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Policy implications of learning from more accurate central bank forecasts

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Abstract

How might central bank communication of its internal forecasts assist the conduct of monetary policy? The literature has shown that heterogeneous expectations may have destabilizing effects on aggregate dynamics. This paper analyzes through adaptive learning the policy implications of central bank influence of private forecasts stemming from more accurate central bank forecasts. In this case, the central bank must only respect the Taylor principle to ensure macroeconomic stability, in contrast to the situation where private agents are learning from less accurate central bank forecasts.

1. Introduction

Since the beginning of the nineties, most of the central banks have become more transparent. The publication of macroeconomic projections has become a widespread practice among them, including inflation targeting central banks, the Federal Reserve, and the ECB. On the other hand, private expectations play a central role in macroeconomics because of their importance in determining ex-ante real interest rates, and both current and future inflation and output by driving consumption and investment decisions.

This paper analyses, under adaptive learning and using the expectational stability (E-stability) criterion, how central bank communication of internal forecasts may affect the conduct of monetary policy when central bank forecasts are more accurate than private forecasts. Muto (2011) analyses the E-stability conditions when private agents condition their forecasts on central bank forecasts. However, the alternative forecast of private agents is the sample average of the inflation rate, meaning that they form their expectations based on a combination of past average inflation and the central bank forecast. This consistently biases private forecasts upward adding their own estimation error to central bank forecast errors.

The contribution of this paper is to extend the analysis of Muto (2011) to a framework in which central bank forecasts are influential because they are more accurate than private forecasts, and in which the forecasting function of private agents is more realistic as it includes their information set, central bank forecasts, and the average past inflation.

More accurate central bank forecasts are defined as forecasts with a smaller mean squared forecast error. This paper's framework is similar to Muto (2011) except that it introduces an additional shock – cost push shocks – into the framework of Muto (2011) with only demand shocks. More accurate forecasts are obtained because the central bank observes both shocks, while private agents only the demand one. Assessing for simplicity the limit case when private expectations converges to central bank ones, our main assumption to model more accurate forecasts is that private agents put a weight equal to unity on central bank forecasts in equilibrium. We abstract from the possibility that central bank forecasts might provide information on the future path of interest rates.

The framework used is the standard New-Keynesian (NK) model (Clarida et al. 1999) which gives a central role for inflation and output expectations. Using adaptive learning, a consequent literature has analysed the effects of heterogeneous expectations (Evans and Honkapohja 1997, Giannitsarou 2003, McCallum and Nelson 2004, Honkapohja and Mitra 2005, 2006, Granato et al. 2008, Preston 2008) and of information asymmetry (Honkapohja and Mitra 2004) on E-stability.

The main result of this paper is that satisfying the Taylor principle is sufficient to reach E-stability and determinacy when central bank forecasts are more accurate than private ones. This outcome is not surprising and is consistent with Bullard and Mitra (2002) since the Taylor principle is a sufficient condition for the E-stability when the central bank and private agents form homogeneous expectations. In this framework, the forecast heterogeneity at the starting point disappears in equilibrium as private agents and the central bank use the same learning algorithm and the former follow central

bank forecasts, due to their lower forecast errors. The present result is different from Muto (2011) because the forecast heterogeneity between the central bank and private agents disappears when private agents do not rely only on the average inflation and the central bank has more accurate forecasts.

The rest of the paper is organized as follows. Section 2 presents the overall framework, while Section 3 the assumptions and conditions for stability and determinacy. Section 4 concludes.

2. The Framework

Compared to rational expectations (RE) based on the hypothesis that agents know the correct equilibrium probabilities laws and the model of the economy, the learning approach assumes that agents form their expectations as econometricians by estimating and updating reduced-form forecasting models in real time.

