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Note on Stable Mergers in a Market with Asymmetric Substitutability

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Abstract

This paper studies the stability of mergers between firms in a Cournot market. Unlike most existing works, we consider a demand structure where the substitutability between firms is asymmetric. We specifically focus on the stability of the grand coalition by analyzing the core allocation. The main result of our analysis shows that the grand coalition becomes stable, as the market is more asymmetric in terms of substitutability.

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1 Introduction

The profitability of horizontal mergers has received much attention in the industrial organization literature. The seminal paper in this line is Salant et al. (1983), who examine a merger between symmetric firms in a simple Cournot oligopoly. They show a somewhat unintuitive result that in the absence of cost synergy, a profitable merger requires the participation of many members in the industry. This is due to a free-riding effect, where in response to an adjustment in the output of the merged firms, the outsiders can expand their output. Since then, many researchers have addressed this issue by introducing relevant models. More recently, Currarini and Marini (2011) discuss the stability of the grand coalition in a symmetric Cournot game by analyzing the existence of the core allocation.

However, as mentioned above, most of these previous studies assume a symmetric market in terms of cost and demand structures. This is clearly due to the analytical tractability of the models employed. Therefore, very few papers model a merger between potentially asymmetric firms. One of them is Belleflamme (2000), who explores coalition-proof Nash equilibrium (CPNE) in a Cournot oligopoly where each player has an asymmetrically different preference for joining an association. Another work is Zhao (2013) who examines the stability of the grand coalition in a situation where three firms have different marginal costs and produce a homogeneous good in a Cournot fashion. Ebina and Shimizu (2009) also explore stable merger structures in a Cournot market consisting of four firms with asymmetric substitutability, provided that only a merger between two firms is allowed.

Following Currarini and Marini (2011) and Zhao (2013), we also analyze the stability of the grand coalition in a Cournot oligopoly by investigating the core allocation. However, like Ebina and Shimizu (2009), we allow the substitutability between firms to be asymmetric. In order to study the monopoly merger in an oligopoly, many papers employ an approach where a normal form game is firstly converted to a partition function game (Thrall and Lucas 1963) by finding a quasi-hybrid solution (Zhao 1991), and then an appropriate core concept is examined. With regard to the latter, we note that although α-core and β-core are well known (Aumann 1959), some refinements including the two recent concepts e-core (Yong 2004) and j-belief core (Lekeas 2013) have been developed. Among them, as in Currarini and Marini (2011) and Zhao (2013), we consider γ-core and δ-core independently. That is, we focus on the situation where no member has an incentive to deviate from the grand coalition, provided that: (i) in response to the individual/coalitional deviation, the remaining members break up into singletons (γ-stable, also see Chander and Tulkens 1997, Rajan 1989, Lardon 2012), and (ii) the remaining members stay together and still cooperate (δ-stable, also see Hart and Kurz 1983, Rajan 1989).

Although we confine our analysis to the case of at most four firms due to analytical tractability, we make some new findings. The grand coalition is necessarily γ-stable even in a market with asymmetric substitutability. In contrast, unlike the negative result for symmetric cases presented by Currarini and Marini (2011) that the grand coalition cannot be δ-stable, we show a positive result that the grand coalition becomes stable, as the market becomes more asymmetric in terms of substitutability.

2 Model

Let $N = \{1, 2, \ldots, n\}$ be the set of firms that produce differentiated goods, where the output and price of firm i's product are denoted by q_i and p_i ($i \in N$), respectively. We define $\boldsymbol{q} = (q_1, \ldots, q_n)$ and $\boldsymbol{p} = (p_1, \ldots, p_n)$ as vectors of output and price, respectively. Let A be an $n \times n$ matrix with its elements, given as (a_{ij}), representing the degree of substitutability between firms i ($i \in N$) and j ($j \in N, i \neq j$). With this matrix, our inverse demand function is given by

$$\boldsymbol{p} = \mathbf{1} - A\boldsymbol{q},$$

where $\mathbf{1}$ is an n-dimensional vector with all elements as 1.

In this study, we consider only the cases of $n = 3$ and $n = 4$ with the following specific forms of substitutability matrices.

