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A model-based ranking of U.S. recessions

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Abstract

A dynamic factor VAR model, estimated by MCMC simulation, is employed to assess the relative severity of post-war U.S. recessions. Joint modeling and estimation of all model unknowns yields rank estimates that fully account for parameter uncertainty. A convenient by-product of the simulation approach is a probability distribution of possible recession ranks that (i) accommodates uncertainty about the exact location of troughs, and (ii) can be used to resolve any potential inconsistencies or ties in the rank estimates. These features distinguish the approach from single-variable measures of downturns that ignore the co-movement and dynamic dependence and could lead to contradictory conclusions about timing and relative severity.

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1 Introduction

A number of recent empirical studies have relied on latent factor models to examine the co-movements and linkages among macroeconomic aggregates. Models with dynamic factors have been employed to study a variety of issues relating to country-specific, regional, and world-wide business cycles (Gregory and Head, 1999; Kose et al., 2003, 2008; Bagliano and Morana, 2009), and have provided a useful analytical framework for forecasting with many macroeconomic predictors (Koop and Potter, 2004; Stock and Watson, 2002, 2005; Belviso and Milani, 2006). Chauvet (1998) and Chauvet and Hamilton (2006) use dynamic factor models with regime switching to examine business cycle turning points and compute recession probabilities. The implementation of these methods in economics can be traced back to important early papers by Sargent and Sims (1977) and Geweke (1977). Applications in finance and exchange rate analysis are considered in Geweke and Zhou (1996), Kim et al. (1998), Aguilar and West (2000), and Chib et al. (2006). A recent extension to hierarchical dynamic factor structures has been pursued in Ng et al. (2008).

In this paper, we examine the dynamic comovement of key macroeconomic aggregates (output, unemployment, interest rates and inflation) in order to compare the relative severity of economic downturns. The issue is of particular policy relevance. Thus far, however, recession ranks have not been produced by formal econometric techniques and comparisons have instead relied on single-variable metrics such as peak-to-trough changes in growth or employment. Unfortunately, univariate comparisons suffer from important drawbacks. One obvious deficiency is that a single macroeconomic aggregate, treated in isolation, necessarily ignores information about business conditions that is contained in the trajectories of other variables. It is well known that for this reason the National Bureau of Economic Research (NBER) relies on a variety of data series, rather than on individual macroeconomic variables, in determining the timing of recessions. This is desirable because recessions do not always manifest themselves the same way through the same variable and any comparisons must accommodate potential discrepancies. For instance, the beginning of the March–November 2001 recession was driven by a drop in employment, while its end was announced as soon as growth increased. However, other measures of economic activity do not support the March–November 2001 dates – industrial production had been falling since October 2000 whereas unemployment continued to increase in the jobless recovery that followed the announced end of the recession. Similarly, labor market conditions and output growth provided divergent evidence in the case of the latest recession. Even though unemployment deteriorated in December 2007, it was not until the third quarter of 2008 that GDP growth turned negative.

A related problem with gauging recessions by single-variable metrics, for example peak-to-trough changes in that variable, is that different ranks can be obtained depending on the chosen variable. For example, the 1973-1975 recession ranks fourth by GDP decline, but second worst by increase in unemployment. A less obvious problem is that univariate comparisons typically do not involve a distinction between idiosyncratic and economy-wide shocks, so that it is not possible to disentangle variable-specific from common shocks. In addition, single-variable methods also ignore the dynamic interdependence in the series.

Motivated by the aforementioned considerations, we employ a dynamic factor vector autoregressive (DF-VAR) model to study the severity and rank-ordering of recessions in the U.S. over the period 1948–2009. In particular, we estimate a dynamic factor common to all macroeconomic variables within the model and an idiosyncratic component that captures variable-specific characteristics, and use the former to gauge downturns. We present evidence on the relative ranks of recessions, which account for information in multiple data series, parameter uncertainty, the distinction between common and idiosyncratic shocks in the economy, and uncertainty about the timing of the minima attained by the factor during each recession. Simulation-based estimation, using recently developed Markov chain Monte Carlo (MCMC) algorithms, allows us to make probabilistic statements about recession ranks which are difficult to obtain from point estimates and point-wise confidence bands alone.