2.1 A standard NK model

The aggregate demand curve obtained by log-linearizing the Euler equation, the aggregate supply curve derived as a linearization of the firms' optimality condition under the price setting constraint, and the forward-looking interest rate rule derived by maximizing a policy objective function of a quadratic form are:

$$x_t = E_t^{PA} x_{t+1} - \varphi(i_t - E_t^{PA} \pi_{t+1}) + g_t \tag{1}$$

$$\pi_t = \lambda x_t + \beta E_t^{PA} \pi_{t+1} + u_t \tag{2}$$

$$i_t = \phi_0 + \phi_\pi E_t^{CB} \pi_{t+1} + \phi_x E_t^{CB} x_{t+1} \tag{3}$$

where x_t is the output gap, π_t the inflation rate, i_t the nominal interest rate, g_t the demand shock and u_t the cost-push shock. E_t^{PA} and E_t^{CB} are private and central bank expectations respectively. φ is the elasticity of intertemporal substitution, λ the elasticity of inflation to output and β the discount factor. ϕ_π and ϕ_x represent the central bank responses to inflation and the output gap. These structural parameters satisfy $\lambda>0$, $\varphi>0$ and $0<\beta<1$.[1] g_t and u_t are uncorrelated and i.i.d. shocks and follow these processes:

$$w_t = \begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \cdot \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix} \text{ with } F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix}$$

The model can be written under the following reduced-form:

$$y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = D + A^{PA} \cdot E_t^{PA} y_{t+1} + A^{CB} \cdot E_t^{CB} y_{t+1} + B \cdot w_t \tag{4}$$

where $D = \begin{pmatrix} -\varphi\phi_0 \\ -\lambda\varphi\phi_0 \end{pmatrix}$, $A^{PA} = \begin{pmatrix} 1 & \varphi \\ \lambda & \lambda\varphi+\beta \end{pmatrix}$, $A^{CB} = \begin{pmatrix} -\varphi\phi_x & -\varphi\phi_\pi \\ -\lambda\varphi\phi_x & -\lambda\varphi\phi_\pi \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$

2.2 Adaptive Learning

In the adaptive learning framework, agents do not know the true values of the structural parameters. Uncertainty regards the reduced-form equilibrium dynamics of aggregate variables following stochastic shocks and how shocks get mapped into inflation and

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[1] We exclude from the analysis the unlikely case in which $\lambda = 0$ and $\Phi x = 0$, so inflation is unaffected by g_t.

output. Agents are assumed to estimate a perceived law of motion (PLM) using recursive least squares (RLS) on past and current data to obtain parameter estimates:

$$y_t = a + bw_t \quad with \ y_t = (\pi_t, x_t)' \ and \ w_t = (g_t, u_t)'$$

Given the parameters estimated and the observed shocks, agents form forecasts::

$$E_t^i y_{t+1} = a + bFw_t \quad with \ y_t = (\pi_t, x_t)' \ and \ w_t = (g_t, u_t)'$$

By introducing homogenous expectations - $E_t^{CB} y_{t+1} = E_t^{PA} y_{t+1}$ - in (4), we obtain the actual law of motion (ALM) that describes a temporary equilibrium of the economy:

$$y_t = D + (A^{PA} + A^{CB})a + ((A^{PA} + A^{CB})bF + B)w_t \tag{5}$$

The conditions of convergence of the model are given by the local stability conditions of the associated ordinary differential equations (ODE). Evans and Honkapohja (2001) shows that the local stability is determined by this ODE:

$$d\theta / d\tau = T(\theta) - \theta \tag{6}$$

where τ is "notional" time and $T(\theta)$ is the mapping function (T-maps) from PLM to ALM. The E-stability depends on the local stability of this ODE under RLS learning, and defines the convergence of estimations under adaptive learning towards the fundamental values of these parameters under RE. The equilibrium is E-stable if all eigenvalues of the Jacobian matrix of the function mapping the PLM to the ALM have negative real parts. The T-maps are:

$$T(a) = D + (A^{PA} + A^{CB})a \tag{7}$$

$$T(b) = (A^{PA} + A^{CB})bF + B \tag{8}$$

Bullard and Mitra (2002) show that with a forward-looking interest rule the derivations of E-stability conditions yield to the following inequality:

$$\lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \tag{9}$$

which is the Taylor principle put forward by Taylor (1993) and Woodford (2001). It is the necessary and sufficient condition for the E-stability of this model.

