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Heteroskedastic Dynamic Factor Models: A Monte Carlo Study

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Abstract

We propose to estimate heteroskedastic dynamic factor models using the Kalman filter, where the state vector is augmented with the heteroskedastic disturbances. Although this model is not conditionally Gaussian, Monte Carlo results show that parameters can be accurately estimated.

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1. Introduction

We use Monte Carlo experiments to analyse quasi-maximum likelihood inference for conditionally heteroskedastic dynamic factor models as the one used in Morales-Arias & Moura (2013). Dynamic factor models were first proposed by Sargent & Sims (1977) and are nowadays widespread in economics and finance (see Forni *et al.* , 2000; Stock & Watson, 2002a,c, 2005; Doz & Reichlin, 2006; Han, 2006; Diebold *et al.* , 2006). However, macroeconomic and financial time-series are often subject to heteroskedasticity (see Engle, 1982; Stock & Watson, 2002b; Cogley & Sargent, 2005; Cogley *et al.* , 2010; Morales-Arias & Moura, 2013), which is not accounted for in the traditional homoskedastic dynamic factor model.

Moreover, modeling large covariance matrices remains a fundamental challenge in econometrics. Many of the the initial attempts to build models for conditional covariances, such as the VEC model of Bollerslev *et al.* (1988) and the BEKK model of Engle & Kroner (1995), among others, suffered from the so-called curse of dimensionality. In these specifications, the number of parameters increase very rapidly as the cross-section dimension grows, thus creating difficulties in the estimation process and entailing a large amount of estimation error in the resulting covariance matrices.

In this context, factor models emerge as promising alternatives to circumvent the problem of dimensionality and to alleviate the burden of the estimation process. The idea behind factor models is to assume that the co-movements of the time-series depend on a small number of underlying factors. This dimensionality reduction allows for a great flexibility in the econometric specification and in the modeling strategy, as shown, for example in Santos & Moura (2012). Many alternative approaches for conditional covariance matrices based on factors models have been proposed in the literature. Generally, these models differ in their assumptions regarding the characteristics of the factors (see Alexander & Chibumba, 1996; Alexander, 2001; Chan *et al.* , 1999; Engle *et al.* , 1990; Aguilar & West, 2000; van der Weide, 2002; Han, 2006; Santos & Moura, 2012).

Here we reinterpret a dynamic factor model as a state space model following Jungbacker & Koopman (2008), and introduce heteroskedastic error terms in the resulting state space model as proposed by Harvey *et al.* (1992). As in Harvey *et al.* (1992), estimation is carried out in one step using the Kalman filter, even though the model is not conditionally Gaussian. Harvey *et al.* (1992) use Monte Carlo experiments to show that this approximation yields precise parameter estimates. However, their simulation experiment is only concerned with univariate models and with ARCH effects. In this paper we generalize their Monte Carlo experiment to dynamic factor models subject to GARCH(1,1) effects.

The paper is organized as follows. In Section 2 we present the dynamic factor model analysed, and its state space form. In Section 3 we show how to use the Kalman filter to obtain an approximation to the likelihood, and how quasi-maximum likelihood inference can be implemented. In Section 4 we discuss the simulation study developed and its results, and Section 5 concludes.

2. Dynamic Factor Model

A general form of the Dynamic Factor Model can be found in Jungbacker & Koopman (2008). This paper departs slightly from the general model and uses a version applied by Han (2005) to model asset returns, which is given by

$$y_t = Bf_t + \kappa_t, \qquad \kappa_t \sim NID(0, \Sigma_{\kappa,t}) \tag{1}$$

where y_t represents a $p \times 1$ vector of observations at time $t = 1, ..., N$, B represents the factor loadings matrix, f_t represents a $k \times 1$ vector of common factors, and κ_t represents the idiosyncratic Gaussian shock terms. The dynamics of the vector of factors is given by:

$$f_{t+1} = c + Af_t + \zeta_{t+1}, \qquad \zeta_{t+1} \sim NID(0, \Sigma_{\zeta,t+1}) \tag{2}$$

where c is a vector of constants, A is a diagonal matrix of autoregressive coefficients and ζ_t represents a vector of idiosyncratic Gaussian error terms. The covariance matrix of the observable time series, y_t, is defined here as $\Sigma_{t+1|t} \equiv \text{var}_t(y_{t+1})$. The elements of $\Sigma_{t+1|t}$ depend on the loading matrix B and on the covariance matrices of κ_t and of ζ_t:

$$\Sigma_t = \Sigma_{\kappa,t} + B\Sigma_{\zeta,t}B' \tag{3}$$

Matrices $\Sigma_{\kappa,t}$ and $\Sigma_{\zeta,t}$ are diagonal and the covariance between $y_{t,i}$ and $y_{t,j}$ is captured via the loading matrix B. All idiosyncratic shock terms are modeled by a GARCH(1,1) processes, thus

$$\Sigma_{\zeta,t} = diag\left\{w_{1,t}, ..., w_{k,t}\right\} \tag{4}$$
$$\Sigma_{\kappa,t} = diag\left\{w_{k+1,t}, ..., w_{r,t}\right\}, \qquad r = k + p \tag{5}$$
$$w_{i,t} = \beta_{0,i} + \beta_{1,i}\zeta_{t-1}^2 + \beta_{2,i}w_{i,t-1}, \qquad i \in 1, ..., k \tag{6}$$
$$w_{j,t} = \beta_{0,j} + \beta_{1,j}\kappa_{t-1}^2 + \beta_{2,j}w_{j,t-1}, \qquad j \in (k+1), ..., r \tag{7}$$

where w is a variance term in the diagonal covariance matrices $\Sigma_{\zeta,t}$ and $\Sigma_{\kappa,t}$. The GARCH(1,1) processes are constrained to ensure stationarity, as in Bollerslev (1986) and Tsay (2005). Thus, $\beta_{k,j} > 0$ for all k, j and $1 - \beta_1 - \beta_2 > 0$.

Matrix A with elements a_{ij} and factor loading matrix B with elements b_{ij} are also subject to constraints. Since current values of a factor are not affect by lagged values of any other factors, $a_{ij} = 0$ for all $i \neq j$. Moreover, we requires that $a_{ii} \in (-1, 1)$ to ensure stationarity. Because of identification problems (rotational indeterminacy problem) in factor models (see Geweke & Zhou, 1996), we set $b_{ii} = 1$ and $b_{ij} = 0$ when $j > i$.

