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Relative profit maximization in asymmetric oligopoly

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Abstract

We analyze Bertrand and Cournot equilibria in an asymmetric oligopoly in which the firms produce differentiated substitutable goods and seek to maximize their relative profits instead of their absolute profits. Assuming linear demand functions and constant marginal costs we show the following results. If the marginal cost of a firm is lower (higher) than the average marginal cost over the industry, its output at the Bertrand equilibrium is larger (smaller) than that at the Cournot equilibrium, and the price of its good at the Bertrand equilibrium is lower (higher) than that at the Cournot equilibrium.

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1. Introduction

We analyze Bertrand and Cournot equilibria in an asymmetric oligopoly in which the firms produce differentiated substitutable goods and seek to maximize their relative profits instead of their absolute profits. Firms in an industry not only seek to improve their own performance but also want to outperform the rival firms. TV audience-rating race and market share competition by breweries, automobile manufacturers, convenience store chains and mobile-phone carriers, especially in Japan, are examples of such behavior of firms[1].

In the next section we present the model, in Section 3 and 4 we investigate the outputs and prices at Bertrand and Cournot equilibria, and in Section 5 we compare Bertrand and Cournot equilibria. In Section 6 we mention some related results in other works.

2. The model

There are $n \geqq 2$ firms. They produce differentiated substitutable goods. The output and the price of the good of Firm i are denoted by x_i and p_i. The inverse demand functions of the goods are

$$p_i = a - x_i - b \sum_{j=1, j \neq i}^{n} x_j,\ i = 1, 2, \dots, n, \tag{1}$$

We assume $a > 0$ and $0 < b < 1$. From (1) we obtain the following ordinary demand functions (See Appendix 1).

$$\begin{aligned} x_i = &\frac{1}{(1-b)[1+(n-1)b]}\Bigg[(1-b)a - [1+(n-2)b]p_i \\ &+ b \sum_{j=1, j \neq i}^{n} p_j\Bigg],\ i = 1, 2, \dots, n. \end{aligned} \tag{2}$$

The inverse and ordinary demand functions are symmetric for the firms.

3. Cournot equilibrium under relative profit maximization

[1]For analyses about relative profit maximization please see Schaffer (1989), Gibbons and Murphy (1990), Lu (2011) and Matsumura et. al. (2013).

In this section we assume that each firm determines its output given the outputs of other firms so as to maximize its relative profit. Let denote the absolute profit of Firm i by π_i. Then,

$$\pi_i = \left(a - x_i - b\sum_{j=1,j\neq i}^{n} x_j\right)x_i - c_i x_i,\ i = 1,2,\dots,n.$$

The relative profit of Firm i is defined as the difference between its absolute profit and the average of the absolute profits of other firms. Denote it by Π_i. Then,

$$\Pi_i = \left(a - x_i - b\sum_{j=1,j\neq i}^{n} x_j\right)x_i - c_i x_i$$
$$- \frac{1}{n-1}\sum_{j=1,j\neq i}^{n}\left[\left(a - x_j - b\sum_{k=1,k\neq j}^{n} x_k\right)x_j - c_j x_j\right],\ i = 1,2,\dots,n.$$

Differentiating Π_i with respect to x_i for each i, the conditions for relative profit maximization are obtained as follows.

$$a - 2x_i - c_i - \frac{(n-2)b}{n-1}\sum_{j=1,j\neq i}^{n} x_j = 0,\ i = 1,2,\dots,n. \tag{3}$$

From this, we have

$$x_i = \frac{n-1}{2(n-1)-(n-2)b}(a - c_i) - \frac{(n-2)b}{2(n-1)-(n-2)b}\sum_{j=1}^{n} x_j,$$

and

$$na - 2\sum_{j=1}^{n} x_i - \sum_{j=1}^{n} c_i - (n-2)b\sum_{j=1}^{n} x_i = 0.$$

The latter equation means

$$\sum_{j=1}^{n} x_i = \frac{1}{2+(n-2)b}\left(na - \sum_{j=1}^{n} c_i\right).$$

Then, we get the equilibrium output of Firm i as follows.