3. Central Bank Influence from More Accurate Forecasts

3.1 Assumptions

Assumption 1: the central bank observes both shocks the economy face, while the private agents only observe demand shocks.

This assumption of asymmetric information in favour of the central enables modelling more accurate central bank forecasts compared to private agents, which is the main assumption of this model.[2] The key corollary is that private agents put a weight equal to unity on central bank forecasts, whereas the weight given to the shock they observe – and share with the central bank – is equal to zero.[3]

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[2] This paper does not support that all central banks have more accurate forecasts, but that some central banks may have (see Romer and Romer 2000, for the US, or Hubert 2014, for inflation targeting countries).

[3] Private agents cannot infer from interest rate setting the shock they do not observe. The interest rate is perfectly observed but not the interest rate rule parameters. This is consistent with both the learning framework and the heterogeneity of monetary policy rule parameters in the literature.

Assumption 2: in each period, the central bank publishes its current-period $E_t^{CB} y_t$ and one-period-ahead $E_t^{CB} y_{t+1}$ expectations and private agents form their forecasts after the publication of central bank ones.

Private agents should be able to incorporate central bank forecasts in their forecasting function (see Figure A). We assume that both PLM are estimated with past data. This is to avoid the simultaneity between the determination of private agents' beliefs and the true data generating process of fundamental variables.

Figure A: the timing of actions
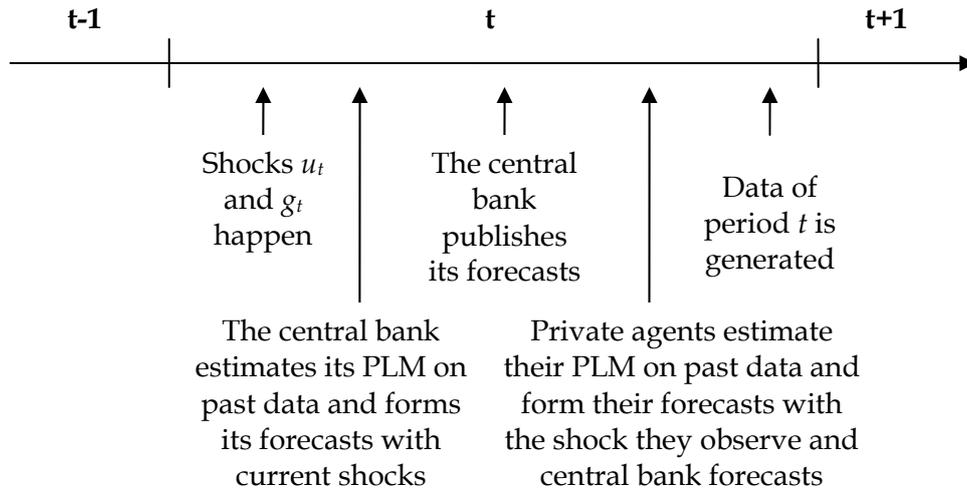
3.2 The Model and E-Stability Conditions

The PLM of the central bank is

$$y_{t-1} = a^{CB} + b^{CB} \cdot w_{t-1} \tag{10}$$

and can be written

$$y_{t-1} = a^{CB} + b_g^{CB} \cdot g_{t-1} + b_u^{CB} \cdot u_{t-1} \tag{11}$$

$$\text{with } a^{CB} = \begin{pmatrix} a_x^{CB} \\ a_\pi^{CB} \end{pmatrix}, \ b_u^{CB} = \begin{pmatrix} b_{xu}^{CB} \\ b_{\pi u}^{CB} \end{pmatrix} \text{ and } b_g^{CB} = \begin{pmatrix} b_{xg}^{CB} \\ b_{\pi g}^{CB} \end{pmatrix}$$

