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How well can business cycle accounting account for business cycles?

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Abstract

The business cycle accounting method introduced by Chari, Kehoe and McGrattan (2007) is a useful tool to decompose business cycle fluctuations into their contributing factors. However, the model estimated by the maximum likelihood method cannot replicate business cycle moments computed from data. Moment-based estimation might be an attractive alternative if the purpose of the research is to study business cycle properties such as volatility, persistence and cross-correlation of variables instead of a specific business cycle episode.

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1 Introduction

The business cycle accounting method introduced by Chari, Kehoe and McGrattan (CKM 2007) is a useful tool to decompose business cycle fluctuations into their contributing factors. A large class of macroeconomic models can be mapped into a prototype business cycle accounting model with time-varying wedges in equilibrium conditions[1]. Therefore, the method provides a useful guide for choices of where to introduce frictions in a detailed quantitative model. By construction, the prototype model with all wedges can perfectly reproduce the observed fluctuations of output, consumption, investment and labor. However, the model does not necessarily reproduce the data moments characterizing the comovement patterns of the data. This paper discusses this discrepancy between the model prediction and the data regarding selected moments that are commonly focused in the business cycle literature.

The accounting procedure of business cycle accounting starts with defining the wedges in equilibrium conditions of a prototype closed economy dynamic stochastic general equilibrium model. Efficiency, government, investment and labor wedges are defined as residuals in the production function, the resource constraint, the capital Euler equation and the labor first order condition. They are assumed to be exogenous and follow a vector autoregressive stochastic process. Since investment wedges are defined in an expectational equation and are not directly computable from data, the entire stochastic process is estimated using maximum likelihood estimation. Once all parameter values are pinned down through calibration and estimation, the model is solved numerically and the value of wedges can be backed out using the data of output, consumption, investment and labor. Finally, selected wedges are fed into the model in order to assess the impact of each type of wedges on the business cycle.

Business cycle accounting has been widely applied to the analysis of specific business cycles episodes in various countries. CKM (2007) focuses on the U.S. Great Depression and early 1980s recession. Kobayashi and Inaba (2006) and Chakraborty (2009) investigates the sources of the boom and bust in Japan during the 1980s and 1990s. Saijo (2008) analyzes the Japanese interwar depression. Kersting (2008) focuses on the UK recession

[1] CKM (2007) derives equivalence results by comparing first order conditions in the prototype model to those in a detailed frictional model. Inaba and Nutahara (2008) shows that equivalence also requires a restriction on the stochastic process of the frictions in the detailed model.

in the 1980s. Cociuba and Ueberfeldt (2008) analyzes the Canadian business cycles over the 1961-2005 period. Lama (2011) focuses on output drops in Argentina, Brazil, Chile, Colombia, Mexico and Peru during the 1990s. Otsu (2010a) studies the 1998 crises in Hong Kong, Korea, and Thailand. Most of these studies show that efficiency and labor wedges are important in accounting for output fluctuations[2]. The method has also been extended from the canonical closed economy framework to various settings. Lama (2011) and Otsu (2010a) apply the method to a small open economy framework. Otsu (2010b) applies the method to a two country setting and investigates the business cycle correlation between Japan and the US. Sustek (2011) extends the method to a monetary model in order to account for fluctuations of the inflation rate and nominal interest rate in the US. Inaba and Nutahara (2012) apply business cycle accounting to a medium-scale New Keynesian model and show that business cycle accounting is empirically useful even though the stochastic process of the wedges are misspecified.

By construction, feeding all wedges into the model can perfectly reproduce the observed fluctuations of the data. This is why we can decompose the effects of each type of wedges on business cycle fluctuations by feeding the measured wedges into the model one-by-one. However, it turns out that the theoretical moments such as the volatility, persistence and cross-correlation of variables computed from the prototype model using frequency-domain based calculations do not match those computed from the observed data. The main reason is because the parameters governing the stochastic process are estimated by the maximum likelihood method which is not intended to match moments. Introducing moments-based estimation for the stochastic process can potentially extend the use business cycle accounting to comparing the roles of wedges on business cycle moments.

The rest of this paper is organized as follows. In section 2, I will describe the prototype model and the business cycle accounting procedure. In section 3, I will present the business cycle moments computed from the model and the data. Section 4 concludes the paper.

[2]Chakraborty (2009) shows that investment wedges also contribute to the output fluctuation in Japan during the boom and bust in the 1980s and 1990s. Christiano and Davis (2006) argues that the existence of capital adjustment costs increase the importance of investment wedges.

