where k_i is capital stock of type i agent and δ denotes the rate of depreciation. By definition, the aggregate capital is expressed as $k = \theta k_1 + (1-\theta) k_2$. In

[2] See also Sarte (1997).

addition to (2)and (3), the household's spending is subject to the cash-in-advance constraint such that

$$c_i + \phi v_i \leq m_i, \qquad 0 \leq \phi \leq 1. \tag{4}$$

When $\phi > 0$, the cash-in-advance constraint applies to the investment spending as well.

The household maximizes U_i subject to (2), (3) , (4) and the initial holdings of real money balances and capital stock. Since households earn capital income alone, $y_i = rk_i = Ak_i$. As a result, the relative income in the tax function is expressed as $y_i/y = k_i/k$.[3] Considering this fact, we set up the Hamiltonian function for the household's optimization problem in such a way that

$$\begin{aligned} H_i &= \frac{c_i^{1-\sigma}}{1-\sigma} + q_i \left\{ \left[1 - \xi \left(\frac{k_i}{k} \right)^{\varepsilon} \right] Ak_i - c_i - v_i - \pi m_i + z \right\} \\ &\quad + \eta_i \left(v_i - \delta k_i \right) + \lambda_i \left(m_i - c_i - \phi v_i \right), \end{aligned}$$

where q_i and η_i respectively denote the shadow values of real money balances and λ_i is a Lagrangian multiplier. It is to be noted that when selecting optimal consumption-saving plan, the household takes future sequences of the average income at the society at large, y, the rate of inflation, π, and personal transfer, z, as given. The necessary conditions for an optimum involve the following:

$$c_i^{-\sigma} = q_i + \lambda_i, \tag{5}$$

$$-q_i + \eta_i - \phi \lambda_i = 0, \tag{6}$$

$$\dot{q}_i = q_i \left(\rho_i + \pi \right) - \lambda_i, \tag{7}$$

$$\dot{\eta}_i = (\rho_i + \delta)\eta_i - q_i \left(1 - \xi \left(1 + \varepsilon \right) \left(\frac{k_i}{k} \right)^{\varepsilon} \right) A, \tag{8}$$

$$\lambda_i \left(m_i - c_i - \phi v_i \right) = 0, \quad \lambda_i > 0 \;\text{ and } m_i - c_i - \phi v_i > 0, \tag{9}$$

$$\lim_{t \to \infty} q_i(t)\, m_i(t)\, e^{-\rho_i t} = 0; \quad \lim_{t \to \infty} \eta_i(t)\, k_i(t)\, e^{-\rho_i t} = 0. \tag{10}$$

Here, (9) presents the Kuhn-Tucker conditions for the cash-in-advance constraint and equations in (10) are the transversality conditions.

[3] In the standard neoclassical growth model, the individual income share is a nonlinear funcion of individual as well as aggregate caoital and labor. The Ak structure of our model drastically simplifies the analysis.

Finally, we assume that the monetary authority keeps the growth rate of nominal money stock at a positive constant rate, μ, and both the tax revenue and the newly issued money are distributed back to each households as a transfer. Hence, the government's flow budget constraint is $z = \theta\tau\left(y_1/y\right)y_1 + \left(1-\theta\right)\tau\left(y_2/y\right)y_2 + \mu m$, where $m = \theta m_1 + \left(1-\theta\right)m_2$.

3 Balanced-Growth Characterization

In the following we focus on the balanced-growth equilibrium where consumption, capital and real money holding of each household grow at a common, constant rate. Namely, on the balanced-growth path it holds that

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = \frac{\dot{m}_i}{m_i} = g, \quad i = 1, 2. \tag{11}$$

for all $t \geq 0$, where g denotes the balanced growth rate. Given those conditions, it is easy to confirm that the shadow values in the each household optimization conditions also satisfy:

$$\frac{\dot{q}_i}{q_i} = \frac{\dot{\eta}_i}{\eta_i} = \gamma, \quad i = 1, 2. \tag{12}$$

for all $t \geq 0$. To see the relation between g and γ, we use (5) and (6) to obtain $c_i^{-\sigma} = \left(1 - \frac{1}{\phi}\right)q_i + \frac{1}{\phi}\eta_i$. Therefore, (11) and (12) mean that

$$g = -\frac{1}{\sigma}\gamma. \tag{13}$$

is held in the balanced-growth equilibrium.