The forecasting function of the central bank, when shocks at date t are available, is then:

$$E_t^{CB} y_{t+1} = a^{CB} + b_g^{CB} \cdot \mu \cdot g_t + b_u^{CB} \cdot \rho \cdot u_t \tag{12}$$

The core difference with Muto (2011)[4] stems from the PLM of private agents which here include their own information set and takes the following form:

$$y_{t-1} = a^{PA} + b^{PA} \cdot g_{t-1} + c^{PA} \cdot E_{t-1}^{CB} y_{t-1} \tag{13}$$

And, since current-period expectations are $E_{t-1}^{CB} y_{t-1} = a^{CB} + b_g^{CB} \cdot g_{t-1} + b_u^{CB} \cdot u_{t-1}$, can be written

$$y_{t-1} = (a^{PA} + c^{PA} \cdot a^{CB}) + (b^{PA} + c^{PA} \cdot b_g^{CB}) \cdot g_{t-1} + c^{PA} \cdot b_u^{CB} \cdot u_{t-1} \tag{14}$$

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[4] See his equation (18). The timing of the central bank's expectation is also different to comply with the timing of information publication and the data generating process of fundamental variables as show in Figure A.

$$\text{with } a^{PA} = \begin{pmatrix} a^{PA}_x \\ a^{PA}_\pi \end{pmatrix}, \ b^{PA} = \begin{pmatrix} b^{PA}_x \\ b^{PA}_\pi \end{pmatrix} \text{ and } c^{PA} = \begin{pmatrix} c^{PA}_{xx} & c^{PA}_{x\pi} \\ c^{PA}_{\pi x} & c^{PA}_{\pi\pi} \end{pmatrix}$$

The PLM of private agents is based on the shock they observe and the central bank forecast. Their forecasting function is then:

$$E^{PA}_t y_{t+1} = a^{PA} + b^{PA} \cdot \mu \cdot g_t + c^{PA} \cdot E^{CB}_t y_{t+1} \tag{15}$$

and can be written[5]

$$E^{PA}_t y_{t+1} = (a^{PA} + c^{PA} \cdot a^{CB}) + (b^{PA} + c^{PA} \cdot b^{CB}_g)\mu \cdot g_t + c^{PA} \cdot b^{CB}_u \cdot \rho \cdot u_t \tag{16}$$

Introducing central bank and private forecasts in (4) and supposing without loss of generality that $\phi_0 = 0$ and so $D = 0$, the ALM of the economy is then:

$$y_t = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} \dots$$
$$\dots + (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b^{CB}_g) + A^{CB} \cdot b^{CB}_g)\mu + B_g) \cdot g_t \tag{17}$$
$$\dots + ((A^{PA} \cdot c^{PA} \cdot b^{CB}_u + A^{CB} \cdot b^{CB}_u)\rho + B_u) \cdot u_t$$

Following Evans and Honkapohja (2003), the T-maps defining E-stability derived from the correspondence between PLM to ALM are then:

$$T(a^{CB}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} \tag{18}$$
$$T(a^{PA}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} - c^{PA} \cdot a^{CB} \tag{19}$$
$$T(b^{CB}_g) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b^{CB}_g) + A^{CB} \cdot b^{CB}_g)\mu + B_g \tag{20}$$
$$T(b^{PA}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b^{CB}_g) + A^{CB} \cdot b^{CB}_g)\mu + B_g - c^{PA} \cdot b^{CB}_g \tag{21}$$
$$T(b^{CB}_u) = (A^{PA} \cdot c^{PA} \cdot b^{CB}_u + A^{CB} \cdot b^{CB}_u)\rho + B_u \tag{22}$$
$$T(c^{PA}) = (A^{PA} \cdot c^{PA} + A^{CB})\rho + B_u (b^{CB}_u)^{-1} \tag{23}$$

