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Herfindahl rule under return flows

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Abstract

The paper investigates the efficient extraction path in a partial equilibrium model with exhaustible resource, the usage of which is characterized by return flows, and a backstop to analyze the sequence of the resource use. It is shown that in the presence of users with different return flow coefficients Herfindahl rule can be violated as the low-cost and the high-cost resources could be used simultaneously.

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1. Introduction

For a long period economists believed that whenever several resources with constant but different marginal extraction costs are present, the least cost resource should be used first. This result, known in economic literature as Herfindahl principle, was further elaborated into the two so-called folk theorems. The first theorem states that two resources with different marginal extraction costs cannot be used simultaneously during some time interval and the second theorem determines the sequence of extraction in accordance with the increase in marginal cost. Later on, it was shown by Kemp and Long (1980) that this result might be violated in general equilibrium context but Lewis (1982) demonstrated that the rule is valid even in this case under additional assumption that the extracted resource can be used for intertemporal reallocation of wealth. Chakravorty and Krulce (1994) found that cheap and expensive resources could be used simultaneously even in partial equilibrium framework if there is some quality difference between the resources that is important for some users and is not important for others. This model with heterogeneous demand was generalized by Chakravorty *et al.* (2005), where the Herfindahl principle of "least cost first" was restated, taking into account the user-specific conversion cost, in terms of "net cost". Gaudet *et al.* (2001) found that in the presence of set-up costs it might be optimal to use high marginal cost deposit first if the opening set up costs associated with the low marginal cost deposit are high. Another violation of Herfindahl principle was discovered by Holland (2003) in the case where extraction capacity constraints are present for some deposits.

This paper proposes one more example of the violation of Herfindahl principle that can arise when the resource extracted is not fully utilized and the non-utilized part returns to the stock. A classic example of such resource is provided by groundwater: a part of water used for irrigation infiltrates back to the groundwater system. The corresponding return flow coefficient could vary from 0 to over 50% (Dewandel *et al.* 2008) depending on irrigation technique and intensity, type of soil and its usage (e.g., agricultural fields, golf lawns, etc.) In this case we might deal with heterogeneous consumption patterns as users are characterized by different return flow coefficients. This results in different user consumption costs as the scarcity component is proportional to the net water consumption. If, in addition to a low-cost exhaustible resource, a high-cost backstop can be used, then users will switch to the backstop sequentially starting from the consumer with the highest return flow coefficient, which implies that low-cost groundwater and high-cost backstop are used simultaneously during some time interval.

The reminder of the paper is organized in the following way. In Section 2, we describe the model and derive the characteristics of efficient extraction path. In Section 3, we study the efficient consumption path and illustrate the violation of the Herfindahl rule. Section 4 concludes the paper with some policy implications.

2. The model

Consider a model with exhaustible resource (for example, a groundwater) with initial stock S_0 and marginal extraction cost c of a backstop substitute (desalinated water), that is characterized by higher constant marginal cost $c_b > c$. There are N groups of users ($i = 1, 2, \ldots, N$) that have different return flow coefficients: $\delta_1 > \delta_2 > \ldots > \delta_N$, where $0 < \delta_i < 1$. If we denote the groundwater consumption of group i at time t by g_{it}, then evolution of groundwater stock is given by $\dot{S}_t = \bar{g} - \sum_{i=1}^{N} \delta_i g_{it}$, where $\bar{g}$ ($\bar{g} \geq 0$) stays for natural water recharge. Denote by b_{it} the consumption of backstop substitute by user i at moment t, then his total consumption is $x_{it} \equiv g_{it} + b_{it}$ that results in gross instantaneous surplus $u_i(x_{it})$, where $u_i' > 0$, $u_i'' < 0$ and $u_i'(\bar{g}/\delta) > c_b$. The first two assumptions of positive but diminishing utility are quite standard; the last assumption says that if stock is exhausted, no user can rely exclusively on groundwater recharge and thus guarantees that every user still has positive water consumption of the perfect substitute provided by the backstop technology. Optimal resource consumption path is given by the social planner maximization problem, where r stays for the social discount rate

$$\begin{aligned} &\max \quad \int_0^{\infty} \left(\sum_i u_i(g_{it} + b_{it}) - c \sum_i g_{it} - c_b \sum_i b_{it} \right) e^{-rt} dt \\ &\dot{S}_t = \bar{g} - \sum_i \delta_i g_{it}, \quad S_t \geq 0, \quad g_{it} \geq 0, \quad b_{it} \geq 0, \quad S_0 - given \end{aligned} \tag{1}$$

