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Classification of the frontier in the three-country, three-good Ricardian model

Takeshi Ogawa
Nagoya University

Abstract

The production assignment problem assumes a central role in the multi-country, multi-good Ricardian trade model. However, resolution of the problem is not sufficient to illustrate the shape of the world production frontier, even if the examination is limited to efficient production of all goods. This study established the frontier pertinent to the three-country, three-good Ricardian model by connecting Ikema's (1993) illustration to McKenzie's (1954) efficient facets in order to classify the shape of the world production frontier. Our analysis indicates that the illustration of the Ricardian model has more explanatory power than the production assignment problem in identifying the shape of the world production frontier, at least in the three-country, three-good model.

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Contact: Takeshi Ogawa - takeshi.ogawa.123@gmail.com.
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1. Introduction

Since Jones (1961), the *production assignment problem*—the determination of which good a country should specialize in for efficient production—has been central to the analysis of the Ricardian model characterized by many countries producing many goods. Jones (1961) demonstrated that within efficient specializations, there is usually only a single pattern of specialization determined by certain inequalities in the n-country, n-good model if every good is considered essential. Before Jones (1961), McKenzie (1954) had analyzed the model in the general form, and had proposed the efficient facet diagram using a three-country or four-country, three-good model. The McKenzie efficient facet diagram is in the quantity space.

After Jones (1961), Ikema (1993) (in Japanese) used another illustration, which is based on that of Amano's (1966), representing the three-country or four-country, three-good model without intermediate goods.[1] Ikema's illustration concentrates on the price space and can analyze the model accurately with the technology parameter of each good for each country, whereas McKenzie's efficient facet diagram is intuitive. Both McKenzie's efficient facets and Ikema's illustration provide useful information for this study.

Ikema (1993) focuses on information that the production assignment problem cannot solve, at least in the three-country, three-good model. The present study focuses on the entire world production frontier. In the frontier, there are many efficient production points satisfying the following parameters: (1) they can be achieved through diversifications, (2) they may occur due to various given prices, and (3) information regarding them cannot be obtained by the production assignment problem. To clarify these points, it is necessary to classify the entire frontier, including all essential information.

This study classifies the frontier by extracting the latent potential of Ikema's illustration. While Jones focused on only the case where each country specializes in a single good, this study uniquely characterizes the entire frontier by connecting Ikema's illustration with McKenzie's efficient facets. Although this study considers only the three-country, three-good model, its method of classification can be extended to a multi-country, three-good model similar to Ikema's illustration, and further explains the entire production frontier.

Section 2 describes the production assignment problem by discussing Ikema's illustration of the three-country, three-good model. Section 3 classifies the frontier in the model, and Section 4 concludes.

2. Model and Basic Illustration

The analysis is based on the Ricardian model with three countries and three goods. The

[1] Amano's (1966) illustration is based on a two-country, three-good model with intermediate goods.

production function is linear with one factor, labor. Let l_j^i be the amount of labor needed to produce one unit of the j^{th} good in the i^{th} country $(i, j = 1,2,3)$, x_j^i be the quantity of the j^{th} good in the i^{th} country, and L^i be the positive labor endowment in the i^{th} country. Then the resource constraint is $l_1^i x_1^i + l_2^i x_2^i + l_3^i x_3^i \le L^i$ for any $i = 1,2,3$. If $(p_1, p_2, p_3) >> 0$ is assumed as the world price[2] and $X = (X_1, X_2, X_3)$ as the world production point, then a point $X = \left(x_1^1 + x_1^2 + x_1^3, x_2^1 + x_2^2 + x_2^3, x_3^1 + x_3^2 + x_3^3\right)$ is on the frontier if and only if there is a vector $(p_1, p_2, p_3) >> 0$ such that X maximizes

$$\max_{x_j^i \ge 0} \sum_{i,j} p_j x_j^i \quad \text{s.t.} \quad \sum_j l_j^i x_j^i \le L^i \quad (i = 1,2,3). \tag{1}$$

To understand Ikema's (1993) illustration, necessary for later analysis, this study analyzes the relationship between the world price and specialization. Specialization is characterized as follows: the i^{th} country specializes in the j^{th} good if and only if the i^{th} country devotes all its labor to the production of the j^{th} good. Thus, the set of prices at which the i^{th} country specializes in the j^{th} good can be expressed as

$$\left\{ (p_1, p_2, p_3) >> 0 \;\middle|\; \frac{p_j}{l_j^i} > \frac{p_k}{l_k^i} \quad (k \ne j) \right\}.$$

Figure 1, which is originally derived from Amano (1966) and plays a key role in Ikema's (1993) illustration, shows the good in which the i^{th} country will specialize given the world price. The first good is defined as the numeraire.