2 Business Cycle Accounting

2.1 The Prototype Model

Efficiency wedges ω^e are defined as the wedges between output y and the composite of inputs, capital k and labor l, in the production function f:

$$\omega_t^e f(k_t, l_t) = y_t. \tag{1}$$

Therefore, they are equivalent to total factor productivity, i.e., the Solow residuals. Capital stock follows the law of motion

$$\Gamma k_{t+1} = i_t + (1-\delta)k_t, \tag{2}$$

where Γ is the growth trend of technology and population.

Government wedges ω^g are defined as the wedges between total resources and private expenditures, consumption c and investment i, in the resource constraint

$$c_t + i_t + \omega_t^g = y_t. \tag{3}$$

Therefore, they are equivalent to the sum of government purchases and the trade balance.

Investment wedges ω^i are defined as the wedges between the expected return of capital and the marginal rate of intertemporal consumption substitution in the capital Euler equation

$$\beta E_t \left[u_c(c_{t+1}, l_{t+1}) \left(f_k(k_{t+1}, l_{t+1}) + \omega_{t+1}^i (1-\delta)\right)\right] = \omega_t^i u_c(c_t, l_t). \tag{4}$$

They operate as distortionary taxes on investment[3].

Labor wedges ω^l are defined as the wedges between the marginal product of labor and the marginal rate of substitution of labor to consumption in the

[3] Christiano and Davis (2006) and Kobayashi and Inaba (2006) demonstrate that the choice of whether to model investment wedges as taxes on investment or taxes on capital income affects the accounting results. Kobayashi and Inaba (2006) and Chakraborty (2009) show that the choice of steady states will affect the quantitative implication of investment wedges. Nonetheless, these choices of how to model investment wedges do not affect the conclusion of this paper.

labor first order condition

$$\omega_t^l \omega_t^e f_l\left(k_t, l_t\right) = -\frac{u_l\left(c_t, l_t\right)}{u_c\left(c_t, l_t\right)}. \tag{5}$$

They operate as distortionary taxes on labor income.

These four wedges are assumed to follow a VAR process

$$\widetilde{\omega_t} = P\widetilde{\omega_{t-1}} + \varepsilon_t, \tag{6}$$

where $\omega_t = \left(\omega_t^e, \omega_t^g, \omega_t^i, \omega_t^l\right)'$ and $\varepsilon_t = \left(\varepsilon_t^e, \varepsilon_t^g, \varepsilon_t^i, \varepsilon_t^l\right)'$. The "~" denotes the log deviation of the variable from its steady state: $\widetilde{x_t} = \ln x_t - \ln x$. The innovations are assumed to be normally distributed: $\varepsilon_t \sim N(0, V)$. Contemporaneous correlation between wedges are allowed, hence, there are no restrictions on the variance-covariance matrix V[4].

The competitive equilibrium is fully characterized by equations (1) to (6).

2.2 Parameterization

In order to conduct a quantitative analysis, we first need to assume the functional forms of production technology and preferences. Following standard business cycle literature, let's assume $f\left(k_t, l_t\right) = k_t^{\theta} l_t^{1-\theta}$ and $u\left(c_t, l_t\right) = \Psi \log c_t + (1-\Psi)\log(1-l_t)$. The choice of these functions will affect the accounting results so it is important that these functional forms are widely accepted as a reasonable representation of the production technology and preferences.

Next, we have to pin down the values of parameters that illustrate the equilibrium. The parameters that characterize the production technology and preferences $(\theta, \delta, \Gamma, \Psi, \beta)$ are calibrated using quarterly data available from national statistics[5]. The calibrated parameters are listed in Table 1. The lag matrix P and the variance-covariance matrix of the innovations V in the stochastic process (6) are estimated using the maximum likelihood

[4]Christiano and Davis (2006) criticize that the wedges are not identified due to the contemporaneous correlation across wedges.

[5]For simplicity, the steady state wedges ω^e, ω^i,and ω^l are assumed to be equal to 1. This simplification saves the trouble of estimating the steady state level of these wedges. This assumption does not affect the conclusion of this paper. The steady state ω^g is computed directly from the data of government purchases to output ratio.

method in Dynare based on the Kalman filter[6]. Investment wedges, unlike the other wedges, cannot be directly computed since it is defined in the expectational equation (4). Therefore, the entire stochastic process is estimated using the data of output, consumption, investment and labor given the entire structure of the model and the calibrated parameters.