The remainder of the paper is organized as follows. In Section 2, we formalize the econometric model and estimation methodology, while Section 3 presents our main results. Section 4 offers concluding remarks.

2 Methodology

We apply a DF-VAR model to analyze the post-war macroeconomic data for the United States which consists of $T = 248$ quarterly observations (1948:Q1 to 2009:Q3) on $n = 4$ general macroeconomic variable series: output (GDP) growth, unemployment rate, interest rate, and inflation. For $t = 1, \ldots, T$, the DF-VAR model is given by

$$y_t = \mu + \Gamma y_{t-1} + Af_t + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \Omega), \tag{1}$$

where $\Omega = \mathrm{diag}(\omega_{11}, \ldots, \omega_{nn})$, and

$$f_t = \gamma f_{t-1} + \nu_t, \qquad \nu_t \sim \mathcal{N}(0, \sigma^2), \tag{2}$$

is a dynamic economy-wide factor which reflects unobserved sources of macroeconomic volatility. Equation (2) is initialized with the stationary distribution $f_1 \sim N\left(0, \sigma^2/\left(1-\gamma^2\right)\right)$, so that the joint distribution for $f = (f_1, \ldots, f_T)'$ becomes $[f|y, \sigma^2] \sim \mathcal{N}(0, \sigma^2 F_0^{-1})$, where the precision F_0 is a symmetric matrix with $(1, 1+\gamma^2, \ldots, 1+\gamma^2, 1)$ on the main diagonal, $(-\gamma, \ldots, -\gamma)$ on one first sub- and super-diagonals, and 0s elsewhere. Because neither the factor f nor the loadings vector A is known, for identification reasons we restrict the first element in A to unity, that is $A = (1, a')'$. The model is capable of reproducing complex dynamic behavior because intertemporal cross-correlations can be captured through the unobserved factor as well as lags of the dependent variable vector.

For the purposes of estimation, the model is written in the form of a seemingly unrelated regression model (Zellner, 1962) as $y_t = X_t\beta + \varepsilon_t$, where $X_t = I_n \otimes \left(1, y_{t-1}'\right)$ and $\beta \equiv \mathrm{vec}\left([\mu : \Gamma]'\right)$, where the $\mathrm{vec}(\cdot)$ operator stacks the columns of a matrix into a vector. The model is estimated efficiently by the collapsed MCMC sampler for state space models developed in Chan and Jeliazkov (2009), which significantly improves the performance of the Markov chain. Under the priors $\beta \sim \mathcal{N}\left(\beta_0, B_0\right)$, $a \sim \mathcal{N}\left(a_0, A_0\right)$, $\omega_{ii} \sim \mathcal{IG}\left(\nu_{i0}/2, r_{i0}/2\right)$, $\sigma^2 \sim \mathcal{IG}\left(\gamma_0, G_0\right)$ and $\gamma \sim \mathcal{TN}_{(-1,1)}\left(\gamma_0, G_0\right)$, and upon stacking $y = (y_1', \ldots, y_T')'$, $X = (X_1', \ldots, X_T')'$, estimation proceeds as follows.

Algorithm 1 ***MCMC Sampling of the DF-VAR Model***

1. *Sample* $[\beta|y, A, \Omega, \gamma, \sigma^2] \sim \mathcal{N}(\hat{\beta}, B)$, *where* $B = (B_0^{-1} + \sum_{t=1}^T X_t'\Omega^{-1}X_t - \tilde{X}'P^{-1}\tilde{X})^{-1}$, $\hat{\beta} = B(B_0^{-1}\beta_0 + \sum_{t=1}^T X_t'\Omega^{-1}y_t - \tilde{X}'P^{-1}\tilde{y})$, $\tilde{y}_t = A'\Omega^{-1}y_t$, $\tilde{X}_t = A'\Omega^{-1}X_t$, $\tilde{y} = (\tilde{y}_1', \ldots, \tilde{y}_T')'$, $\tilde{X} = (\tilde{X}_1', \ldots, \tilde{X}_T')'$ *and* $P = [\sigma^{-2}F_0 + I_T\left(A'\Omega^{-1}A\right)]$.