2.1. State-space Representation

The dynamic factor model can be rewritten in state space form. The linear Gaussian state space form is a general representation of a variety of different linear Gaussian time-series models. It consists of two equations, the observation equation represented in (9) and the state, or transition, equation represented in (8). The general linear Gaussian state space model is given by

$$\alpha_{t+1} = T_t\alpha_t + R_t\eta_t, \qquad \eta_t \sim NID(0, Q_t) \tag{8}$$
$$y_t = Z_t\alpha_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, H_t) \tag{9}$$

in which α_t is the state-vector, T_t, R_t and Z_t are system-matrices and y_t is the data-vector at time t.

The transformation of equations (1) and (2) into this setup is straightforward. However, modeling heteroskedasticity with GARCH processes require some modifications. Since the dynamic factor model is an unobserved component model, past values of κ_t and ζ_t are not directly observable, as knowledge of past observations does not imply knowledge of past disturbance terms in this class of models. Therefore, equations (6) and (7) cannot be computed. Harvey *et al.* (1992) propose to substitute the square of the error terms in (6) and (7) by their expectation, thus (6) and (7) become:

$$w_{i,t} = \beta_{0,i} + \beta_{1,i}E[\zeta_{t-1}^2|Y_{t-1}] + \beta_{2,i}w_{i,t-1}, w_{j,t} = \beta_{0,j} + \beta_{1,j}E[\kappa_{t-1}^2|Y_{t-1}] + \beta_{2,j}w_{j,t-1} \tag{10}$$

Harvey *et al.* (1992) also note that this expectation can be computed by the Kalman filter recursions if we augment the state vector with the disturbances κ_t and ζ_t, as

$$E[\zeta_{t-1}^2|Y_{t-1}] = \widehat{\zeta}_{t-1|t-1}^2 + P_{t-1|t-1}^{\zeta}, \tag{11}$$

where $\widehat{\zeta}_{t-1|t-1}$ are filtered estimates of ζ_{t-1}, and $P_{t-1|t-1}^{\zeta}$ is the covariance matrix of ζ_{t-1} given the observations at period $t-1$[1], which are computed for all states during the Kalman filter recursions. The resulting

[1] An analogous expression can be rewritten for $E[\kappa_{t-1}^2|Y_{t-1}]$ as well.

augmented state space model for our heteroskedastic dynamic factor model is then written as

$$w_{t+1,i} = \beta_{0,i} + \beta_{1,i}\eta_{t,i}^2 + \beta_{2,i}w_{t,i}, \qquad i \in 1,...,r \tag{12}$$

$$Q_{t+1}^* = diag\{w_{t+1}\}, \qquad \eta_t^* = \begin{bmatrix} \zeta_t \\ \kappa_t \end{bmatrix} \tag{13}$$

$$\alpha_{t+1} = T^*\alpha_t^* + R^*\eta_{t+1}^*, \qquad \eta_{t+1}^* \sim NID(0, Q_{t+1}^*) \tag{14}$$

$$y_t = Z\alpha_t^*, \tag{15}$$

where (12) describes the heteroskedasticity captured in Q_{t+1}^*, and the measurement equation is singular, since all disturbances are included in the state vector α^* in order to compute $E[\zeta_{t-1}^2|Y_{t-1}]$ and $E[\kappa_{t-1}^2|Y_{t-1}]$ within the Kalman filter recursions as in Harvey *et al.* (1992).

3. Quasi-Maximum Likelihood Estimation

Although the conditional distribution of $\kappa_t|t-1$ and $\zeta_t|t-1$ is assumed to be Gaussian with mean zero and variance $\Sigma_{\kappa,t}$ and $\Sigma_{\zeta,t}$, the distribution of these error terms conditional on past observations is unknown. Nevertheless, Harvey *et al.* (1992) suggest to treat the augmented state space (12)-(15) as if it were conditionally Gaussian and to use the Kalman filter to obtain an approximation to the likelihood based on the prediction error decomposition:

$$\ell(y|\theta) = \log L(y_1, ..., y_N|\theta) = -\frac{Np}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{N}\left(log|F_t| + v_t' F_t^{-1} v_t\right) \tag{16}$$

where $v_t = y_t - Z\alpha_t^*$ are the one-step ahead prediction errors and F_t is their variance (see, for instance, Durbin & Koopman, 2001, for further details). The parameter vector θ contains all parameters in the model. Maximization of (16) is performed numerically using the Broyden-Fletcher-Goldfarb-Shanno algorithm.

All the computations are carried out in *SsfPackAff*, which is a suite of C routines for computations involving the statistical analysis of time series models in state space form. *SsfPackAff* can be used in the OxMetrics[2] environment. For more information on *SsfPack* suites and their implementation, see Koopman *et al.* (1999).

4. Simulation study

This section will discuss simulation results for the dynamic factor model with GARCH(1,1) errors as is presented in section 2. The underlying theoretical model is not conditionally Gaussian, since lagged values of the error terms are not observed. However, when constructing the likelihood and estimating the parameters, the model is treated as if it were Gaussian. Harvey *et al.* (1992) present Monte Carlo simulations showing that the approximation error is negligible for univariate unobserved component models with a relatively large sample. Here we investigate the degree of error caused by this approximation in a multivariate model, with a larger number of time series and factors.

4.1. Parameter Estimation results for N = 1500 observations

The version of the factor model initially analysed has k = 2 factors and p = 16 time-series. This model has 87 parameters, whose true values used to simulate samples can be seen in Tables 1 - 7. We simulate $M = 1000$ samples of size $N = 1500$ and estimate the parameters via QML for each of these samples. The sample size was selected based on the financial application of Han (2005).

A summary of the parameter values and statistics for this simulation study, can be found in Tables 1 - 5. It is interesting to note that the medians of the estimates are close to the true parameter values. Moreover, the standard deviation of these estimated parameters are small, indicating that the approximation error

[2]OxMetrics is developed by Jurgen Doornik. More information can be found at www.doornik.com

induced by assuming that the model is conditionally Gaussian when it is not is small. The true parameter values all lie within two standard deviations from the median of the empirical distribution of parameters estimates.

For a closer inspection of the empirical distributions of parameter estimates, see Figures Figures 1 - 7. Some asymmetries can be seen for GARCH parameters, but these are due to the fact that the true parameter values are close to their boundaries.

The filtered states and their filtered variances play an important role in this model, since they are used to compute the approximation to the GARCH equations in (10), which enter the multivariate time-varying covariance matrix of the observations. If the states can be estimated precisely, the approximation to the true likelihood is going to be good. Figure 8 shows how the filtered variance is captures the heteroskedasticity present in the first factor. The filteres factor can be seen in figure 9. These figures show that the model precisely estimates the states, and their variances. Moreover, all variations are followed quite closely by the estimated factor and variance.