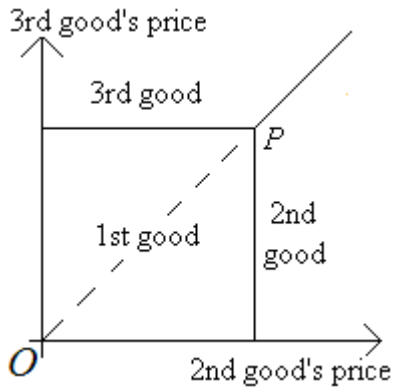

Figure 1

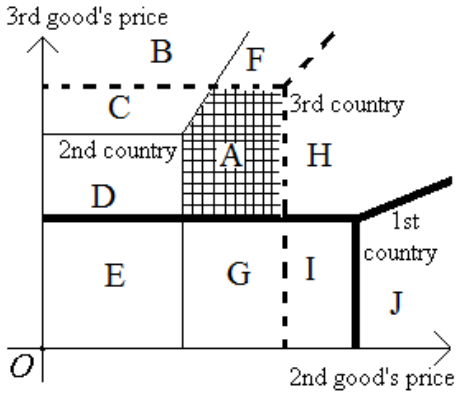

Figure 2

[2] Inequality signs are $\ge, >, >>$.

The shape of Figure 1 crucially depends on the information regarding the technology parameter in the normalized world price $P = (1, p_2, p_3)$. Thus, Figure 1 is in a one-to-one correspondence to the normalized world price *P*, where the value of every good's marginal productivity is identical. Figure 2 presents the case for all three countries. Within area *A* of Figure 2, each country specializes in each good in the following manner:

(first country, second country, third country) = (third good, second good, first good).

Figure 2 demonstrates Ikema's illustration. Similar to the manner of one-to-one correspondence above, Jones (1961) considered the resolution of the production assignment problem to determine the assignment satisfying the condition that each country specializes in a different good at a world production point located on the frontier. To illustrate the frontier using Figure 2, the efficient facets developed by McKenzie (1954) are beneficial.

In Figure 2, each country specializes in one good within each domain labeled *A* through *J*. These domains correspond to extreme points on the world production frontier, where an extreme point is defined as one that cannot serve as an internal dividing point between different production points. When two domains in Figure 2 are adjacent to each other in a definite straight line, such as *H* and *I*, there is a one-dimensional degree of freedom in the price where both corresponding world production points represent maximization of the world output, leading the frontier to bend along this straight line. In contrast, a case in which two domains are tangential to each other at a point, such as *C* and *F*, there is only one price vector at which both world production points corresponding to the domains yield maximum world output. Thus, two world production points corresponding to *C* and *F* are in the same plane on the frontier. However, in a case in which two domains are neither adjacent nor tangential to each other, such as *A* and *J*, there is no price at which both world production points maximize world output. Thus, two corresponding world production points cannot occupy the same plane on the frontier.