We now denote: $x_i = \eta_i/q_i$ and $\kappa_i = k_i/k$. Then on the balanced-growth path (6), (7) and (13) yield

$$\sigma g = \frac{1}{\phi}\left(x_i - 1\right) - \rho_i - \pi \quad i = 1, 2. \tag{14}$$

Similarly, the steady state expression of (8) is

$$\sigma g = \frac{1}{x_i}\left[1 - \xi(1+\varepsilon)\left(\kappa_i\right)^{\varepsilon}\right]A - \rho_i - \delta, \quad i = 1, 2. \tag{15}$$

Notice that the real money balances grow at the rate of g so that $\pi = \mu - g$ holds on the balanced-growth path. Thus (14) gives

$$x_i = \phi\left[(\sigma - 1)\, g + \rho_i + \mu\right] + 1, \quad i = 1, 2. \tag{16}$$

Using (15) and (16), we obtain the following:[4]

$$(\sigma g + \rho_i + \delta)\left\{\phi[(\sigma - 1)\, g + \rho_i + \mu] + 1\right\} = A\left[1 - \xi(1 + \varepsilon)\left(\kappa_i\right)^{\varepsilon}\right], \quad i = 1, 2. \tag{17}$$

By definition, it holds that

$$\theta\kappa_1 + (1 - \theta)\,\kappa_2 = 1. \tag{18}$$

Equations (17) and (18) may determine the steady state level of relative capital holdings (relative income), κ_1 and κ_2, and the balanced-growth rate, g.

4 Growth and Distributional Effects of Inflation

If the time discount rate is identical $(\rho_1 = \rho_2)$, the balanced-growth conditions reduce to those established in the representative-agent economy. In fact, if $\rho_1 = \rho_2 = \rho$, then (17) and (18) indicate that $\kappa = 1$. As a result, the balanced-growth rate is determined by

$$(\sigma g + \rho + \delta)\left\{\phi[(\sigma - 1)\, g + \rho + \mu] + 1\right\} = A[1 - \xi(1 + \varepsilon)]. \tag{19}$$

In this case it is easy to confirm that if $\phi > 0$ and $\sigma \geq 1$, the balanced-growth rate satisfying (19) is uniquely given and a rise in money growth rate, μ, depresses g.[5] In addition, if $\sigma < 1$, then there may exist dual balanced-growth paths. In this case a rise in μ increases the growth rate of the higher-growth steady state, while it decreases the growth rate of the steady state with a lower growth rate.

[4] We offer the detailed derivation of (19) upon request.

[5] If there are two balanced-growth paths, one with a higher growth rate is locally indeterminate and the other with a lower growth rate is locally determinate. See Chen and Guo (2008), Meng (2002), Jha, Wang and Yip (2002), and Suen and Yip (2005) for detailed discussion on the representative-agent Ak growth models with cash-in-advance constraint.

If there is no cash constraint on investment $(\phi = 0)$, equation (17) reduces to

$$\sigma g + \rho_i + \delta = A\left[1 - \xi(1+\varepsilon)\left(\kappa_i\right)^{\varepsilon}\right], \quad i = 1, 2$$

and thus the growth rate of money supply will not affect the long-run growth and distribution.

When $\rho_1 > \rho_2$ and $\phi > 0$, we can also confirm that there may exist dual balanced-growth paths if $\sigma < 1$. In what follows, we assume that $\sigma \geq 1$ to focus on the case of unique balanced growth equilibrium. When $\sigma \geq 1$ the left-hand sides in (17) monotonically increases with g. We also see that the right-hand side of (17) is a strictly increasing function of κ_i. Hence, in view of (18), if the balanced-growth path exists, it must be unique. In this case it is easy to show that a rise in the money growth rate, μ, depresses the balanced-growth rate, that is, a higher inflation tax has a negative impact on growth in our two-class economy as well. It is also seen that the effect of inflation tax on income distribution on the balanced-growth path is ambiguous.