For $n = 3$, we consider the following:

$$A = \begin{pmatrix} 1 & \theta & \theta \\ \theta & 1 & \rho \\ \theta & \rho & 1 \end{pmatrix}.$$

For $n = 4$, we consider the following three cases:

$$Case1 : A = \begin{pmatrix} 1 & \theta & \theta & \theta \\ \theta & 1 & \rho & \rho \\ \theta & \rho & 1 & \rho \\ \theta & \rho & \rho & 1 \end{pmatrix}, Case2 : A = \begin{pmatrix} 1 & \rho & \theta & \theta \\ \rho & 1 & \theta & \theta \\ \theta & \theta & 1 & \rho \\ \theta & \theta & \rho & 1 \end{pmatrix}, Case3 : A = \begin{pmatrix} 1 & \theta & \theta & \theta \\ \theta & 1 & \theta & \theta \\ \theta & \theta & 1 & \rho \\ \theta & \theta & \rho & 1 \end{pmatrix}.$$

We assume that $0 \leq \theta \leq \rho \leq 1$ for ensuring relational consistency in the degree of substitutability[1]. This implies that the market is organized by some groups each consisting of symmetric firms, where the degree of substitutability between groups is θ whereas that between firms within a group is ρ. We note that these four are the only possible cases as such type of asymmetric market structure with three or four firms. Without loss of generality, we normalize constant marginal costs to be zero. Therefore, the profit of firm i is given by $\pi_i = p_i(q_1, \ldots, q_n)q_i$. Each firm i chooses its output level q_i and thus engages in Cournot competition.

Throughout the paper, we assume that the Cournot equilibrium is obtained as an interior solution: all outputs are positive in the equilibrium. The feasible range of parameters (θ, ρ) is further constrained by this assumption.

[1]For example, if $a_{12} = a_{13} = 1$ for $n = 3$, that is, if products 1 and 2 and products 2 and 3 are identical, then $a_{23} = 1$.

We describe the structures of our games. We define coalitional form games $G_\gamma = \{N, v^\gamma\}$ and $G_\delta = \{N, v^\delta\}$. The value of coalition for each game is specifically given as follows.

First, in game G_γ, we assume that for an individual/coalitional deviation, the remaining members are assumed to break up into singletons and maximize their individual profit. Namely, for any $S \subset N$, v^γ_S is defined as the total profit earned by S in the equilibrium of the following Cournot game:

$$\max_{q_i,\ i\in S} \pi_S = \sum_{i\in S} q_i p_i(q_1, \ldots, q_n),\ and\ \max_{q_i} \pi_i = q_i p_i(q_1, \ldots, q_n),\ i \in N - S.$$

In contrast, in game G_δ, we assume that for an individual/coalitional deviation, the remaining members remain loyal to each other and continue to cooperate. Formally, v^δ_S is defined as the total profit earned by S in the equilibrium of the following two-person game between S and $N - S$:

$$\max_{q_i,\ i\in S} \pi_S = \sum_{i\in S} q_i p_i(q_1, \ldots, q_n)\ and\ \max_{q_i,\ i\in N-S} \pi_{N-S} = \sum_{i\in N-S} q_i p_i(q_1, \ldots, q_n).$$

Finally, since we clearly have $v^\gamma_N = v^\delta_N$, we denote the value of the grand coalition by v_N.

In this study, we investigate the existence of the core as the set satisfying Pareto efficiency and group rationality. To do this, following Zhao (2001, 2013), we deem that it suffices to compute the minimum no-blocking payoff $MNBP$ given as

$$MNBP_t = \begin{cases} Min\ \ \sum x_i \\ subject\ to\ x \in R^n_+\ ;\ \sum_{i\in S} x_i \geq v^t_S\ \ for\ all\ \ S \subset N, \end{cases}$$

where $t = \gamma, \delta$. It is clear that $Core(G_t) \neq \emptyset$ is equivalent to $v_N \geq MNBP_t$, where $Core(G_t)$ is the core of game $G_t (t = \gamma, \delta)$. Interchangeably, we refer to a game as *t-stable*, if $Core(G_t) \neq \emptyset$.