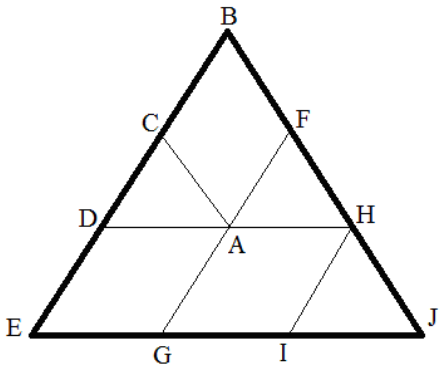

Figure 3

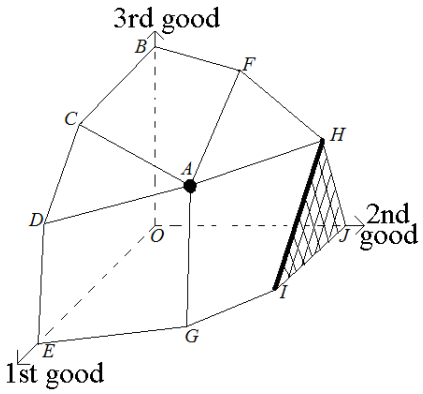

Figure 4

The abovementioned points are simplified in Figure 3 using McKenzie's (1954) efficient facets. First, a triangle is drawn to identify each extreme point. Second, where two domains are identified as adjacent, such as *H* and *I* in Figure 2, they are connected with a

straight line through the corresponding points shown in Figure 3. In this manner, one can illustrate McKenzie's (1954) efficient facets in Figure 3, thus simplifying the original frontier as shown in Figure 4.

With reference to the production assignment problem, point *A* can be characterized in Figure 4. Since Point *A* is located on various planes composing the frontier, information about the frontier can be obtained when point *A* is characterized. However, the case of triangle *HIJ* and the line segment *HI* must be considered. Although triangle *HIJ* composes one part of the frontier, point *A* is not located on *HIJ*, implying that no information can be obtained regarding *HIJ* by focusing only on point *A*. Moreover, the interior points of *HI* compose a part of the frontier, and these interior points can be achieved only by including diversification, because world output is maximized at these points with various prices. Figure 5, viewed from the direction of line *HI*, illustrates this case. The points also satisfy the condition that every good is produced. However, because point *A* is not on *HI* , it is important to focus on classifying the aspect of the frontier for which the resolution of the production assignment problem seldom provides information.

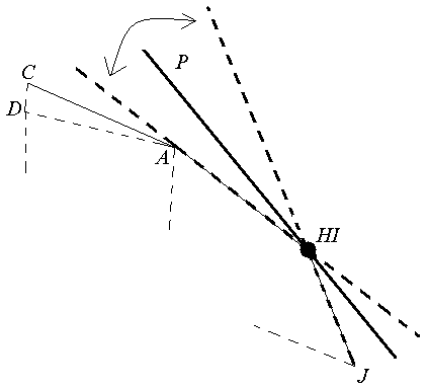

Figure 5

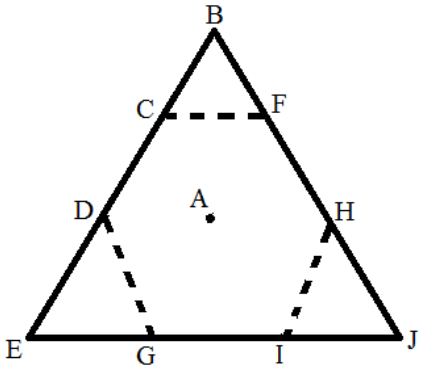

Figure 6

The production assignment problem derives the efficient specialization pattern from the three-country, three-good model. As Jones (1961) argued, without the case of ties between different specializations, the pattern exists and is unique. Area *A* shows the assignment in Figure 2. Moreover, in Figure 2, there are areas like *E* (all countries specializing in the first good), *J* (all countries specializing in the second good), and *B* (all countries specializing in the third good), where a specific good's price is very high. Because three countries exist in the model, three borders and two areas typically exist between areas *B* and *E*, a situation similar to that between *EJ* and *BJ*. Thus, there are 10 areas in Ikema's figure describing the three-country, three-good model. Area *A* is surrounded by nine other areas. Each area can be connected to the neighboring areas by a line segment. If now the efficient facets shown in Figure 6 are considered, it can be observed that *CF*, *DG*, and *HI* are the only possible pairs that can be linked with each other without passing through point *A*. The frontier can be classified regardless of whether or not *CF*, *DG*, and *HI* are linked. Consider Figure 3 as an

example. While *C* and *F*, and *D* and *G* are not linked to each other, *H* and *I* are.

Focusing solely on Ikema's (1993) illustration, the difference in the shape of the frontier cannot be analyzed. However, if the focus is solely on McKenzie's (1954) efficient facets, even if the difference can be discerned intuitively, an effective tool cannot be determined for analyzing it. The classification of the frontier can be analyzed only by connecting Ikema's illustration and McKenzie efficient facets. In the following section, the shape of the frontier is classified using Ikema's illustration in a manner beyond its original scope.