In order to inspect growth and distributional effects of inflation more clearly, we now assume that the utility function is logarithmic $(\sigma = 1)$. Then (17) and (18) give the following equation:

$$\begin{aligned} & \frac{A}{\phi\left(\rho_1 + \mu\right) + 1}\left[1 - \xi(1+\varepsilon)\left(\frac{1}{\theta + (1-\theta)\,\kappa}\right)^{\varepsilon}\right] - \rho_1 \\ = & \frac{A}{\phi\left(\rho_2 + \mu\right) + 1}\left[1 - \xi(1+\varepsilon)\left(\frac{\kappa}{\theta + (1-\theta)\,\kappa}\right)^{\varepsilon}\right] - \rho_2, \end{aligned} \tag{20}$$

where $\kappa = \kappa_2/\kappa_1 \left(= k_2/k_1\right)$. The left-hand side of (20) monotonically increases with κ, while the right-hand side monotonically decreases with κ. In addition, when $\kappa = 0$, our assumption, $\rho_1 > \rho_2$, ensures that

$$\frac{A}{\phi\left(\rho_1 + \mu\right) + 1}\left[1 - \xi(1+\varepsilon)\theta^{-\varepsilon}\right] - \rho_1 < \frac{A}{\phi\left(\rho_2 + \mu\right) + 1} - \rho_2.$$

Therefore, there exists a unique positive level of κ satisfying (20) and thus the balanced-growth path is uniquely given. As before, it is easy to show that a rise in the money growth rate, μ, lowers the balanced-growth rate. On the other hand, the effect of a change in the money growth rate on the long-run level of relative income, κ, depends on the parameter magnitudes involved in (20).

We present some numerical examples. The benchmark parameter values concerning the real side of the economy are the following:

$$\begin{aligned} A &= 0.12, \quad \rho_1 = 0.04, \quad \rho_2 = 0.03, \quad \xi = 0.17, \quad \varepsilon = 0.6, \\ \phi &= 0.2, \quad \delta = 0.04, \quad \theta = 0.5. \end{aligned}$$

The magnitudes of A, ξ, ε and δ are basically follow those used by by Li and Sarte (2004).[6] Table 1 (a) shows the benchmark case using the parameter values displayed above. We change the growth rate of money, μ, from 0.02 up to 0.20. The table indicates that a rise in inflation tax depresses the long-run growth rate and increases the relative income share of the household with a lower time discount rate.

Panels (b) and (c) set $\phi = 0.5$ and 1.0, respectively (the other parameters are the same as those given above.). A rise in ϕ means that the cash-in-advance constraint for investment becomes tighter. This directly reduces the long-run growth rate of income, while it increases the relative income share of type 2 households. In panel (d) we lower ε from 0.6 to 0.4. A decline in the progressiveness of income tax raises both the balanced-growth rate and the income share of type 2 households. Panel (e) displays the case where the time discount rate of type 2 household is 0.02 instead of 0.03. This small increase in preference divergence produces a considerable change in the long-run levels of relative income. Finally, Table (f) treats the case where $\rho_1 = \rho_2 = 0.03$, so that the steady-state level of relative income, κ, is always unity.

Table 1

μ	κ	g
0.02	1.636	0.0188
0.04	1.639	0.0184
0.10	1.648	0.0174
0.15	1.659	0.0165
0.20	1.665	0.0154

(a) Bench mark

μ	κ	g
0.02	1.672	0.0173
0.04	1.689	0.0165
0.10	1.704	0.0139
0.15	1.724	0.0118
0.20	1.745	0.0099

(b) $\phi = 0.5$

μ	κ	g
0.02	1.810	0.0088
0.04	1.823	0.0065
0.10	1.884	0.0023
0.15	1.933	− 0.0009
0.20	1.984	− 0.0039

(c) $\phi = 1.0$

[6] We set the values of ξ, ε and δ that are slightly different from those used by Li and Sarte (2004) in order to obtain plausible balanced growth rate in the presence of cash-in-advance constraint.

μ	κ	g
0.02	2.404	0.0274
0.04	2.408	0.0234
0.10	2.433	0.0221
0.15	2.454	0.0218
0.20	2.475	0.0203

(d) $\varepsilon = 0.4$

μ	κ	g
0.02	2.802	0.0249
0.04	2.814	0.0243
0.10	2.852	0.0234
0.15	2.884	0.0223
0.20	2.917	0.0215

(e) $\rho_1 = 0.04,\ \rho_2 = 0.02$

μ	κ	g
0.02	1.0	0.0304
0.04	1.0	0.0291
0.10	1.0	0.0266
0.15	1.0	0.0244
0.20	1.0	0.0223

(f) $\rho_1 = \rho_2 = 0.03,\ \phi = 0.5$

Our numerical exercises reveal that a monetary expansion have a negative impact on long-run growth rate of income and a positive impact on the relative income share of the agents with a lower time discount rate. It is shown that although the degree of cash constraint for investment (the level of ϕ) has a relatively large effects on growth, the quantitative effect of a change in money growth (so the long-run inflation) is considerably small.[7] In contrast, the degree of heterogeneity of households (difference in time discount rates) and the progressiveness of income tax may produce much larger impacts on growth and distribution. However, it is needless to add that our finding depends on a specific modelling of inflation, growth and distribution. Further investigations based on more general formulations would be relevant.