4.2. Effect of the sample size N

Although $N = 1500$ is only a moderately large sample size in finance, it is huge when we consider macroeconomic data. Therefore, it is interesting to ask how sensitive are the parameter estimates to different sample sizes. Tables 6 and 7 show the declining MC standard deviation when the sample size is increased. A sample size of 1500 trading days is something between the 5 or 6 years, and was used by Han (2005) in a financial application for portfolio optimization. Santos & Moura (2012), on the other hand, use $N = 2766$ observations for their portfolio optimization application, and Morales-Arias & Moura (2013) only have $N =$ 196 observations in their macroeconomic aplication to model inflation volatility. Tables 6 and 7 show that with a sample of $N = 500$ observations, the median of the estimates are mostly close to the true values, and given the large standard errors, the true values of the parameters are all within two standard errors of the true parameter value. Increasing the sample to $N = 1000$ brings the median of the parameter estimates much closer to the true parameter values, and reduces the Monte Carlo standard errors of the estimates. Increasing the sample size further to $N = 1500$ improves again the median of the estimates, although they were already very close to the true values. The Monte Carlo standard errors are further reduced, as expected. All in all, the results suggest that sample sizes bellow $N = 500$ should be used with caution. Although most parameters of the model are still precisely estimated, estimates of the constant in GARCH equations, $\beta^{\kappa}_{i,0}$, are subject to larger errors. Moreover, increasing the sample from $N = 1000$ to $N = 1500$ does not improve the median estimates much further. Nevertheless, the Monte Carlo standard errors will be always reduced when sample size increases.

5. Conclusion

In this paper we use Monte Carlo simulations to analyse quasi-maximum likelihood inference for conditionally heteroskedastic dynamic factor models. Although this model is not conditionally Gaussian, we apply the strategy of Harvey *et al.* (1992) and treat the model as if it were conditionally Gaussian, and asses the accuracy of this approximation by means of a Monte Carlo experiment.

Results have shown that estimation is precise for moderately large samples and encourages the use of heteroskedastic dynamic factor models to estimate, for example, high dimensional time-varying covariance matrices.

The possibility to use the Kalman filter to construct the likelihood of such a model, and the precision of the QML parameter estimates indicates that heteroskedastic dynamic factor models can be easily estimated in one step using a much simpler approach than, for example, the approach of Han (2005).

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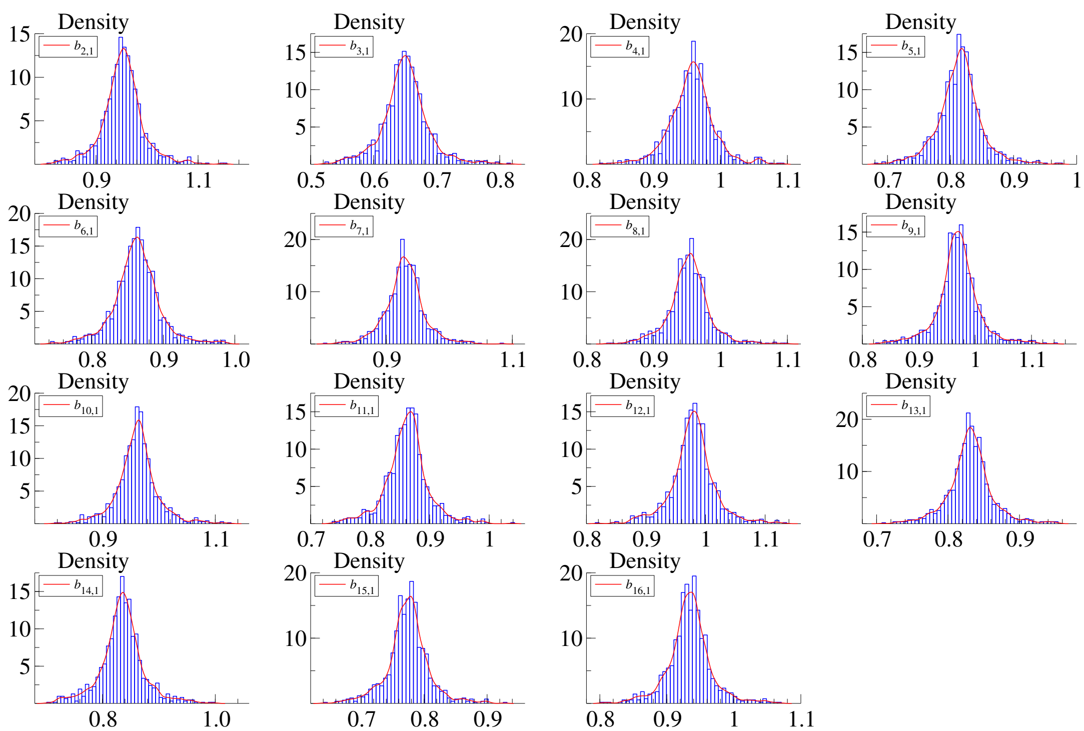

Figure 1: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters in the first column of the loading matrix B. See table 1 for the corresponding true parameter values.