3. Classification

First, it should be noted that several characteristics of the model are shown in Figure 1. Each country has three borders whose location has a one-to-one correspondence to the price (denoted by *P*) at which the value of the marginal product for any good is the same. In Figure 7, therefore, the location of another country's point *P* can be found within six areas labeled a through f. As the analysis of d is similar to that of a and the analysis of c, e, and f is similar to that of b, the results of the analysis of a and b can be generalized to the other areas.

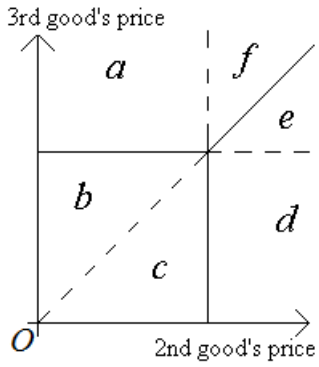

Figure 7

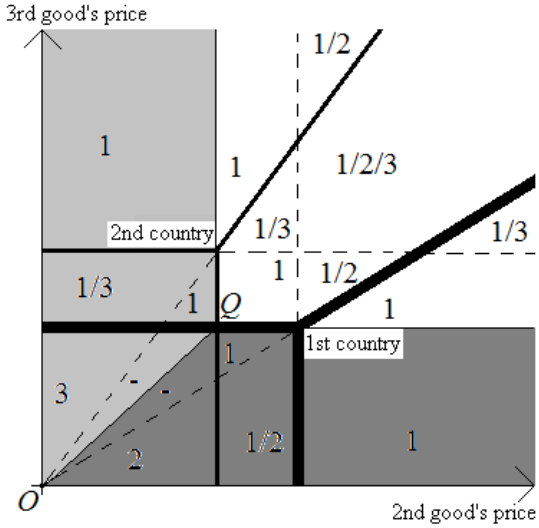

Figure 8

In case a, the shapes of the frontiers can be classified as shown in Figure 8. Specifically, given the borders that define the specialized production of goods for two countries in Figure 7, the frontiers can be classified with the position of point *P* of the third country, using notations such as 1/3, 2, and 1/2 to illustrate the shape of the frontier.

For example, if one considers that point *P* of the third country falls within the area denoted by 1/3, then *E* (all countries specializing in the first good) and *A* (derived by the resolution of the production assignment problem) would occupy the same plane on the frontier. Similarly, *B* (all countries specializing in the third good) and *A* would occupy the same plane on the frontier. However, *J* (all countries specializing in the second good) and *A* would not occupy the same plane on the frontier. When neither "*A* and *B*" nor "*A* and *E*" nor

"*A* and *J*" occupy the same plane, the symbol "-" is used in Figure 8. The detail corresponding to the notations is presented in Table 1.

Notations	*A* and *E*	*A* and *J*	*A* and *B*
1	Same Plane	Not	Not
2	Not	Same Plane	Not
3	Not	Not	Same Plane
-	Not	Not	Not
1/2	Same Plane	Same Plane	Not
1/3	Same Plane	Not	Same Plane
2/3	Not	Same Plane	Same Plane
1/2/3	Same Plane	Same Plane	Same Plane

Table 1: Notations (1)

These conditions correspond to the conditions shown in Figure 6. For example, in the case of notation 1/3, *C* and *F* are linked and *D* and *G* are linked, but *H* and *I* are not linked. There are 8 such cases. While the detailed correspondence to notations is presented in Table 2, the rules regarding Figure 6 are given below. The three rules are as follows:

(1) When *A* and *E* occupy (do not occupy) the same plane on the frontier, *D* and *G* are not linked (are linked) to each other.

(2) When *A* and *J* occupy (do not occupy) the same plane on the frontier, *H* and *I* are not linked (are linked) to each other.

(3) When *A* and *B* occupy (do not occupy) the same plane on the frontier, *C* and *F* are not linked (are linked) to each other.