2.3 Solving the Model

Linearized decision rules for output, consumption, investment and labor are defined as

$$\widetilde{q}_t = A\widetilde{k}_t + B\widetilde{\omega}_t,$$

where $\widetilde{q}_t = \left(\widetilde{y}_t, \widetilde{c}_t, \widetilde{i}_t, \widetilde{l}_t\right)'$. The coefficients in the linear decision rule are obtained through the method of undetermined coefficients following Uhlig (1999). Since investment is observable, capital stock can be computed using the linear capital law of motion

$$\Gamma\widetilde{k_{t+1}} = \frac{i}{k}\widetilde{i}_t + (1-\delta)\widetilde{k}_t,$$

given the initial value of capital, $\widetilde{k}_1$. Then, the wedges can be computed from the observable variables and capital stock computed above:

$$\widetilde{\omega}_t = B^{-1}\left(\widetilde{q}_t - A\widetilde{k}_t\right).$$

It is straight forward to show that the simulation with all wedges $\widetilde{q_t^{\omega}}$ can perfectly replicate the data:

$$\widetilde{q_t^{\omega}} = A\widetilde{k}_t + B\widetilde{\omega}_t = A\widetilde{k}_t + BB^{-1}\left(\widetilde{q}_t - A\widetilde{k}_t\right) = \widetilde{q}_t. \tag{7}$$

It is also straight forward to show that the simulations with each wedge alone $\widetilde{q_t^{\omega^j}}$ sum up to the simulation with all wedges:

$$\widetilde{q_t^{\omega^e}} + \widetilde{q_t^{\omega^g}} + \widetilde{q_t^{\omega^i}} + \widetilde{q_t^{\omega^l}} = \widetilde{q_t^{\omega}}. \tag{8}$$

Therefore, business cycles can be decomposed using wedges. Notice that (7)

[6] A detailed description of the estimation method built into Dynare can be found in Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto and Villemot (2011).

and (8) hold regardless of the choice of parameter values.

3 Computing the Moments

In this section, following Kydland and Prescott (1982), I compute business cycle moments of HP filtered output, consumption, investment and labor. The moments reported are the standard deviation of each variable, those relative to the standard deviation of output, and the cross-correlation between each variable and output with leads and lags.

Table 2a and 2b present data moments for the US and Japan over the 1980-2010 period using the dataset constructed in Otsu (2010b). There are several common business cycles features across the two countries. Consumption, investment and labor are all procyclical. Investment is much more volatile than output while consumption is less volatile. There are also a few notable differences. The volatility of labor and investment are much lower in Japan than in the US. In addition, the contemporaneous correlation between consumption and output is much lower in Japan than in the US. Moreover, as discussed in Otsu (2011), the Japanese labor leads output by 1 quarter while in the US the fluctuations of the two series are coincidental.

Tables 3a and 3b report the theoretical moments computed from the model using the frequency-domain based calculations which is built into the toolkit developed by Uhlig (1999)[7]. The results show that there are considerable discrepancies between the data moments and the theoretical moments. In the US, comparing the model to the data, the output volatility and persistence are much lower; correlation between output and consumption is considerably lower; the volatility of investment is slightly higher; and the volatility of labor and its correlation with output are considerably lower. In Japan, the output volatility is slightly lower; the volatility of consumption is much higher; the volatility of investment is slightly lower and the correlation between output and investment is considerably lower; the volatility of labor is much higher and the correlation of output with labor is much lower[8].

[7] Simulation based moments calculation is also built into the toolkit. The average moments computed from 10,000 simulations are approximately equal to moments computed from frequency-domain based calculations.

[8] There are concerns that these results may be distorted because the maximum likelihood estimation has produced bad estimates due to issues such as local maximization and linearization. I have experimented with several different initial values and found that the

The main reason why the model cannot reproduce the selected moments computed from data is because the stochastic process is estimated using the maximum likelihood method. The aim of maximum likelihood estimation is to utilize all of the information that the observable variables convey in order to infer the 'true' stochastic process that is generating the data. Therefore, there is no guarantee that the selected moments computed from the model based on these estimates will match those computed from data[9].