$$
\begin{aligned}
x_i^C =& \frac{n-1}{2(n-1)-(n-2)b}(a-c_i) \\
&- \frac{(n-2)b}{[2(n-1)-(n-2)b][2+(n-2)b]}\left(na-\sum_{j=1}^{n} c_i\right),\ i=1,2,\dots,n.
\end{aligned}
$$

C indicates *Cournot*. The equilibrium price of the good of Firm i is

$$
\begin{aligned}
p_i^C =& a - x_i^C - b\sum_{j=1,j\neq i}^{n} x_j^C = \frac{n-1+b}{2(n-1)-(n-2)b}(a-c_i) \\
&- \frac{nb}{[2(n-1)-(n-2)b][2+(n-2)b]}\left(na-\sum_{j=1}^{n} c_j\right)+c_i,\ i=1,2,\dots,n.
\end{aligned}
$$

4. Bertrand equilibrium under relative profit maximization

In this section each firm determines the price of its good given the prices of the goods of other firms so as to maximize its relative profit. The absolute profit of Firm i is written as

$$
\pi_i = \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a-[1+(n-2)b]p_i+b\sum_{j=1,j\neq i}^{n} p_j\right](p_i-c_i).
$$

The relative profit of Firm i is

$$
\begin{aligned}
\Pi_i =& \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a-[1+(n-2)b]p_i+b\sum_{j=1,j\neq i}^{n} p_j\right](p_i-c_i) \\
&- \frac{1}{(1-b)(n-1)[1+(n-1)b]}\sum_{j=1,j\neq i}^{n}\Bigg\{(1-b)a-[1+(n-2)b]p_j \\
&+ b\sum_{k=1,k\neq j}^{n} p_k\Bigg\}(p_j-c_j).
\end{aligned}
$$

Differentiating Π_i with respect to p_i, the conditions for relative profit maximization are obtained as follows.

$$(1-b)a - 2[1+(n-2)b]p_i + b\sum_{j=1,j\neq i}^{n} p_j + [1+(n-2)b]c_i \tag{4}$$

$$-\frac{b}{n-1}\sum_{j=1,j\neq i}^{n}(p_j - c_j) = 0,\ i = 1, 2, \ldots, n.$$

Then, we get the equilibrium price of the good of Firm i as follows (See Appendix 2).

$$p_i^B = \frac{(n-1)[1+(n-1)b]}{2(n-1)+(n-2)(2n-1)b}(a-c_i)$$
$$-\frac{nb[1+(n-2)b]}{[2(n-1)+(n-2)(2n-1)b][2+(n-2)b]}\left(na - \sum_{j=1}^{n} c_j\right) + c_i,$$
$$i = 1, 2, \ldots, n.$$

B indicates *Bertrand*. The equilibrium output of Firm i is

$$x_i^B = \frac{1}{(1-b)[1+(n-1)b]}\Bigg\{[1+(n-1)b](a-c_i) - [1+(n-1)b](p_i - c_i)$$
$$-b\left(na - \sum_{j=1}^{n} c_j\right) + b\sum_{j=1}^{n}(p_j - c_j)\Bigg\}$$
$$= \frac{[n-1+(n^2-3n+1)b]}{(1-b)[2(n-1)+(n-2)(2n-1)b]}(a-c_i)$$
$$-\frac{(n-2)[1+(n-1)b]b}{(1-b)[2(n-1)+(n-2)(2n-1)b][2+(n-2)b]}\left(na - \sum_{j=1}^{n} c_j\right),$$
$$i = 1, 2, \ldots, n.$$

5. Comparison of Cournot and Bertrand equilibria

Let us compare the outputs and prices at the Bertrand equilibrium and those at the Cournot equilibrium. Comparing the output of Firm i at the Bertrand equilibrium and that at the Cournot equilibrium,

$$x_i^B - x_i^C = \frac{n(n-2)b^2(\sum_{j=1}^n c_j - nc_i)}{(1-b)[2(n-1)+(n-2)(2n-1)b][2(n-1)-(n-2)b]}. \quad (5)$$

Comparing the price of the good of Firm i at the Bertrand equilibrium and that at the Cournot equilibrium,

$$p_i^B - p_i^C = \frac{n(n-2)b^2(nc_i - \sum_{j=1}^n c_j)}{[2(n-1)+(n-2)(2n-1)b][2(n-1)-(n-2)b]}. \quad (6)$$

Assume that $n \geq 3$. From (5) we find that $x_i^B = x_i^C$ if and only if $c_i = \frac{\sum_{j=1}^n c_j}{n}$. Also from (6) $p_i^B = p_i^C$ if and only if $c_i = \frac{\sum_{j=1}^n c_j}{n}$.