Parameters		θ_{TRUE}		Median θ_{ML}		$\bar{\theta}_{ML}$		MC SD		Min		Max	
$b_{2,1}$		0.95502		0.95243		0.95263		0.041406		0.80587		1.1520	
$b_{3,1}$	$b_{3,2}$	0.65199	0.89994	0.64994	0.90061	0.65015	0.90102	0.037188	0.026240	0.52477	0.81794	0.81608	1.0104
$b_{4,1}$	$b_{4,2}$	0.95915	0.67465	0.95783	0.67439	0.95703	0.67442	0.033408	0.023572	0.82427	0.59451	1.0925	0.76980
$b_{5,1}$	$b_{5,2}$	0.81648	0.87611	0.81498	0.87573	0.81428	0.87546	0.035031	0.023042	0.68330	0.79206	0.97253	0.96893
$b_{6,1}$	$b_{6,2}$	0.86517	0.70851	0.86273	0.70771	0.86259	0.70807	0.032562	0.021928	0.74240	0.60079	0.98993	0.80312
$b_{7,1}$	$b_{7,2}$	0.93129	0.63930	0.92928	0.63963	0.92938	0.64011	0.031317	0.021263	0.80257	0.56878	1.0846	0.71505
$b_{8,1}$	$b_{8,2}$	0.95533	0.59969	0.95350	0.59947	0.95300	0.59945	0.031152	0.022056	0.83066	0.49890	1.1006	0.69148
$b_{9,1}$	$b_{9,2}$	0.97172	0.88418	0.96977	0.88392	0.96978	0.88442	0.037821	0.023255	0.82831	0.79575	1.1415	0.98468
$b_{10,1}$	$b_{10,2}$	0.96332	0.85841	0.96171	0.85816	0.96144	0.85810	0.037161	0.022268	0.81624	0.79175	1.1256	0.94571
$b_{11,1}$	$b_{11,2}$	0.86393	0.88133	0.86266	0.88084	0.86181	0.88124	0.036684	0.022633	0.73664	0.78913	1.0370	0.96829
$b_{12,1}$	$b_{12,2}$	0.98189	0.88212	0.97949	0.88273	0.97889	0.88244	0.037362	0.021416	0.82050	0.81838	1.1337	0.95917
$b_{13,1}$	$b_{13,2}$	0.83334	0.70651	0.83076	0.70578	0.83084	0.70638	0.031132	0.020246	0.70832	0.63510	0.95370	0.83086
$b_{14,1}$	$b_{14,2}$	0.83730	0.99595	0.83549	0.99613	0.83503	0.99649	0.038977	0.023763	0.70266	0.90884	0.99980	1.0917
$b_{15,1}$	$b_{15,2}$	0.77652	0.85259	0.77489	0.85288	0.77415	0.85274	0.033769	0.020712	0.64128	0.78003	0.92706	0.92634
$b_{16,1}$	$b_{16,2}$	0.93700	0.70411	0.93387	0.70398	0.93388	0.70447	0.032453	0.019120	0.80247	0.64091	1.0693	0.77379

Table 1: Parameter estimates of simulated DFM - GARCH(1,1) model for k=2 factors, p=16 series and N=1500 observations per series with corresponding median, the mean, the MC standard deviation of the estimates, the minimum and the maximum of M=1000 simulations respectively. These parameters are the parameters in the loading matrix.

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Parameters	θ_{TRUE}	Median θ_{ML}	$\bar{\theta}_{ML}$	MC SD	Min	Max
$\beta^{\kappa}_{1,0}$	0.050000	0.050261	0.053053	0.018473	0.013142	0.21932
$\beta^{\kappa}_{2,0}$	0.048000	0.048578	0.053795	0.018710	0.016390	0.18442
$\beta^{\kappa}_{3,0}$	0.046000	0.046452	0.050774	0.018003	0.0084336	0.19349
$\beta^{\kappa}_{4,0}$	0.044000	0.044613	0.050031	0.017684	0.011126	0.16333
$\beta^{\kappa}_{5,0}$	0.042000	0.042520	0.047755	0.018040	0.0088716	0.19809
$\beta^{\kappa}_{6,0}$	0.040000	0.040558	0.046079	0.018260	0.011447	0.21385
$\beta^{\kappa}_{7,0}$	0.038000	0.038389	0.042435	0.014319	0.013171	0.15489
$\beta^{\kappa}_{8,0}$	0.036000	0.036379	0.039506	0.013968	0.0044431	0.15143
$\beta^{\kappa}_{9,0}$	0.034000	0.034442	0.037921	0.013494	0.010052	0.12469
$\beta^{\kappa}_{10,0}$	0.032000	0.032379	0.035904	0.012319	0.0083292	0.13453
$\beta^{\kappa}_{11,0}$	0.030000	0.030361	0.033643	0.011277	0.0046348	0.10259
$\beta^{\kappa}_{12,0}$	0.028000	0.028338	0.030685	0.009362	0.0074224	0.099852
$\beta^{\kappa}_{13,0}$	0.026000	0.026233	0.028830	0.010270	0.0052653	0.13400
$\beta^{\kappa}_{14,0}$	0.024000	0.024213	0.025875	0.009085	0.0051755	0.11589
$\beta^{\kappa}_{15,0}$	0.022000	0.022207	0.024499	0.008481	0.0064711	0.080547
$\beta^{\kappa}_{16,0}$	0.020000	0.020165	0.021917	0.006963	0.0051121	0.065885

Table 2: Parameter estimates of simulated DFM - GARCH(1,1) model for k=2 factors, p=16 series and N=1500 observations per series with corresponding median, the mean, the MC standard deviation of the estimates, the minimum and the maximum of M=1000 simulations respectively. These parameters are part of the GARCH(1,1) parameters in the observation equations.

Parameters	θ_{TRUE}	Median θ_{ML}	$\bar{\theta}_{ML}$	MC SD	Min	Max
$\beta^{\kappa}_{1,1}$	0.098308	0.11763	0.11910	0.027014	0.044972	0.26444
$\beta^{\kappa}_{2,1}$	0.099698	0.099468	0.10025	0.018306	0.051787	0.15797
$\beta^{\kappa}_{3,1}$	0.098810	0.10064	0.10084	0.018979	0.046066	0.18548
$\beta^{\kappa}_{4,1}$	0.10050	0.10076	0.10141	0.017937	0.035864	0.16835
$\beta^{\kappa}_{5,1}$	0.10007	0.10066	0.10087	0.018290	0.021620	0.16545
$\beta^{\kappa}_{6,1}$	0.10123	0.10035	0.10114	0.018133	0.058320	0.16617
$\beta^{\kappa}_{7,1}$	0.10061	0.10243	0.10255	0.017645	0.059184	0.16038
$\beta^{\kappa}_{8,1}$	0.10233	0.10462	0.10470	0.019047	0.052249	0.17694
$\beta^{\kappa}_{9,1}$	0.10143	0.10332	0.10379	0.018980	0.049198	0.17746
$\beta^{\kappa}_{10,1}$	0.099754	0.10185	0.10221	0.019525	0.029968	0.17491
$\beta^{\kappa}_{11,1}$	0.099092	0.098657	0.099843	0.018759	0.045091	0.17219
$\beta^{\kappa}_{12,1}$	0.10014	0.10317	0.10394	0.018894	0.047030	0.16436
$\beta^{\kappa}_{13,1}$	0.099234	0.098617	0.099073	0.018076	0.049713	0.16070
$\beta^{\kappa}_{14,1}$	0.098844	0.10567	0.10628	0.020500	0.031851	0.17344
$\beta^{\kappa}_{15,1}$	0.10040	0.10502	0.10577	0.019598	0.047513	0.18482
$\beta^{\kappa}_{16,1}$	0.10015	0.10267	0.10430	0.020260	0.030791	0.19088

Table 3: Parameter estimates of simulated DFM - GARCH(1,1) model for k=2 factors, p=16 series and N=1500 observations per series with corresponding median, the mean, the MC standard deviation of the estimates, the minimum and the maximum of M=1000 simulations respectively. These parameters are part of the GARCH(1,1) parameters in the observation equations.