2. *Sample* $[a, f|y, \beta, \Omega, \gamma, \sigma^2]$ *in one block as follows*

   (a) *Sample* $[a|y, \beta, \Omega, \gamma, \sigma^2]$ *marginally of* f *by a Metropolis-Hastings step with tailored proposal* $a^\dagger \sim q(\hat{a}, V)$, *and accept the proposed draw* $a^\dagger$ *with probability*

$$\alpha_{MH}(a, a^\dagger) = \min\left\{1, \frac{\pi(a^\dagger|y, \beta, A, \Omega, \gamma, \sigma^2)q(a|\hat{a}, V)}{\pi(a|y, \beta, A, \Omega, \gamma, \sigma^2)q(a^\dagger|\hat{a}, V)}\right\}$$

   (b) *Sample* $[f|y, \beta, A, \Omega, \gamma, \sigma^2] \sim \mathcal{N}(\hat{f}, F)$, *where* $\hat{f} = F\,(I_T \otimes A)'\,(I_T \otimes \Omega^{-1})\,(y - X\beta)$ *and* $F = \left(F_0/\sigma^2 + (I_T \otimes A)'(I_T \otimes \Omega^{-1})(I_T \otimes A)\right)^{-1}$.

3. *Sample* $[\Omega|y, \beta, A, f]$ *by drawing* $\omega_{ii} \sim \mathcal{IG}((\nu_{i0} + T)/2, (r_{i0} + e_i'e_i)/2)$ *for* $i = 1, \ldots, n$, *where* e_i *is a* T*-vector of residuals from the* i*th observation equation.*

4. *Sample* $[\gamma|f, \sigma^2]$ *by a Metropolis-Hastings step with proposal* $\gamma^\dagger \sim \mathcal{N}(\hat{\gamma}, G)$, *where* $G = (G_0^{-1} + f_{1:T-1}'f_{1:T-1}/\sigma^2)$ *and* $\hat{\gamma} = G(G_0^{-1}\gamma_0 + f_{1:T-1}'f_{2:T}/\sigma^2)$, *and the proposed value* $\gamma^\dagger$ *is accepted with probability*

$$\alpha_{MH}(\gamma, \gamma^\dagger) = \min\left\{1, \frac{f_{\mathcal{N}}(f_1|0, \sigma^2/(1 - \gamma^{\dagger 2}))}{f_{\mathcal{N}}(f_1|0, \sigma^2/(1 - \gamma^2))}\right\}.$$

5. *Sample* $[\sigma^2|f, \gamma] \sim \mathcal{IG}\left(\frac{s_0+T}{2}, \frac{\delta_0 + (f^* - \hat{f})'(f^* - \hat{f})}{2}\right)$, *where* $f^* = (f_1\sqrt{1 - \gamma^2}, f_2, \ldots, f_T)'$ *and* $\hat{f} = (0, \gamma f_1, \ldots, \gamma f_{T-1})'$.

It is important to note that because Algorithm 1 produces draws from the joint posterior distribution of all model unknowns, subsequent inferences based on the simulated factors fully account for all parameter uncertainty (unlike plug-in approaches). Moreover, the framework is quite flexible and can easily accommodate systems with variable-specific lag lengths, more complex factor dynamics, or many macroeconomic predictors (Stock and Watson, 2002).

3 Results

The estimated dynamic macroeconomic factor is shown in Figure 1, together with the timing of officially announced U.S. recessions. Figure 1 reveals that the factor captures the timing of recessions quite well, and that it also gives an idea of the relative severity and dynamic evolution of each recession. The key finding from this figure is that the latest recession that started in December 2007 appears to be the worst in the post-war sample. However, two other recessions (those in 1957-1958 and 1973-1975) exhibit similar severity. For this reason, in comparing these recessions to each other, we must account for the uncertainty in the estimated factor. This, however, is a difficult problem that can not be directly addressed by looking at point estimates and point-wise confidence bands. Moreover, due to the estimation uncertainty, it is quite possible that the factor could actually achieve its local minima at points other than the ones depicted in Figure 1. To address these concerns and provide a ranking of the 11 post-war recessions, we proceed as follows. For each MCMC draw of f, we determine the minima in the factor corresponding to each recession. We compare and rank these 11 extrema, and record the number of times each recession achieves a given rank over the course of MCMC sampling. Doing so allows us to make comparisons and probabilistic statements about recession ranks while simultaneously accounting for the uncertainty in the location of each trough.