If $c_i < \frac{\sum_{j=1}^n c_j}{n}$ we have $x_i^B > x_i^C$ and $p_i^B < p_i^C$. And if $c_i > \frac{\sum_{j=1}^n c_j}{n}$ we have $x_i^B < x_i^C$ and $p_i^B > p_i^C$.

But if $n = 2$, we have $x_i^B = x_i^C$ and $p_i^B = p_i^C$ for $i = 1, 2$, even if $c_1 > c_2$ or $c_1 < c_2$.

Therefore, we obtain the following results[2].

Proposition 1 *For $n = 2$, that is, in a duopoly the Cournot equilibrium and the Bertrand equilibrium are equivalent.*

For $n \geqq 3$, that is, in an oligopoly with more than two firms if the marginal cost of a firm is lower than the average marginal cost over the industry, its output at the Bertrand equilibrium is larger than that at the Cournot equilibrium, and the price of its good at the Bertrand equilibrium is lower than that at the Cournot equilibrium.

On the other hand, if the marginal cost of a firm is higher than the average marginal cost over the industry, its output at the Bertrand equilibrium is smaller than that at the Cournot equilibrium, and the price of its good at the Bertrand equilibrium is higher than that at the Cournot equilibrium.

Comparing the first order conditions for relative profit maximization in the Cournot oligopoly and those in the Bertrand oligopoly, we can provide the reason why the equivalence of the Cournot equilibrium and the Bertrand equilibrium holds in a duopoly and a symmetric oligopoly. In a duopoly, since $n = 2$ the

[2]Note that if firms are symmetric with respect to c, Cournot and Bertrand equilibria coincide.

first order conditions for relative profit maximization in the Cournot oligopoly, (3), are reduced to

$$\frac{\partial \Pi_1}{\partial x_1} = a - 2x_1 - c_1 = 0, \text{ and } \frac{\partial \Pi_2}{\partial x_2} = a - 2x_2 - c_2 = 0, \tag{7}$$

and the first order conditions for relative profit maximization in the Bertrand oligopoly, (4), are reduced to

$$\frac{\partial \Pi_1}{\partial p_1} = (1-b)a - 2p_1 + c_1 + bc_2 = 0, \tag{8–1}$$

$$\frac{\partial \Pi_2}{\partial p_2} = (1-b)a - 2p_2 + c_2 + bc_1 = 0. \tag{8–2}$$

Alternatively these are written as

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_1}{\partial x_1}\frac{\partial x_1}{\partial p_1} + \frac{\partial \Pi_1}{\partial x_2}\frac{\partial x_2}{\partial p_1} = 0, \ \frac{\partial \Pi_2}{\partial p_2} = \frac{\partial \Pi_2}{\partial x_1}\frac{\partial x_1}{\partial p_2} + \frac{\partial \Pi_2}{\partial x_2}\frac{\partial x_2}{\partial p_2} = 0. \tag{9}$$

From the property of the relative profits the following relation holds.

$$\Pi_2 = -\Pi_1. \tag{10}$$

Then, (9) is rewritten as

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_1}{\partial x_1}\frac{\partial x_1}{\partial p_1} - \frac{\partial \Pi_2}{\partial x_2}\frac{\partial x_2}{\partial p_1} = 0, \ \frac{\partial \Pi_2}{\partial p_2} = -\frac{\partial \Pi_1}{\partial x_1}\frac{\partial x_1}{\partial p_2} + \frac{\partial \Pi_2}{\partial x_2}\frac{\partial x_2}{\partial p_2} = 0. \tag{11}$$

Substituting the inverse demand functions into (8–1) and (8–2) yields

$$(1-b)a-2(a-x_1-bx_2)+c_1+bc_2 = 0, \text{ and } (1-b)a-2(a-x_2-bx_1)+c_2+bc_1 = 0.$$

Arranging the terms, we obtain

$$a-2x_1-c_1+b(a-2x_2-c_2) = 0, \text{ and } a-2x_2-c_2+b(a-2x_1-c_1) = 0. \tag{12}$$

They are equivalent to (11) because, if $n = 2$, $\frac{\partial x_1}{\partial p_1} = \frac{\partial x_2}{\partial p_2} = -\frac{1}{1-b^2}$ and $\frac{\partial x_2}{\partial p_1} = \frac{\partial x_2}{\partial p_1} = \frac{b}{1-b^2}$. Since $b \neq 1$, (12) implies (7).