Parameters	θ_{TRUE}	Median θ_{ML}	$\bar{\theta}_{ML}$	MC SD	Min	Max
$\beta^{\kappa}_{1,2}$	0.89169	0.87177	0.86947	0.027068	0.72716	0.94126
$\beta^{\kappa}_{2,2}$	0.89030	0.88819	0.88702	0.019292	0.81200	0.94105
$\beta^{\kappa}_{3,2}$	0.89119	0.88759	0.88725	0.019414	0.77692	0.94052
$\beta^{\kappa}_{4,2}$	0.88950	0.88777	0.88598	0.018553	0.81290	0.94888
$\beta^{\kappa}_{5,2}$	0.88993	0.88727	0.88655	0.018650	0.79429	0.95744
$\beta^{\kappa}_{6,2}$	0.88877	0.88720	0.88583	0.019714	0.78579	0.93715
$\beta^{\kappa}_{7,2}$	0.88939	0.88568	0.88517	0.017948	0.81361	0.92917
$\beta^{\kappa}_{8,2}$	0.88767	0.88427	0.88301	0.019398	0.79970	0.93695
$\beta^{\kappa}_{9,2}$	0.88857	0.88566	0.88387	0.019376	0.80187	0.93511
$\beta^{\kappa}_{10,2}$	0.89025	0.88654	0.88533	0.020008	0.76586	0.94719
$\beta^{\kappa}_{11,2}$	0.89091	0.88816	0.88738	0.019401	0.79303	0.94009
$\beta^{\kappa}_{12,2}$	0.88986	0.88517	0.88394	0.019083	0.80427	0.93969
$\beta^{\kappa}_{13,2}$	0.89077	0.88997	0.88846	0.018437	0.80112	0.94211
$\beta^{\kappa}_{14,2}$	0.89116	0.88300	0.88195	0.020864	0.78606	0.94238
$\beta^{\kappa}_{15,2}$	0.88960	0.88383	0.88234	0.020047	0.78007	0.93703
$\beta^{\kappa}_{16,2}$	0.88985	0.88467	0.88358	0.020139	0.78850	0.96055

Table 4: Parameter estimates of simulated DFM - GARCH(1,1) model for k=2 factors, p=16 series and N=1500 observations per series with corresponding median, the mean, the MC standard deviation of the estimates, the minimum and the maximum of M=1000 simulations respectively. These parameters are part of the GARCH(1,1) parameters in the observation equations.

Parameters	θ_{TRUE}	Median θ_{ML}	$\bar{\theta}_{ML}$	MC SD	Min	Max
$\beta^{\zeta}_{1,0}$	0.060000	0.060346	0.066291	0.026000	0.014485	0.27715
$\beta^{\zeta}_{2,0}$	0.055000	0.055399	0.060826	0.024418	0.016075	0.30923
$\beta^{\zeta}_{1,1}$	0.14895	0.17887	0.18188	0.036977	0.064193	0.42517
$\beta^{\zeta}_{2,1}$	0.14973	0.19097	0.19508	0.044748	0.050530	0.42928
$\beta^{\zeta}_{1,2}$	0.84105	0.80730	0.80402	0.038107	0.55360	0.91687
$\beta^{\zeta}_{2,2}$	0.84027	0.79440	0.79055	0.044862	0.56474	0.92187
c_1	0.041416	0.044085	0.043798	0.045608	-0.12862	0.18952
c_2	0.012197	0.014726	0.014819	0.045121	-0.17504	0.18167
a_1	0.75000	0.74587	0.74454	0.021228	0.65774	0.80222
a_2	0.65000	0.64526	0.64408	0.027855	0.53473	0.73160

Table 5: Parameter estimates of simulated DFM - GARCH(1,1) model for k=2 factors, p=16 series and N=1500 observations per series with corresponding median, the mean, the MC standard deviation of the estimates, the minimum and the maximum of M=1000 simulations respectively. These parameters define the factor process.

		N = 500		N = 1000		N = 1500	
Parameters	θ_{TRUE}	Median	SD	Median	SD	Median	SD
$\beta^{\zeta}_{1,0}$	0.083168	0.10410	0.12941	0.088041	0.065768	0.084388	0.048317
$\beta^{\zeta}_{1,1}$	0.14895	0.15816	0.055621	0.16168	0.036837	0.16210	0.030935
$\beta^{\zeta}_{1,2}$	0.84105	0.81937	0.066690	0.82101	0.042164	0.82286	0.034608
c_1	0.041416	0.035943	0.089824	0.041461	0.059387	0.042169	0.047840
a_1	0.75000	0.74252	0.034598	0.74568	0.025066	0.74568	0.020946
$b_{2,1}$	0.62537	0.62508	0.031837	0.62602	0.019432	0.62660	0.016114
$b_{3,1}$	0.70390	0.70433	0.034620	0.70347	0.022230	0.70451	0.017245
$b_{4,1}$	0.86035	0.86022	0.036496	0.86106	0.022323	0.86012	0.017306
$b_{5,1}$	0.98229	0.98121	0.041489	0.98244	0.026067	0.98219	0.020453
$b_{6,1}$	0.96050	0.96170	0.036317	0.96070	0.023702	0.96118	0.019138
$b_{7,1}$	0.76795	0.76897	0.033576	0.76912	0.021286	0.76775	0.017067
$b_{8,1}$	0.84139	0.83898	0.034746	0.84207	0.022384	0.84080	0.017163

Table 6: Parameter estimates of simulated DFM - GARCH(1,1) model for k=1 factors, p=8 series and for N = 500, 1000 and 1500 observations per series with corresponding median and MC standard deviation of the estimates for M=1000 simulations respectively.