3 Equilibrium and Analysis

This section is devoted to our results. The detailed proofs of both the propositions and the derivations of the equations are given in the Appendix.

3.1 Stability in the Case of Three Firms

The values of coalitions can specifically be derived as follows (we omit superscripts for cases where two firms are merged, since clearly, the values of the coalitions are identical for both games G_γ and G_δ):

$$v_N = \frac{3+\rho-4\theta}{4(1+\rho-2\theta^2)}, v^\gamma_1 = \frac{(2+\rho-2\theta)^2}{4(2+\rho-\theta^2)^2}, v^\gamma_2 = v^\gamma_3 = \frac{(2-\theta)^2}{4(2+\rho-\theta^2)^2}, v_{23} = \frac{(2-\theta)^2(1+\rho)}{2(2+2\rho-\theta^2)^2},$$
$$v_{12} = v_{13} = \frac{(2-\rho)^2(2+\rho-3\theta)^2+2\theta(2-\theta)(2-\rho)(2+\rho-3\theta)(2-\theta-\rho)+(2-\theta)^2(2-\theta-\rho)^2}{4(4+2\rho\theta^2-\rho^2-5\theta^2)^2},$$
$$v^\delta_1 = \frac{(1+\rho-\theta)^2}{(2+2\rho-\theta^2)^2}, v^\delta_3 = v^\delta_2 = \frac{(1-\theta)^2(2+\theta-\rho)^2}{(4+2\rho\theta^2-\rho^2-5\theta^2)^2}.$$

For these values of the coalitions, we compute $MNBP_t(t = \rho, \delta)$ and find the parameter region (θ, ρ) that satisfies $v_N \geq MNBP_t$.

We first derive the result for stability in game G_γ.

Proposition 3.1 *When $n = 3$, game G_γ always has the core.*

Proof Let $d_{12} \equiv v_{12} - v_1^\gamma - v_2^\gamma$ and $d_{23} \equiv v_{23} - v_2^\gamma - v_3^\gamma$, respectively. Then, we can directly show that depending on these values, $MNBP_\gamma$ is specifically given as in Table 1. Note that for the case of $d_{12} \geq 0$ and $d_{23} \geq 0$, $MNBP_\gamma$ is determined depending on the relation between $v_1^\gamma + \frac{v_{23}}{2}$ and v_{12}. Further, note that under our feasible region of parameters, $d_{12} \geq 0$ and $d_{23} \leq 0$ do not coincide.

Table 1: $MNBP_\gamma$

	$d_{12} \geq 0$	$d_{12} \leq 0$
$d_{23} \geq 0$	$v_1^\gamma + v_{23}$ or $v_{12} + \frac{v_{23}}{2}$	$v_1^\gamma + v_{23}$
$d_{23} \leq 0$	–	$v_1^\gamma + 2v_2^\gamma$

For all the three cases in Table 1, by direct calculations, we can show that $v_N - MNBP_\gamma \geq 0$ necessarily holds. **Q.E.D.**

Proposition 3.1 shows that γ-stability is guaranteed even in cases where the substitutability among the three firms is asymmetric. In game G_γ, outsiders' reaction led to the breaking up of the remaining members into singletons, and consequently, a deviation from the grand coalition yields intense competition. The proposition proves that this negative impact always makes any deviation unattractive, regardless of the level of substitutability.

We next analyze the stability in game G_δ. We start the analysis by stating the following lemma.

Lemma 3.1 *When $n = 3$, $Core(G_\delta) \neq \emptyset$ if and only if $v_N \geq v_1^\delta + 2v_2^\delta$.*

Proof $v_1^\delta + v_2^\delta (= v_1^\delta + v_3^\delta) \geq v_{12} (= v_{13})$ always holds if $\rho \geq \theta$. Therefore, since $v_2^\delta = v_3^\delta$, it is sufficient to consider the two cases where $MNBP_\delta$ are attained as $v_1^\delta + v_{23}$ and $v_1^\delta + 2v_2^\delta$, respectively. However, under $\rho \geq \theta$, we always have $v_N \geq v_1^\delta + v_{23}$. Therefore, the existence of the core is determined by the relation between v_N and $v_1^\delta + 2v_2^\delta$. **Q.E.D.**