In the case where $n \geqq 3$ a relation such as (10) does not holds.

But, if the marginal costs of all firms are equal, a similar relation holds at the equilibrium. At the Cournot equilibrium all x_i's are equal, and then the first order conditions are reduced to

$$a - [2 + (n-2)b]x_i - c_i = 0. \tag{13}$$

At the Bertrand equilibrium all p_i's are equal, and then the first order conditions are reduced to

$$(1-b)a - [2+(n-2)b]p_i + [1+(n-1)b]c_i = 0. \tag{14}$$

Alternatively this is written as

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{\partial \Pi_i}{\partial x_i}\frac{\partial x_i}{\partial p_i} + (n-1)\frac{\partial \Pi_i}{\partial x_j}\frac{\partial x_j}{\partial p_i} = 0,\ j \neq i. \tag{15}$$

From the property of the relative profits the following relation holds.

$$(n-1)\frac{\partial \Pi_j}{\partial x_i} = (n-1)\frac{\partial \Pi_i}{\partial x_j} = -\frac{\partial \Pi_i}{\partial x_i} = -\{a - [2+(n-2)b]x_i - c_i\},$$

and we have

$$\frac{\partial x_i}{\partial p_i} = -\frac{1+(n-2)b}{(1-b)[1+(n-1)b]},\ \frac{\partial x_j}{\partial p_i} = \frac{b}{(1-b)[1+(n-1)b]}$$

for $j \neq i$ at the equilibrium of a symmetric oligopoly. Thus, (15) is rewritten as

$$\left\{\frac{1+(n-1)b}{(1-b)[1+(n-1)b]}\right\}\frac{\partial \Pi_i}{\partial x_i} = 0,\ j \neq i. \tag{16}$$

From the inverse demand functions

$$p_i = a - [1+(n-1)b]x_i.$$

Substituting this into (14) yields

$$(1-b)a - [2+(n-2)b]\{a - [1+(n-1)b]x_i\} + [1+(n-1)b]c_i = 0.$$

Arranging the terms, we obtain

$$a - [2+(n-2)b]x_i - c_i + (n-1)b\{a - [2+(n-2)b]x_i - c_i\} = 0,\ j \neq i. \tag{17}$$

This is equivalent to (16). Since $1+(n-1)b \neq 0$, it implies (13).

6. Related results

Absolute profit maximization If firms in an oligopoly seek to maximize their absolute profits, the Bertrand and Cournot equilibria do not coincide whether the goods of firms are differentiated or homogeneous. It was widely known that in a duopoly if the goods of the firms are substitutes, the equilibrium outputs at the Cournot equilibrium are larger than those at the Bertrand equilibrium, and if the goods are complements, we have the converse results.

In contrast to these results in absolute profit maximization case, in the current paper we have shown that when firms maximize their relative profits, even if the goods of the firms are substitutes, the equilibrium output at the Cournot equilibrium may be larger than or smaller than or equal to that at the Bertrand equilibrium depending on the relationship among marginal costs of the firms.

Relative profit maximization with a homogeneous good By Vega-Redondo (1997), in a framework of evolutionary game theoretic model, it was shown that in an oligopoly in which firms produce a homogeneous good and seek to maximize their relative profits, the Cournot equilibrium coincide with the outcome of perfect competition. Referring to Alchian (1950) and Friedman (1953) he argued that it is relative rather than absolute performance which should in the end prove decisive in the long run.

With differentiated goods, however, the Cournot equilibrium under relative profit maximization is not equivalent to perfect competition.