Parameters	θ_{TRUE}	N = 500		N = 1000		N = 1500	
		Median	SD	Median	SD	Median	SD
$\beta^{\kappa}_{1,0}$	0.055193	0.070803	0.10230	0.057198	0.042046	0.056134	0.025580
$\beta^{\kappa}_{2,0}$	0.057069	0.075709	0.11907	0.061629	0.043779	0.058247	0.027781
$\beta^{\kappa}_{3,0}$	0.068946	0.096373	0.11454	0.074222	0.053885	0.070194	0.035164
$\beta^{\kappa}_{4,0}$	0.053802	0.072887	0.079951	0.056091	0.038000	0.054321	0.028825
$\beta^{\kappa}_{5,0}$	0.070872	0.095201	0.10573	0.077484	0.050691	0.072294	0.034474
$\beta^{\kappa}_{6,0}$	0.048267	0.060494	0.10995	0.049933	0.034662	0.048928	0.023528
$\beta^{\kappa}_{7,0}$	0.053197	0.070456	0.10149	0.057166	0.039752	0.054229	0.029613
$\beta^{\kappa}_{8,0}$	0.051727	0.066305	0.13403	0.055138	0.036989	0.052494	0.025748
$\beta^{\kappa}_{1,1}$	0.098308	0.10400	0.042625	0.10557	0.027445	0.10645	0.020355
$\beta^{\kappa}_{2,1}$	0.099698	0.099067	0.034825	0.097783	0.023909	0.10108	0.017989
$\beta^{\kappa}_{3,1}$	0.098810	0.098800	0.035548	0.098694	0.021728	0.099323	0.018440
$\beta^{\kappa}_{4,1}$	0.10050	0.10089	0.039010	0.10440	0.026506	0.10652	0.021425
$\beta^{\kappa}_{5,1}$	0.10007	0.10204	0.039882	0.10357	0.026196	0.10478	0.020390
$\beta^{\kappa}_{6,1}$	0.10123	0.10621	0.041288	0.10941	0.028988	0.11072	0.021957
$\beta^{\kappa}_{7,1}$	0.10061	0.10114	0.037024	0.10212	0.024756	0.10355	0.019727
$\beta^{\kappa}_{8,1}$	0.10233	0.10231	0.038866	0.10684	0.026385	0.10831	0.020002
$\beta^{\kappa}_{1,2}$	0.89169	0.87756	0.059075	0.88151	0.030172	0.88109	0.020725
$\beta^{\kappa}_{2,2}$	0.89030	0.88413	0.057528	0.88855	0.027973	0.88662	0.019217
$\beta^{\kappa}_{3,2}$	0.89119	0.88186	0.048389	0.88646	0.025806	0.88862	0.020547
$\beta^{\kappa}_{4,2}$	0.88950	0.87800	0.048430	0.88077	0.029113	0.88229	0.023095
$\beta^{\kappa}_{5,2}$	0.88993	0.87886	0.049166	0.88166	0.029072	0.88375	0.021962
$\beta^{\kappa}_{6,2}$	0.88877	0.87564	0.052559	0.87756	0.031640	0.87735	0.022910
$\beta^{\kappa}_{7,2}$	0.88939	0.87937	0.045713	0.88405	0.028613	0.88370	0.022041
$\beta^{\kappa}_{8,2}$	0.88767	0.87997	0.063702	0.87919	0.029747	0.87990	0.021769

Table 7: Parameter estimates of simulated DFM - GARCH(1,1) model for k=1 factors, p=8 series and for N = 500, 1000 and 1500 observations per series with corresponding median and MC standard deviation of the estimates for M=1000 simulations respectively.

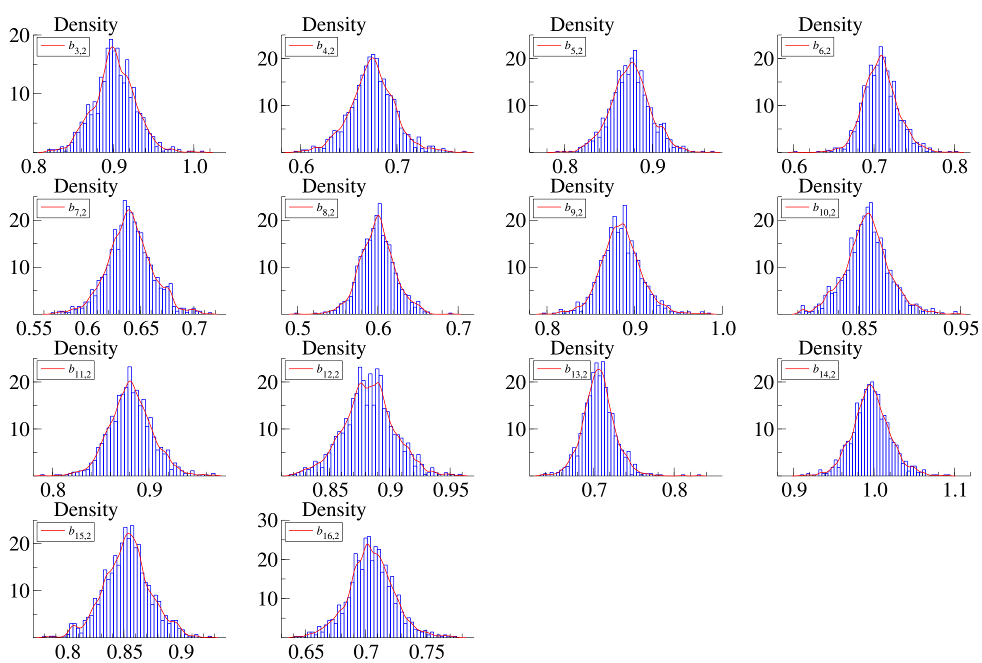

Figure 2: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters in the second column of the loading matrix B. See table 1 for the corresponding true parameter values.