From Lemma 3.1, we can find the parameter region of θ and ρ where the core is nonempty. However, instead, we now introduce the mean and variance of a_{ij} over $i, j = 1, 2, 3,\ i < j$, which indicate the overall degree of substitutability in the market, and display the region where the core is nonempty in the mean-variance space. Let μ be the mean and σ^2 be the variance. These are explicitly calculated as follows:

$$\mu = \frac{2\theta + \rho}{3}, \sigma^2 = \frac{2(\rho - \theta)^2}{9}.$$

From some algebra, we can verify that the shaded area in Figure 1 is the region where the core is nonempty. Therefore, we immediately obtain the following proposition:

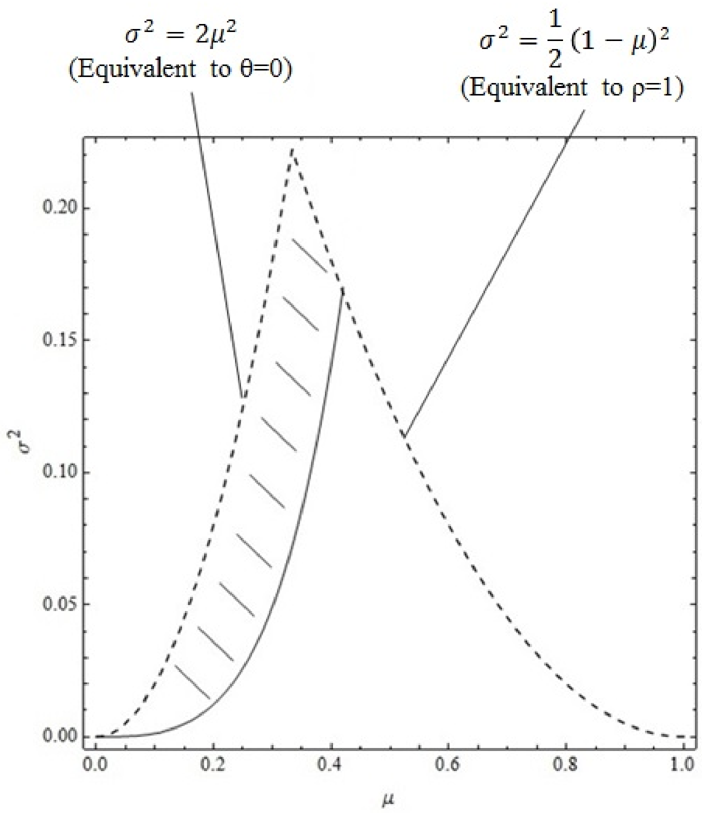

Figure 1: δ-stable region in Case 1.

Proposition 3.2 *When $n = 3$, the core of game G_δ is nonempty if and only if the average of the degree of substitutability is low and its variance is high; that is, the market has highly asymmetric substitutability.*

In game G_δ, for the deviation by S, members in $N - S$ are assumed to continue cooperation. Therefore, the deviation by coalition S is not basically beneficial, because following the deviation, S faces intense competition with $N - S$. In contrast, however, members in $N - S$ are compelled to adjust their output in order to keep higher prices in response to the deviation by S. This in turn enables S to expand its output, resulting in the equilibrium output of $N - S$ decreasing. Because of this free-riding effect, firms are likely to have an incentive to deviate from the grand coalition. In fact, as mentioned in the Introduction, this effect is so strong that the core is always empty in cases of symmetric substitutability. However, if θ and ρ are lower and the difference between them is larger, then firm 1 does not benefit from free riding on the coalition formed by firms 2 and 3. This is because firm 1 is highly differentiated from the other firms, and in addition, the output adjustment by firms 2 and 3 after firm 1's deviation from the grand coalition becomes minimal as they are less differentiated. Therefore, individual deviation by firm 1, and coalitional deviation by firms 2 and 3, are not beneficial. In addition, for example, coalitional deviation by firms 1 and 2, and individual deviation by firm 2, are also not beneficial, because firm 2 is less differentiated from firm 3, and thus will face severe competition from firm 3. Therefore, in such a situation, the grand coalition becomes stable.