The equations for (a^{CB}, a^{PA}), (b^{CB}_g, b^{PA}) and (b^{CB}_u, c^{PA}) are independent of each other. Following Honkapohja and Mitra (2006), the E-stability of the subsystems is satisfied if and only if all the eigenvalues of the Jacobian of M_1, M_2 and M_3 have negative real parts.

$$M_1 = \begin{pmatrix} T(a^{CB}) - a^{CB} \\ T(a^{PA}) - a^{PA} \end{pmatrix}, \ M_2 = \begin{pmatrix} T(b^{CB}_g) - b^{CB}_g \\ T(b^{PA}) - b^{PA} \end{pmatrix}, \text{ and } M_3 = \begin{pmatrix} T(b^{CB}_u) - b^{CB}_u \\ T(c^{PA}) - c^{PA} \end{pmatrix}$$

Proposition: The model is E-stable under learning if all eigenvalues of the following matrix $A^{PA} + A^{CB} - I$ have negative real parts. It corresponds to the following inequality:

$$\lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \tag{24}$$

The proof is in the Appendix.[6] This condition is the Taylor principle, the condition for stability with homogenous forecasts. At the steady state, homogeneous expectations between private agents and the central bank are the same as those of the MSV solution of the model. The analysis of central bank influence from more accurate forecasts is then

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[5] Note that u_t now appears in the equation (16) because of central bank forecasts.
[6] If private agents only observe the cost-push shock u_t (rather than g_t), the condition for stability related to central bank responses is the same as above and the second condition related to the economy structure shown in the Appendix is also similar, with μ replacing ρ.

directly related to the analysis of heterogeneous expectations and how central bank may render expectations homogeneous.[7]

3.3 Determinacy

We now focus on the determinacy condition defining the uniqueness of the equilibrium. This issue matters when the policy rule comprises forward-looking components as it may raise the possibility of sunspot equilibrium (see Bernanke and Woodford, 1997). Blanchard and Kahn (1980) show that the equation *(4)* can be rewritten:

$$y_t = D + (A^{PA} + A^{CB}) \cdot E_t \, y_{t+1} + B \cdot w_t \tag{25}$$

and the standard RE determinacy condition is $| A^{PA} + A^{CB} | < 1$, and leads to:

$$\phi_x (\beta + 1) > \lambda (\phi_\pi - 1) > \phi_x (\beta - 1) \tag{26}$$

In this framework, the determinacy condition is similar to *(26)*, and is a sufficient condition for the E-stability of the equilibrium. This is consistent with McCallum (2007) that shows that in a forward-looking model where the current period information set is available to agents to form their forecasts through adaptive learning, the determinacy condition is sufficient for E-stability.

4. Conclusion

This paper shows that, in a situation of central bank influence from more accurate forecasts, the central bank must only respect the Taylor principle and need not be more restrictive to ensure macroeconomic stability. A policy implication of this work is that influencing private expectations should not be an objective *per se* and central banks should rather increase the quality of their macroeconomic forecasts they communicate to the public to reach macroeconomic stability at a lower cost.

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[7] The properties of heterogeneous expectations on stability also arise from the fact that the determinant of $(A^{PA} - I)$ has one eigenvalue with a positive real part, so private expectations have a destabilizing effect on the economy, whereas central bank forecasts have a stabilizing effect. When private expectations deviate from the fundamental value, the actual inflation or output deviates in the same because all the values of A^{PA} are positive. At the opposite, when the central bank forecasts deviate upward, the actual inflation or output decreases, because all the values of A^{CB} are negative.