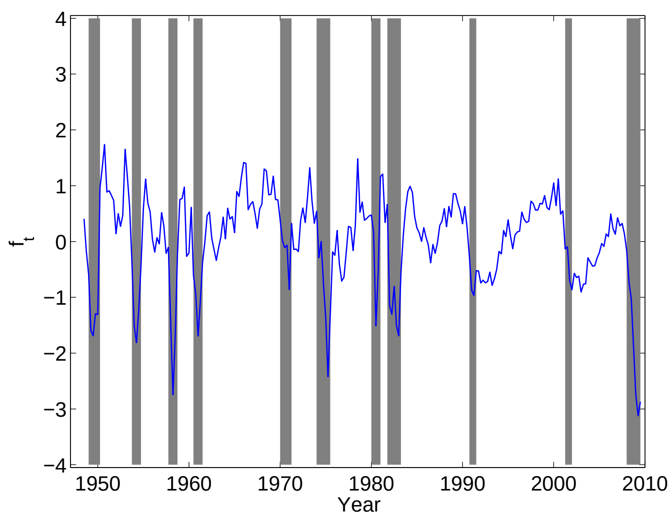

Figure 1: Estimated factor in the dynamic factor VAR model of the U.S. economy together with the timing of officially announced U.S. recessions (shaded regions).

Before continuing with the discussion of recession ranks, we also mention an interesting observation based on Figure 1. In particular, the figure shows that while the officially announced recessions of 1990-1991 and 2001 (during the "Great Moderation" following the mid-1980s) were shorter, shallower, and further apart than recessions earlier in the sample, their ends might have been announced prematurely. The ability of the model to capture these features is notable because the 1990-1991 recession was followed by a period of jobless growth when unemployment actually increased and remained elevated for several years. Recovery from the 2001 recession was stymied by the immediate aftermath of the 9/11 attacks, which affected consumer behavior, investor sentiment, and demand in key industries.

We now turn our attention to a discussion of the relative recession ranks obtained by the proposed approach. Our main results, based on a sample of $10,000$ MCMC draws, are given in Figure 2 and Table 1. Figure 2 illustrates the marginal rank probabilities associated with each recession. The figure confirms that the latest recession is indeed the worst in the sample with probability 76%. The two closest contenders (the 1957-1958 and 1973-1975 recessions) rank first with much lower probability (17% and 6% respectively). There is a 53% probability that the recession of the late 50s is the second deepest and that the 1973-1975 recession ranks third with probability 39%. The rank probabilities for the full set of U.S. recessions is given in Table 1. However, judging recession ranks by these marginal probabilities causes some potential ties that must be resolved. In particular, a closer inspection of the rows of Table 1 indicates that both the 1969-1970 and 2001 recessions exhibit highest marginal probabilities for rank 11 (mildest recession in the post-war period). In addition, looking at the maximum probability ranks in each column of the Table, we run into the problem that the 1960-1961 recession was the most likely outcome in columns 6 and 7. Such ties are resolved by a closer