3.2 Stability in the Three Cases of Four Firms

We next show the results for the three cases of four firms. Unfortunately, some of the analyses are analytically intractable. Therefore, we provide an analytical result only for Case 1, while numerically showing the results for Cases 2 and 3.

Case 1

In Case 1, we can obtain the following two propositions with regard to γ- and δ-stability, which are quite similar to those for the case of three firms.

Proposition 3.3 *In Case 1, game G_γ always has the core.*

Proof Regarding the members who deviate from the grand coalition, we deem that it suffices to consider the following three cases: (a) The coalition formed by $k(1 \leq k \leq 3)$ firms in $N_1 = \{2,3,4\}$ deviates. Let $\pi^\gamma_{2,a,k}$ be the individual profit earned by firm 2 (and by firms 3 and 4) in the equilibrium after the deviation. (b) The coalition formed by firm 1 and $h(1 \leq h \leq 2)$ firms in $N_1 = \{2,3,4\}$ deviates. Let $\pi^\gamma_{1,b,h}$ and $\pi^\gamma_{2,b,h}$ be the individual profit earned by firm 1 and firm 2 (and firms 3 and 4) in the equilibrium after the deviation, respectively. (c) Only firm 1 deviates. Let $\pi^\gamma_{1,c}$ be firm 1's profit in the equilibrium after the deviation.

In contrast, let $\pi^N = \{\pi^N_1, \pi^N_2, \pi^N_3, \pi^N_4\}(v^N = \sum_{i=1}^4 \pi^N_i)$ be the individual profit earned by each firm under the grand coalition. Then, the profit values can specifically be obtained as follows:

$$\pi^N_1 = \frac{-2\rho+3\theta-1}{4(-3\theta^2+2\rho+1)}, \pi^N_2 = \pi^N_3 = \pi^N_4 = \frac{1-\theta}{4(-3\theta^2+2\rho+1)}, \pi^\gamma_{1,c} = \frac{(2\rho-3\theta+2)^2}{(4\rho-3\theta^2+4)^2},$$
$$\pi^\gamma_{2,a,1} = \frac{(2-\theta)^2}{(4\rho-3\theta^2+4)^2}, \pi^\gamma_{2,a,2} = \frac{(2-\theta)^2(2-\rho)^2(\rho+1)}{2(-2\rho^2+\theta^2\rho+4\rho-3\theta^2+4)^2}, \pi^\gamma_{2,a,3} = \frac{3(2-\theta)^2(2\rho+1)}{(8\rho-3\theta^2+4)^2},$$

$$\pi^\gamma_{1,b,1} = \frac{(2-\rho)(\rho-2\theta+1)\left(-\rho^2+\theta\rho+\rho+\theta^3-2\theta^2-2\theta+2\right)}{4(-\rho^2+\theta^2\rho+\rho-3\theta^2+2)^2}, \pi^\gamma_{1,b,2} = \frac{\left(-\rho^2+2\theta\rho+2\rho-5\theta+2\right)\left(-\rho^2+\theta^2\rho+2\rho+\theta^3-4\theta^2-\theta+2\right)}{(-2\rho^2+3\theta^2\rho+4\rho-9\theta^2+4)^2},$$
$$\pi^\gamma_{2,b,1} = \frac{\left(-\rho+\theta^2-2\theta+2\right)\left(-\theta\rho^2+2\theta^2\rho+\theta\rho-\rho-3\theta^2+2\right)}{4(-\rho^2+\theta^2\rho+\rho-3\theta^2+2)^2}, \pi^\gamma_{2,b,2} = \frac{(2-\theta)(-\rho-\theta+2)\left(-\theta\rho^2-2\rho^2+5\theta^2\rho+\theta\rho+2\rho-9\theta^2+4\right)}{4(-2\rho^2+3\theta^2\rho+4\rho-9\theta^2+4)^2}.$$