5 Remarks

This paper addresses the relation between inflation and long-term growth in an endogenously growing economy with heterogeneous agents. The source of heterogeneity in our model is the difference in the time preference. We have assumed that there are two types of households each of which has a specifically fixed rate of time preference. As Chen et al. (2008) demonstrate, the real impact of inflation would be modified if the time discount rate is endogenous. It would be interesting to see how our findings are modified if the rate of time preference of each household is endogenously determined.

As well as in the most of the existing literature on money and endogenous growth, our numerical examples show that a higher monetary expansion

[7] As claimed by Temple (2000), the empirical investigations on inflation and growth have not reach a consensus. Many studies, however, indicate that the relation between inflation and growth is relatively weak in countries with moderate inflation: see, for example, Barro (1996). Our numerical examples confirm this finding even in the presence of income distributional effect of inflation.

reduces the balanced-growth rate of real income. Recent empirical investigations such as Espinoza et al. (2010) revealed that there would be a threshold level of inflation under which a monetary expansion may accelerate growth, while a higher inflation depresses growth when the rate of inflation exceeds the threshold level. To capture such a non-monotonic relation between inflation and growth, we should extend the baseline model. Introducing endogenous time discount rates and endogenous labor supply would be useful for that purpose. We intend to pursue these lines of extensions in our future research.

References

[1] Barro, R.J., (1996), "Inflation and Growth", *Federal Reserve Bank of St. Louis Review* **78**, 153-169.

[2] Chen, B-L, Hsu, M. and Lu, C-H. (2008), "Inflation and Growth: Impatience and a Qualitative Equivalence", *Journal of Money, Credit, and Banking* **40**, 1309-1323.

[3] Chen, Shu-Hua & Guo, Jang-Ting, (2008), "Velocity of Money, Equilibrium (In)determinacy and Endogenous Growth", *Journal of Macroeconomics* **30**, 1085-1096.

[4] De Gregorio, J., (1993), "Inflation Taxation, and Long-run Growth", *Journal of Monetary Economics* **31**, 271-298.

[5] Espinoza, R., Leon. and Prasad, A., (2010) "Estimating The Inflation-Growth Nexus—A Smooth Transition Model", IMF Working Paper 10-76, International Monetary Fund.

[6] Guo, J.-T., and Lansing, K. J., (1998), "Indeterminacy and Stabilization Policy", *Journal of Economic Theory* **82**, 481-490.

[7] Jha, S.K., Wang, P., and Yip, C.K. (2002), "Dynamics in a Transactions-based Monetary Growth Model", *Journal of Economic Dynamics and Control* **26**, 611-635.

[8] Jones, L.E. and Manuelli, R.E., (1995), "Growth and Effects of Inflation", *Journal of Economic Dynamics and Control* **19**, 1405-1428.

[9] Meng, Q., (2002), "Monetary Policy and Multiple Equilibria in a Cash-in-Advance Economy", *Economics Letters* **74**, 165-170.

[10] Suen, M-H. and Yip, C-K., (2005), "Superneutrality, Indeterminacy and Endogenous Growth", *Journal of Macroeconomics* **27**, 579-595.

[11] Li, W. and Sarte, P-D. (2004), "Progressive Taxation and Long-Run Growth", *American Economic Review* **94**, 1705-1716.

[12] Marquis, M.H. and Reffett, K.L., (1995), "The Inflation Tax in a Convex Model of Equilibrium Growth", *Economica* **62**, 109-121.

[13] Mino, K., (1997), "Long-run Effects of Monetary Expansion in a Two-Sector Model of Endogenous Growth", *Journal of Macroeconomics* **19**, 635-655.

[14] Sarte, P-D., (1997). "Progressive Taxation and Income Inequality in Dynamic Competitive Equilibrium", *Journal of Public Economics* **66**, 145-171.

[15] Temple, J., (2000), "Inflation and Growth: Stories Short and Tall", *Journal of Economic Surveys* **14**, 395-426.