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Appendix

The T-maps defining E-stability and derived from PLM to ALM are then:

$$T(a^{CB}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} \tag{18}$$

$$T(a^{PA}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} - c^{PA} \cdot a^{CB} \tag{19}$$

$$T(b_g^{CB}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g \tag{20}$$

$$T(b^{PA}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g - c^{PA} \cdot b_g^{CB} \tag{21}$$

$$T(b_u^{CB}) = (A^{PA} \cdot c^{PA} \cdot b_u^{CB} + A^{CB} \cdot b_u^{CB})\rho + B_u \tag{22}$$

$$T(c^{PA}) = (A^{PA} \cdot c^{PA} + A^{CB})\rho + B_u (b_u^{CB})^{-1} \tag{23}$$

The expectational stability (E-stability) of the ALM is satisfied if these T-maps are locally stable, what is satisfied if and only if all eigenvalues of the Jacobian of M_1, M_2 and M_3 have negative real parts. Those Jacobian matrices are computed at the equilibrium values $b_u^{CB} = b_g^{CB} = J_{2,1}$, $J_{n,m}$ being the nxm unit matrix, deriving from the assumption that $a^{PA} = b^{PA} = 0_{2,1}$ and $c^{PA} = J_{2,2}$, $0_{n,m}$ being the nxm zero matrix.

$$J_1 = \begin{pmatrix} A^{PA} + A^{CB} - I & A^{PA} \\ A^{PA} + A^{CB} - I & A^{PA} - I \end{pmatrix}$$

$$J_2 = \begin{pmatrix} \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} \\ \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} - I \end{pmatrix}$$

$$J_3 = \begin{pmatrix} \rho(A^{PA} + A^{CB}) - I & \rho A^{PA} \\ -B_u & \rho A^{PA} - I \end{pmatrix}$$

Following Honkapohja and Mitra (2006), the determinant for computing the eigenvalues of J_1, J_2 and J_3 may be simplified as follows

$$\det(J_1) = \begin{vmatrix} A^{PA} + A^{CB} - I & A^{PA} \\ A^{PA} + A^{CB} - I & A^{PA} - I \end{vmatrix}$$

$$\det(J_1) = \begin{vmatrix} 0 & I \\ A^{PA} + A^{CB} - I & A^{PA} - I \end{vmatrix}$$

After subtracting the second row from the first, the computation shows that J_1 has eigenvalues with negative real parts if and only if $A^{PA} + A^{CB} - I$ has the same property. Similarly, we obtain:

$$\det(J_2) = \begin{vmatrix} \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} \\ \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} - I \end{vmatrix}$$

$$\det(J_2) = \begin{vmatrix} 0 & I \\ \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} - I \end{vmatrix}$$

After subtracting the second row from the first, the computation shows that J_2 has eigenvalues with negative real parts if and only if $\mu(A^{PA} + A^{CB}) - I$ has the same property. Because $0 < \mu < 1$, it suffices to have only the eigenvalues of $A^{PA} + A^{CB} - I$ for E-stability. The necessary and sufficient condition of J_2 is therefore similar to the one for J_1.

As the system of M_3 is not linear, the Jacobian J_3 is analyzed differently. For b_u^{CB}, the standard E-stability arguments apply and yield to the same property than for J_1 and J_2, because $0 < \rho < 1$. For c^{PA}, the E-stability condition is $\rho A^{PA} - I$.

For the special case of a 2×2 matrix A, it can be shown that the condition that both roots of A have negative real parts is equivalent to the condition that the trace of A is negative and the determinant of A is positive. Thus, all the eigenvalues of $A^{PA} + A^{CB} - I$ have negative real parts if and only if the two conditions apply. It corresponds to the following inequalities:

$$\varphi\phi_x - \lambda\varphi + \lambda\varphi\phi_\pi - \beta\varphi\phi_x > 0 \qquad (27)$$

$$\varphi\phi_x - \lambda\varphi - \beta + \lambda\varphi\phi_\pi + 1 > 0 \qquad (28)$$

If (27) holds then (28) holds. The E-stability condition corresponds to (27) and can be written:

$$\lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \qquad (24)$$

The second condition needs ρ the autocorrelation of the cost push shock to be sufficiently small as A^{PA} has an eigenvalue higher than one (see Honkapohja and Mitra (2004) for more details), but has no effect on the policy responses to inflation or output to reach E-stability.