One way to improve the prediction of selected moments is to increase the number of parameters to be estimated. We can do that by increasing the lags in the VAR stochastic process. However, it is not clear how many lags we have to add in order to replicate the selected moments. An alternative way is to use moments-based estimation such as the generalized method of moments instead of maximum likelihood estimation. The model with alternative parameter estimates can still be used to decompose the sources of business cycles as the replication result (7) holds for all parameter values. Moreover, the theoretical moments should match the moments used to estimate the parameters. Therefore, one can compare the roles of each type of wedges on business cycle moments such as volatility, persistence and cross-correlation of the variables.

4 Conclusion

In this paper, I showed that theoretical business cycle moments computed from a prototype business cycle accounting model estimated by maximum likelihood estimation do not match those computed directly from data. The main reason of this discrepancy is because the stochastic process is not estimated to match any selected moments to begin with but to infer the 'true' data generating process. Since the business cycle accounting method is intended to account for the observed fluctuation patterns of the data series and not the theoretical moments, this discrepancy is not necessarily a flaw in the

results are similar. Therefore, distortions due to the local maximum issue do not seem to be a problem. In terms of nonlinear estimation, Chari, Kehoe and McGrattan (2007) avoid this because it is 'computationally demanding'. I leave testing for the estimation error stemming from linearization as a future research agenda. Nonetheless, even after adjusting for nonlinearity the fundamental issue discussed in this paper will remain.

[9] Since some of the business cycle accounting literature use Bayesian estimation rather than maximum likelihood estimation, I have also checked the results using Bayesian estimation. This does not necessarily improve the model prediction of the selected moments.

method but rather a virtue. Nonetheless, moments-based estimation might be an attractive alternative to the maximum likelihood method if the purpose of the research is to study business cycle properties such as volatility, persistence and cross-correlation of variables instead of a specific business cycle episode.

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Table 1. Calibrated Parameters

	Parameter	US	Japan
θ	Capital Share	0.371	0.407
δ	Depreciation	0.011	0.029
Γ	Growth Trend	1.007	1.006
Ψ	Preference Weight	0.203	0.234
β	Discount Factor	0.988	0.986

Table 2a. Business Cycle Facts: US 1980Q1-2010Q4

	volatility		Cross-correlation between $y(t)$ and $v(t+j)$				
v	$std(\%)$	$\frac{std(v)}{std(y)}$	$j=-2$	$j=-1$	$j=0$	$j=1$	$j=2$
Output	1.34	1.00	0.67	0.86	1.00	0.86	0.67
Consumption	0.84	0.63	0.65	0.78	0.87	0.78	0.65
Investment	4.97	3.70	0.67	0.84	0.94	0.78	0.53
Labor	1.50	1.12	0.53	0.75	0.90	0.90	0.80

Table 2b. Business Cycle Facts: Japan 1980Q1-2010Q4

	volatility		Cross-correlation between $y(t)$ and $v(t+j)$				
v	$std(\%)$	$\frac{std(v)}{std(y)}$	$j=-2$	$j=-1$	$j=0$	$j=1$	$j=2$
Output	1.46	1.00	0.56	0.78	1.00	0.78	0.56
Consumption	0.74	0.51	0.39	0.44	0.57	0.38	0.33
Investment	3.76	2.57	0.50	0.67	0.88	0.76	0.57
Labor	0.83	0.57	0.45	0.63	0.58	0.51	0.33

Table 3a. Theoretical Moments: US

	volatility		Cross-correlation between $y(t)$ and $v(t+j)$				
v	$std(\%)$	$\frac{std(v)}{std(y)}$	$j=-2$	$j=-1$	$j=0$	$j=1$	$j=2$
Output	0.92	1.00	0.46	0.71	1.00	0.71	0.46
Consumption	0.59	0.64	0.34	0.51	0.70	0.51	0.35
Investment	3.61	3.92	0.42	0.63	0.89	0.63	0.41
Labor	0.92	1.00	0.34	0.55	0.80	0.66	0.52

Table 3b. Theoretical Moments: Japan

	volatility		Cross-correlation between $y(t)$ and $v(t+j)$				
v	$std(\%)$	$\frac{std(v)}{std(y)}$	$j=-2$	$j=-1$	$j=0$	$j=1$	$j=2$
Output	1.37	1.00	0.54	0.75	1.00	0.75	0.54
Consumption	0.95	0.69	0.30	0.42	0.56	0.42	0.30
Investment	3.33	2.43	0.39	0.58	0.80	0.65	0.50
Labor	1.09	0.80	0.09	0.08	0.07	0.04	0.02