With these values, we can directly show the following relations for each case:
(a) $\pi^N_2 \geq \pi^\gamma_{2,a,1}$ and $\pi^N_2 \geq \pi^\gamma_{2,a,3} \geq \pi^\gamma_{2,a,2}$ hold.
(b) $\pi^N_1 \geq \pi^\gamma_{1,b,1}$ and $\pi^N_1 \geq \pi^\gamma_{1,b,2}$ hold. Further, $\pi^N_2 \geq \pi^\gamma_{2,b,1}$ and $\pi^N_2 \geq \pi^\gamma_{2,b,2}$ hold.
(c) $\pi^N_1 \geq \pi^\gamma_{1,c}$ holds.
These relations ensure that π^N is a core allocation. **Q.E.D.**

As in the case of three firms, the grand-coalition is γ-stable, since any deviating coalition faces intense competition from the remaining singletons. We next investigate the δ-stability of the grand-coalition. Unfortunately, we immediately have a negative result as follows.

Proposition 3.4 *In Case 1, the core of game G_δ is always empty.*

Proof We consider the three relations (a)–(c) obtained in the proof of Proposition 3.3. Further, we define the profit earned by each member in the equilibrium after the deviation in the same manner as in the proof of Proposition 3.3, except for replacing the superscript γ with δ. Then, the profit values are specifically given as follows:

$$\pi_{1,c}^{\delta} = \frac{(4\,\rho-3\,\theta+2)^2}{(8\,\rho-3\,\theta^2+4)^2},\ \pi_{2,a,1}^{\delta} = \frac{\left(\theta\,\rho-2\,\theta^2-\theta+2\right)^2}{(-2\,\rho^2+3\,\theta^2\,\rho+4\,\rho-9\,\theta^2+4)^2},\ \pi_{2,a,2}^{\delta} = \frac{(1-\theta)^2\,(-\rho+\theta+2)^2\,(\rho+1)}{4\,(-\rho^2+2\,\rho-3\,\theta^2+2)^2},$$
$$\pi_{2,a,3}^{\delta} = \frac{(2-\theta)^2\,(2\,\rho+1)}{(8\,\rho-3\,\theta^2+4)^2},\ \pi_{1,b,1}^{\delta} = \frac{\left(-\rho^2+\theta\,\rho+2\,\rho-4\,\theta+2\right)\left(-\rho^2-\theta^2\,\rho+\theta\,\rho+2\,\rho+\theta^3-2\,\theta^2-2\,\theta+2\right)}{4\,(-\rho^2+2\,\rho-3\,\theta^2+2)^2},$$

$$\pi_{1,b,2}^{\delta} = \frac{\left(-\rho^2+2\,\theta\,\rho+2\,\rho-5\,\theta+2\right)\left(-\rho^2+\theta^2\,\rho+2\,\rho+\theta^3-4\,\theta^2-\theta+2\right)}{(-2\,\rho^2+3\,\theta^2\,\rho+4\,\rho-9\,\theta^2+4)^2},\ \pi_{2,b,1}^{\delta} = \frac{\left(-\theta\,\rho+\theta^2-2\,\theta+2\right)\left(-\theta\,\rho^2+\theta^2\,\rho+\theta\,\rho-3\,\theta^2+2\right)}{4\,(-\rho^2+2\,\rho-3\,\theta^2+2)^2},$$

$$\pi_{2,b,2}^{\delta} = \frac{(2-\theta)\,(-\rho-\theta+2)\left(-\theta\,\rho^2-2\,\rho^2+5\,\theta^2\,\rho+\theta\,\rho+2\,\rho-9\,\theta^2+4\right)}{4\,(-2\,\rho^2+3\,\theta^2\,\rho+4\,\rho-9\,\theta^2+4)^2}.$$