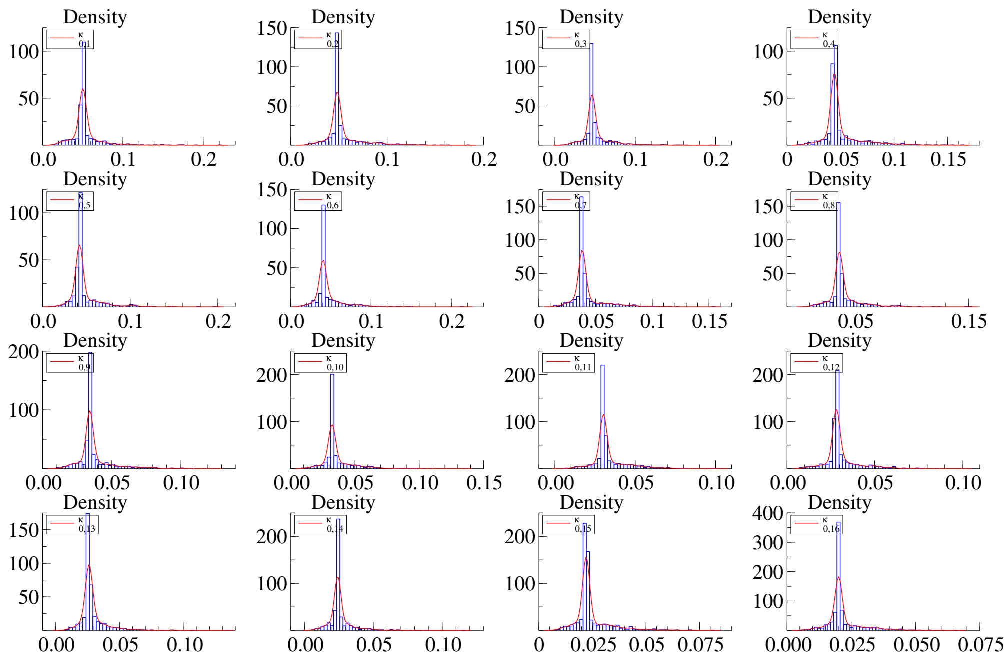

Figure 3: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters $\beta^{\kappa}_{0,i}$ of the GARCH(1,1) processes in the observation equations with $i \in 1, ..., p$. See table 2 for the corresponding true parameter values.

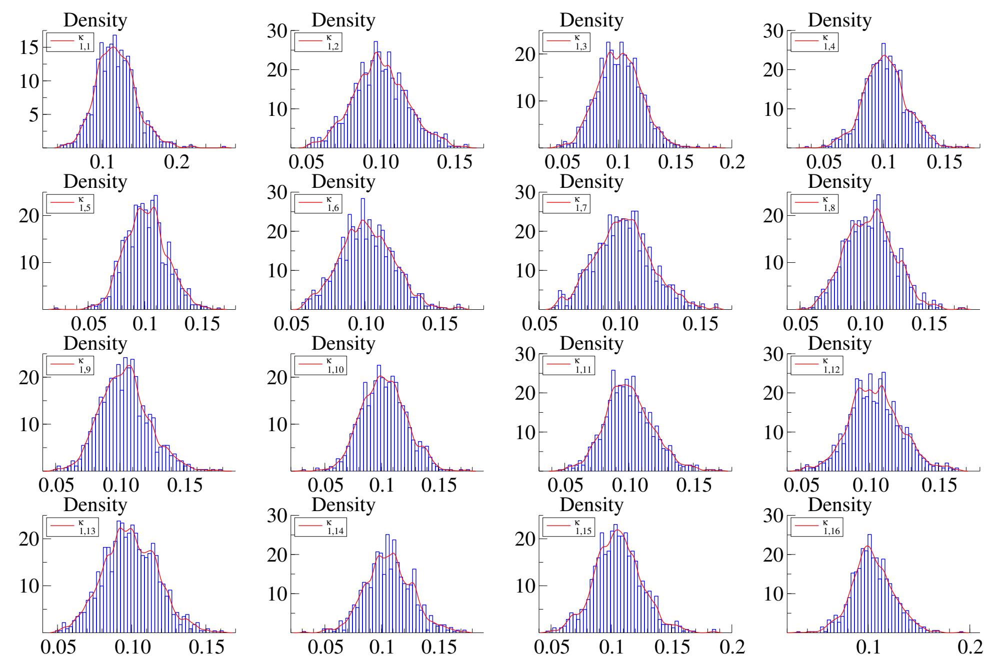

Figure 4: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters $\beta^{\kappa}_{1,i}$ of the GARCH(1,1) processes in the observation equations with $i \in 1, ..., p$. See table 3 for the corresponding true parameter values.

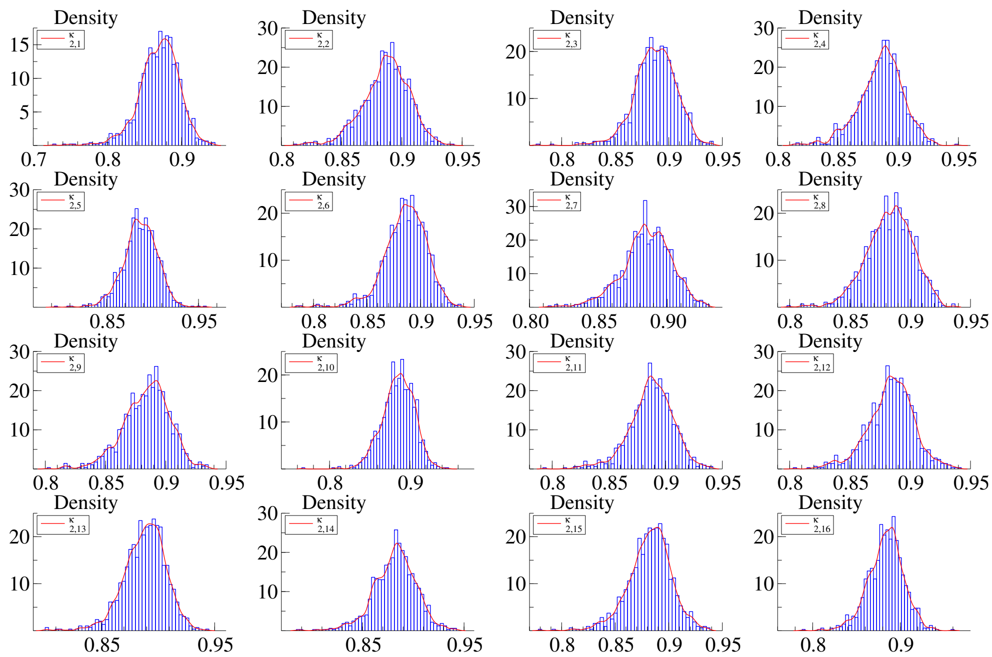

Figure 5: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters $\beta^{\kappa}_{2,i}$ of the GARCH(1,1) processes in the observation equations with $i \in 1, ..., p$. See table 4 for the corresponding true parameter values.