Notations	*D* and *G*	*H* and *I*	*C* and *F*
1	Not linked	Linked	Linked
2	Linked	Not linked	Linked
3	Linked	Linked	Not linked
-	Linked	Linked	Linked
1/2	Not linked	Not linked	Linked
1/3	Not linked	Linked	Not linked
2/3	Linked	Not linked	Not linked
1/2/3	Not linked	Not linked	Not linked

Table 2: Notations (2)

Figure 8 (area *a*)	1st Country	2nd Country	3rd Country
White Area	3rd Good	2nd Good	1st Good
Light-Gray Area	3rd Good	1st Good	2nd Good
Dark-Gray Area	1st Good	2nd Good	3rd Good

Table 3: Explanations of Figure 8

In accordance with Ikema (1993), Figure 8 shows which good each country specializes in per the resolution of the production assignment problem. The result is shown in Table 3.

Q is the point of intersection of these three borders. Therefore, with reference to Figure 8, this study can classify the shape of the frontier in area a.

The shape of the frontiers can be classified for area b by reference to Figure 9, which uses the same symbols used in Figure 8.

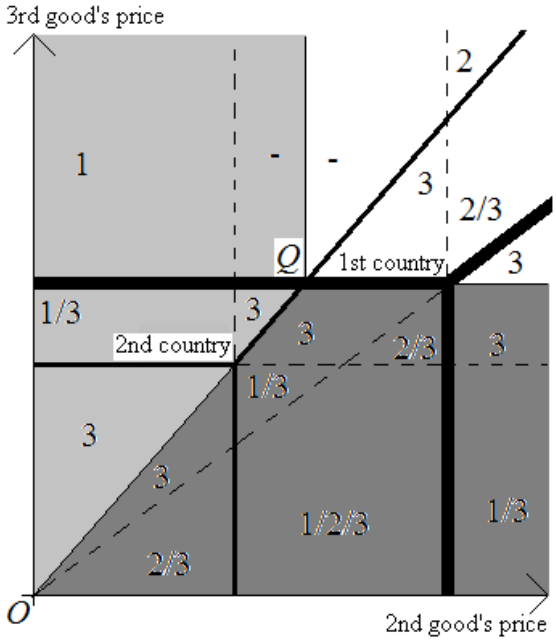

Figure 9

Similar to Figure 8, Figure 9 shows which good each country specializes in per the resolution of the production assignment problem. The result is shown in Table 4.

Figure 9 (area b)	1st Country	2nd Country	3rd Country
White Area	3rd Good	2nd Good	1st Good
Light-Gray Area	1st Good	3rd Good	2nd Good
Dark-Gray Area	1st Good	2nd Good	3rd Good

Table 4: Explanations of Figure 9

With reference to Figure 9, this study can classify the shape of the frontier in area b, and with reference to both Figures 8 and 9, one can classify the shape of the entire frontier.

By using this means of classification, essential information can be derived regarding the frontier in the three-country, three-good Ricardian model. Ikema (1993) shows the difference of efficient specialization patterns derived by the production assignment problem, which is illustrated in different shades (white, light gray, and dark gray) in Figures 8 and 9. However, he does not mention these classifications, which are important for ascertaining the shape of the frontier. By following the approach described in this study, Ikema's (1993) illustration can be extended to the multi-country, three-good model.

4. Conclusion

This study focused on addressing a problem that cannot be resolved solely by the production assignment problem in the three-country, three-good Ricardian model. Specifically, this problem was addressed by introducing Ikema's (1993) illustration and relating it to McKenzie's (1954) efficient facets to classify the shape of the world production frontier. Following Jones (1961), the present study discussed the production assignment problem as a useful means of analyzing the Ricardian model featuring many countries and many goods. However, if the production assignment problem was solely addressed, as did Jones (1961), all the information could not have been clarified regarding the frontier. Ikema (1993) focused on this point but did not provide an explanation. In response to Ikema's argument, the present study focused on classifying the shape of the frontier using Ikema's illustration.

Although the present study is based on Ikema's illustration and on the benchmark of the multi-country, multi-good Ricardian model, it uniquely applies Ikema's diagram to the entire frontier in the three-country, three-good case. Thus, this study's approach is more powerful than not only Jones' (1961) production assignment problem but also Ikema's (1993) original explanation of the illustration.

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