With these values, we have $\pi_{2,a,1}^{\delta} \geq \pi_{2,a,2}^{\delta}$, $\pi_{2,a,1}^{\delta} \geq \pi_{2,a,3}^{\delta}$, $\pi_{2,a,1}^{\delta} \geq \pi_{2,b,1}^{\delta} \geq \pi_{2,b,2}^{\delta}$, and $\pi_{1,c}^{\delta} \geq \pi_{1,b,1}^{\delta}$. This indeed implies that $MNBP_\delta$ must be $MNBP_\delta = \pi_{1,c}^{\delta} + 3\pi_{2,a,1}^{\delta}$ or $MNBP_\delta = \pi_{1,b,2}^{\delta} + 3\pi_{2,a,1}^{\delta}$. However, we can directly verify that $v_N - (\pi_{1,c}^{\delta} + 3\pi_{2,a,1}^{\delta}) \leq 0$ and $v_N - (\pi_{1,b,2}^{\delta} + 3\pi_{2,a,1}^{\delta}) \leq 0$, which completes the proof. **Q.E.D.**

The intuition behind the result of Proposition 3.4 is that because three firms (firms 2–4) are symmetric in Case 1, an individual deviation by one of them can always enjoy free-riding as in the case of $n = 3$ with symmetric firms.

Cases 2 and 3

We now conduct a numerical simulation to examine the existence of the core by computing $MNBP_t$ $(t = \gamma, \delta)$ and comparing it with v_N under a feasible parameter region of θ and ρ. We change these parameters from 0 to 1 in steps of 0.01, satisfying $\theta \leq \rho$ and the condition for ensuring the interior solution. As a result, we first find that the core of game G_γ is nonempty for all parameter values examined for both cases 2 and 3. We next show the result with regard to δ-stability. To do this, we compute both the mean and variance of a_{ij} over $i = 1, 2, 3, 4,\ i < j$, as in the case of three firms. The mean μ and variance σ^2 for Case 2 are explicitly given as follows:

$$\mu = \frac{2\theta + \rho}{3}, \sigma^2 = \frac{2(\rho - \theta)^2}{9}.$$

For Case 3, these are derived as follows:

$$\mu = \frac{5\theta + \rho}{6}, \sigma^2 = \frac{5(\rho - \theta)^2}{36}.$$

The shaded areas in Figures 2 and 3 display the regions in $\mu - \sigma^2$ space where $v_N \geq MNBP_\delta$ holds for Cases 2 and 3, respectively. Indeed, we can verify that as stated in Proposition 3.2, as the market becomes more asymmetric with regard to substitutability, the grand coalition becomes δ-stable. Unlike in Case 1, for both cases, the number of symmetric firms within a group is at most two, which implies that no firm benefits much from a deviation and free-riding on the remaining coalition. In particular, the market structure in Case 2 can mostly avoid this free-riding problem, because it is just split into two groups each consisting of two firms.

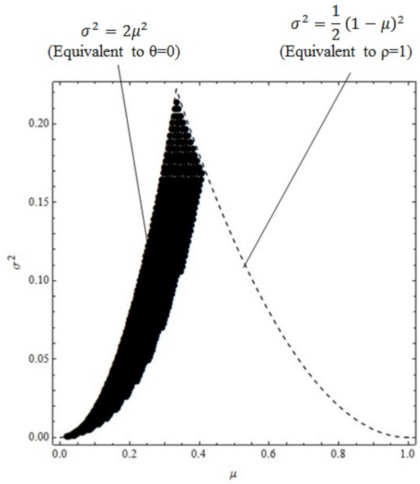

Figure 2: δ-stable region in Case 2

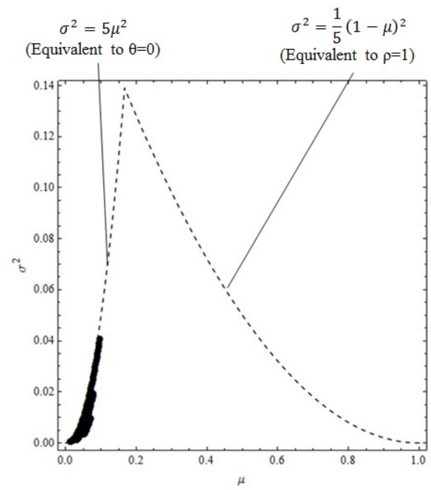

Figure 3: δ-stable region in Case 3

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