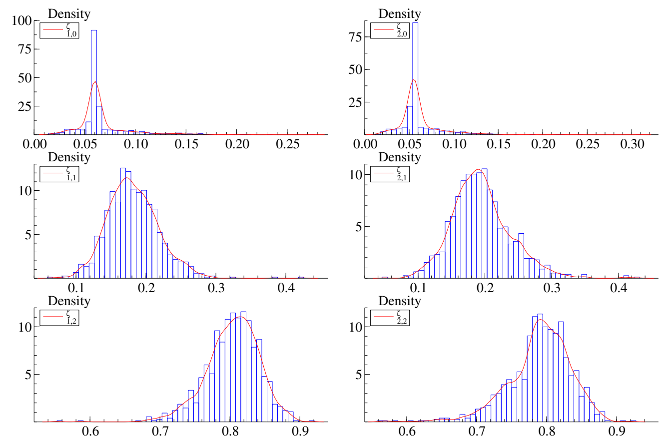

Figure 6: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters $\beta^{\zeta}_{i,j}$ of the GARCH(1,1) processes in the factor equations with $i \in 1, ..., k$ and $j \in 0, 1, 2$. See table 5 for the corresponding true parameter values.

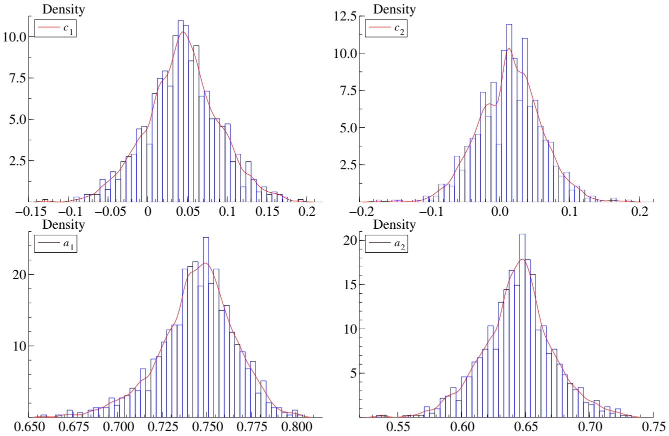

Figure 7: Data: Summary of estimated parameters for a 1000 samples for k=2 factors, p=16 series and N=1500 observations per series. These histograms correspond to the estimated parameters a_i of the AR(1) processes and the constant c_i in the factor equation with $i \in 1, ..., k$. See table 5 for the corresponding true parameter values.

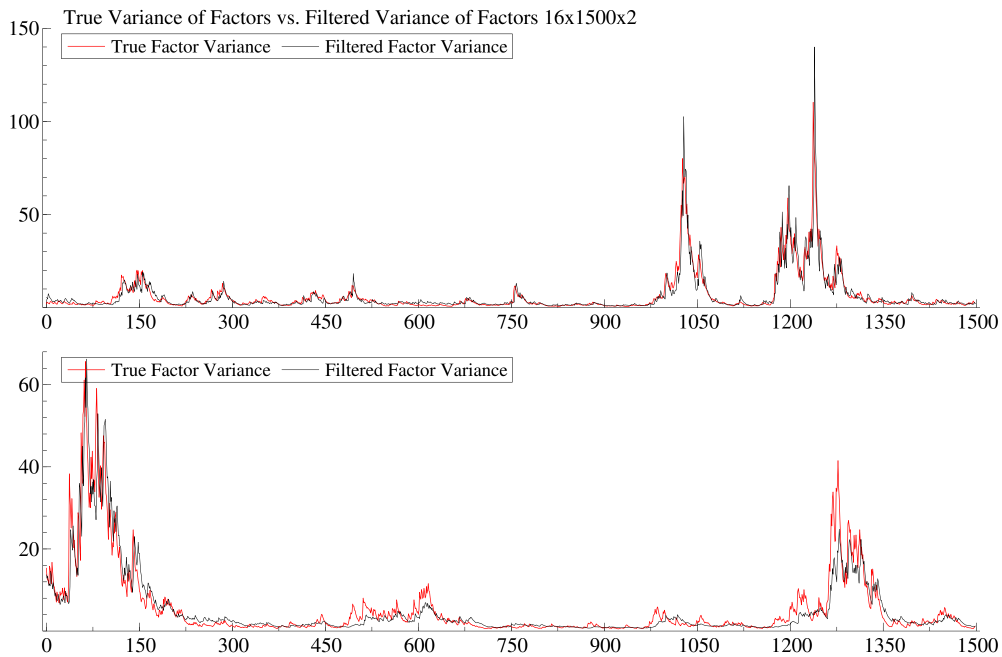

Figure 8: Data: Simulated GARCH(1,1)-process in Factors vs. Estimated GARCH(1,1)-process in Factors for k=2 factors, p=16 series and N=1500 observations per series.

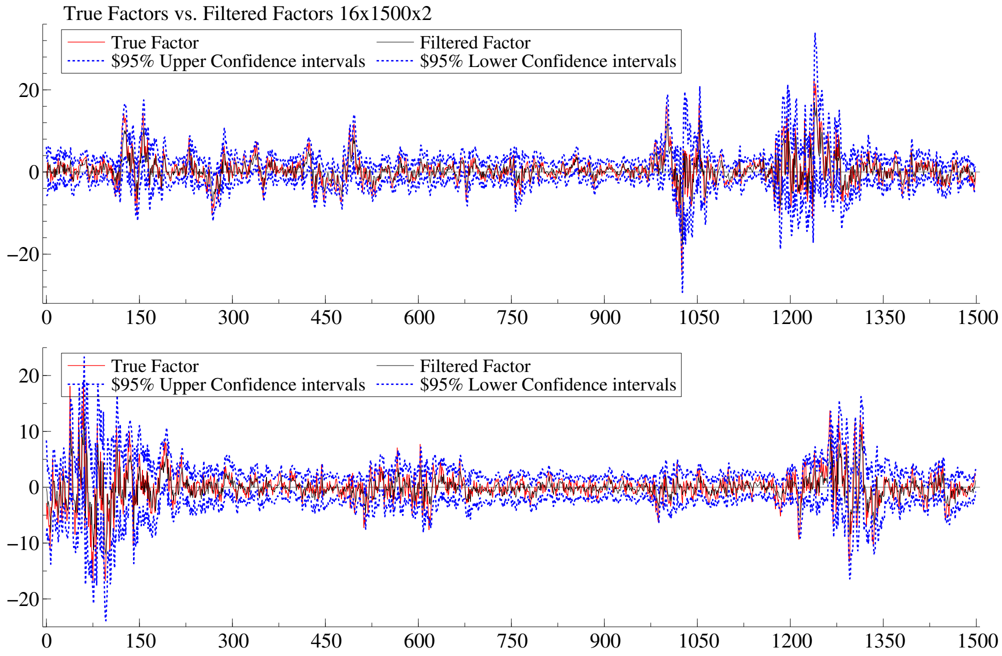

Figure 9: Data: Simulated Factors vs. Filtered Factors for k=2 factors, p=16 series and N=1500 observations per series.