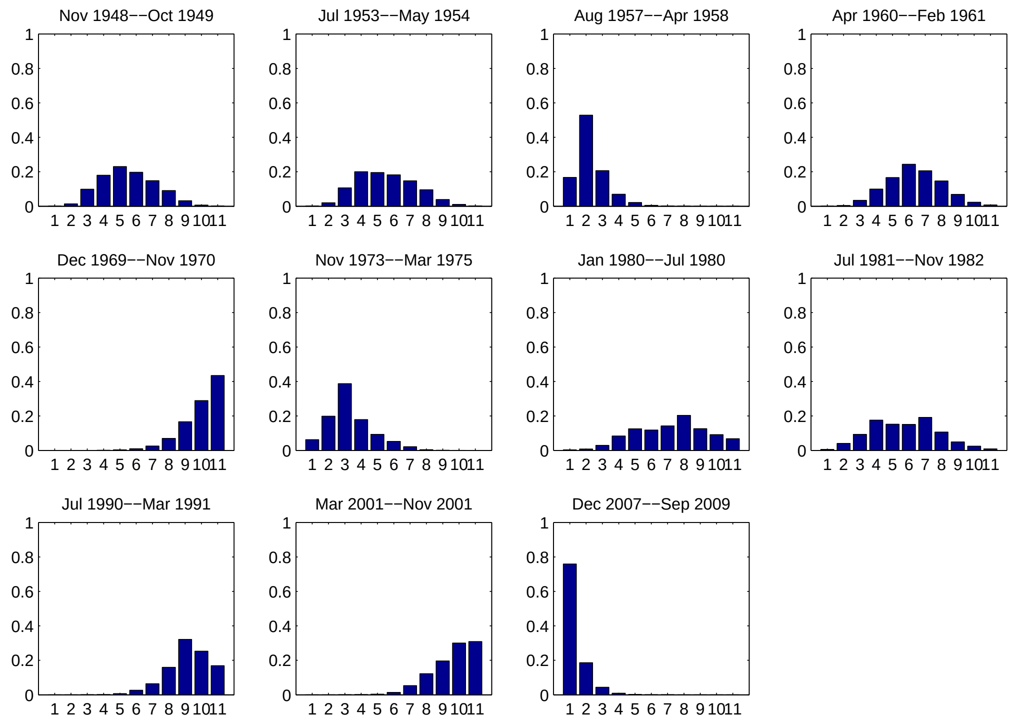

Figure 2: Recession rank probabilities (1=most severe, 11=mildest).

examination of the MCMC simulation output. Pairwise comparisons between the 1969-1970 and 2001 recessions reveal that indeed with probability 59% the 1969-1970 is milder than the one in 2001. A similar comparison also indicates that with probability 58% the 1960-1961 recession is milder than the one in 1981-1982.

Table 1: Severity rank probabilities of U.S. recessions (1 = most severe, 11 = mildest).

	Recession Ranks										
Recession Timing	1	2	3	4	5	6	7	8	9	10	11
Nov 1948–Oct 1949	0	0.01	0.10	0.18	0.23	0.20	0.15	0.09	0.03	0.01	0
Jul 1953–May 1954	0	0.02	0.11	0.20	0.20	0.18	0.15	0.09	0.04	0.01	0
Aug 1957–Apr 1958	0.17	0.53	0.20	0.07	0.02	0.01	0	0	0	0	0
Apr 1960–Feb 1961	0	0	0.03	0.10	0.17	0.24	0.21	0.15	0.07	0.02	0.01
Dec 1969–Nov 1970	0	0	0	0	0	0.01	0.03	0.07	0.17	0.29	0.43
Nov 1973–Mar 1975	0.06	0.20	0.39	0.18	0.09	0.05	0.02	0.01	0	0	0
Jan 1980–Jul 1980	0	0.01	0.03	0.08	0.13	0.12	0.14	0.20	0.13	0.09	0.07
Jul 1981–Nov 1982	0.01	0.04	0.09	0.18	0.15	0.15	0.19	0.11	0.05	0.02	0.01
Jul 1990–Mar 1991	0	0	0	0	0.01	0.03	0.06	0.16	0.32	0.25	0.17
Mar 2001–Nov 2001	0	0	0	0	0	0.02	0.05	0.12	0.20	0.30	0.31
Dec 2007–Sep 2009	0.76	0.19	0.04	0.01	0	0	0	0	0	0	0

4 Concluding Remarks

We have presented a model-based approach for ranking the relative severity of recessions. From a statistical standpoint, the method is desirable because it involves joint modeling and estimation of all model unknowns, thereby producing estimates that fully reflect parameter uncertainty, while also accounting for the co-movement and dynamic interdependence in the series. From a practical perspective, the method is useful because it accommodates uncertainty about the location of troughs. Another advantage of the simulation-based methodology is that it naturally provides rank probabilities and comparisons which are difficult to obtain from point estimates and point-wise confidence bands. Our results indicate that the latest recession is indeed the deepest in the sample, whereas the 1969-1970 recession is the mildest. Probabilistic measures of uncertainty associated with these and other ranks can be found in Table 1.

In future research we intend to apply the proposed techniques to models containing additional macroeconomic indicators in order to capture a broader range of measures of output, income and employment. Moreover, the adequacy of more general dynamic specifications, such as models with variable-specific lags, will also be examined using formal model choice techniques. Finally, even though current data limitations do not allow extension of the sample prior to World War II, it would be useful to examine versions of the model that do make comparisons with pre-war recessions possible.

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