Delegation problem Miller and Pazgal (2001) has shown the equivalence of price strategy and quantity strategy in a delegation game when owners of firms control managers of firms seek to maximize an appropriate combination of absolute and relative profits. Also Kockesen et. al. (2000) showed that in a two-stage game where the owners of firms choose the weight on the relative profit of the objective functions of their firms and then firms face quantity competition, the owners choose positive weight on the relative profit, but pure relative profit maximization yields the lowest equilibrium (absolute) profits.

In their analyses the owners of firms themselves still seek to maximize absolute profits of their firms. On the contrary, we have interest in the case where the owners of firms themselves seek to maximize the (pure) relative profits. The relative profit is not a means to control the firms, but itself is an object of the owners.

Symmetric and asymmetric duopoly In Tanaka (2013), assuming linear demand functions and constant marginal costs, it was shown that in a duopoly with differentiated goods, if firms have the same cost function and maximize their relative profits, Bertrand and Cournot equilibria are equivalent in the sense that the output and the price of each firm's good at the Bertrand equilibrium are equal to those at the Cournot equilibrium. Satoh and Tanaka (2014) has extended this result to a case where firms have different cost functions.

The result of this paper is an extension and generalization of these results in a duopoly to an asymmetric oligopoly.

Appendix 1: Calculations of the ordinary demand functions

For $j \neq i$, we have

$$p_j = a - x_j - bx_i - b \sum_{k=1, k\neq i,j}^{n} x_k.$$

Thus,

$$\sum_{j=1, j\neq i}^{n} p_j = (n-1)a - (n-1)bx_i - [1+(n-2)b] \sum_{j=1, j\neq i}^{n} x_j.$$

From this

$$\sum_{j=1, j\neq i}^{n} x_j = \frac{1}{1+(n-2)b}\left[(n-1)a - (n-1)bx_i - \sum_{j=1, j\neq i}^{n} p_j\right].$$

Substituting this into (1),

$$x_i = a - p_i - \frac{b}{1+(n-2)b}\left[(n-1)a - (n-1)bx_i - \sum_{j=1, j\neq i}^{n} p_j\right].$$

Then, we obtain the following ordinary demand functions.

$$x_i = \frac{1}{(1-b)[1+(n-1)b]}\left[(1-b)a - [1+(n-2)b]p_i + b\sum_{j=1, j\neq i}^{n} p_j\right],$$
$$i = 1, 2, \dots, n.$$

Appendix 2: Calculations of the Bertrand equilibrium prices

(4) is rewritten as

$$[1+(n-2)b](a-c_i)-2[1+(n-2)b](p_i-c_i)+b\sum_{j=1,j\neq i}^{n}(p_j-c_j)$$

$$-b\left[(n-1)a-\sum_{j=1,j\neq i}^{n}c_j\right]-\frac{b}{n-1}\sum_{j=1,j\neq i}^{n}(p_j-c_j)=0,\ i=1,2,\dots,n.$$

From this we obtain

$$p_i-c_i=\frac{n-1}{2(n-1)+(n-2)(2n-1)b}\left\{[1+(n-1)b](a-c_i)-b\left(na-\sum_{j=1}^{n}c_j\right)\right\}$$

$$+\frac{(n-2)b}{2(n-1)+(n-2)(2n-1)b}\sum_{j=1}^{n}(p_j-c_j),$$

and

$$[1+(n-2)b]\left(na-\sum_{j=1}^{n}c_i\right)-2[1+(n-2)b]\sum_{j=1}^{n}(p_i-c_i)+(n-1)b\sum_{j=1}^{n}(p_i-c_i)$$

$$-(n-1)b\left(na-\sum_{j=1}^{n}c_i\right)-b\sum_{j=1}^{n}(p_i-c_i)=0.$$

The latter equation means

$$\sum_{j=1}^{n}(p_i-c_i)=\frac{1-b}{2+(n-2)b}\left(na-\sum_{j=1}^{n}c_i\right).$$

Then, we get the equilibrium price of the good of Firm i as follows.

$$p_i^B=\frac{(n-1)[1+(n-1)b]}{2(n-1)+(n-2)(2n-1)b}(a-c_i)$$

$$-\frac{nb[1+(n-2)b]}{[2(n-1)+(n-2)(2n-1)b][2+(n-2)b]}\left(na-\sum_{j=1}^{n}c_j\right)+c_i,$$

$$i=1,2,\dots,n.$$

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