The present value Hamiltonian for (1) is $H_t = \left(\sum_i u_i(g_{it} + b_{it}) - c \sum_i g_{it} - c_b \sum_i b_{it} \right) e^{-rt} + \lambda_t \left(\bar{g} - \sum_i \delta_i g_{it} \right)$ where λ_t $(\lambda_t \geq 0)$ stays for the shadow value of the stock at time t. The necessary conditions for the problem (1) are

$$u_i'(x_{it}) \leq c + \delta_i \lambda_t e^{rt} \qquad (= \text{if } g_{it} > 0) \quad i = 1, 2, \ldots, N, \ \forall t. \tag{2}$$

$$u_i'(x_{it}) \leq c_b \qquad (= \text{if } b_{it} > 0) \quad i = 1, 2, \ldots, N, \ \forall t. \tag{3}$$

$$\dot{\lambda}_t \leq 0 \ (= \text{if } S_t > 0) \quad \forall t \tag{4}$$

$$\lim_{t \to \infty} \lambda_t S_t = 0. \tag{5}$$

If the efficient path is decentralized, then the price for user i at time t (p_{it}) should be equal to the value of his marginal benefit: $p_{it} \equiv u_i'(x_{it})$. Conditions (2) and (3) suggest that if resource is used by agent i, then price should be equal to the value of user cost, which in case of exhaustible resource includes scarcity component in addition to marginal extraction cost. Moreover, this scarcity component differs between the agents because consumption patterns are different so that one unit extraction results in different stock reduction due to differences in recharge flows. Condition (4) suggests that until exhaustion, the shadow value of the stock is constant over time, and condition (5) is a standard transversality condition.

3. The main results

It is useful to obtain some characteristics of optimal extraction and consumption paths implied by (2)-(5). The following claim shows that users switch to the backstop in accordance with the recharge coefficients starting with the lowest ones, i.e. the more efficient user will never switch to the backstop before the less efficient one.

Claim 1. If agent i uses exhaustible resource than all users $j > i$ do not use backstop.
Proof. See Appendix.

Now we can proceed to the full characterization of the optimal extraction and consumption path.

Proposition 1.
Optimal path is characterized by the following conditions:

(a) $p_{it} \equiv u_i'(x_{it}) = \begin{cases} c + \delta_i \lambda e^{rt}, \, t \le T_i \\ c_b, \quad t \ge T_i \end{cases}$ for $i = 1, 2, \ldots, N$;

(b) $x_{it} = g_{it}, t < T_i$ for $i = 1, 2, \ldots, N$; $x_{it} = b_{it}, t > T_i$ for $i \le N - 1$ $x_{iN} = \bar{g} / \delta_N + b_{Nt}, t > T_N$;

(c) if $T_i > 0$ and $T_j > 0$ then $T_i - T_j = ln(\delta_i / \delta_j) / r$,

(d) $\int_0^\infty \left(\sum_{i=1}^{N} \delta_{it} g_{it} - \bar{g} \right) dt = S_0$.

Proof. See Appendix.

The resulting price paths are illustrated at figure 1 for the case of two users. Initially, both users exploit groundwater but at time T_1 the groundwater user cost for the first user is equal to the cost of substitute so that he switches to the more expensive resource while the second user still extracts groundwater until its full exhaustion at T_2. After T_2 the second user still exploits groundwater but the amount used is restricted by the natural recharge combined with return flows. As by assumption this amount is not enough to satisfy all the needs it is combined with renewable substitute.

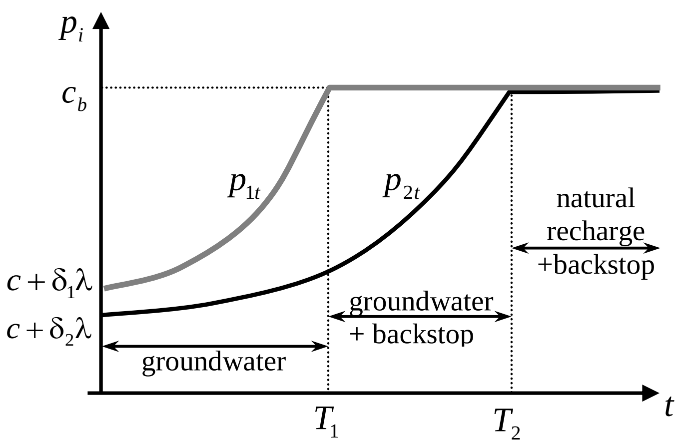

Figure 1. Optimal price paths for the two users case

It should be noted that with small initial resource stock the shadow value of the stock might be so high that the least efficient water user (user 1) never extracts groundwater and uses the more expensive substitute from the very beginning, i.e. $T_1 = 0$.

Thus we observe two intervals: one from T_1 to T_2 and the other one starts after T_2 when two resources with different marginal costs are used simultaneously. However, the explanation for these two intervals is quite different. From T_2 both resources are used by the same agent as the least cost resource has capacity constraint and thus has to be accompanied by the more expensive substitute. This possibility of violation of the Herfindahl rule was demonstrated by Holland (2003).

The explanation for first interval (from T_1 to T_2) does not deal with capacity constraint as groundwater stock is not exhausted up to T_2 and thus natural recharge constraint is not binding here. It should be noted that at the individual (user) level resources are used in accordance with marginal costs - starting from the least expensive one - but at the aggregate level the Herfindahl rule is violated. The reason for this violation comes from the external effects that are associated with the water return flows which differ for different users. Each user has the same private marginal extraction cost but social costs are different as a unit of resource consumed by the user with lower return flows brings higher net extraction and thus higher social cost for this user. Efficiency dictates that the resources should be used in accordance with social (not private) marginal extraction costs. It means that, when user-specific cost achieves the level of the backstop cost of production, the corresponding user should switch to this substitute but for the other user with higher return flows and thus lowers social extraction cost it is still optimal to use less expensive groundwater.

The standard argument for Herfindahl rule that deals with reallocation of extraction of a more expensive resource for future period in favor of current extraction of a least expensive one does not work in this case. Indeed, if we consider two moments of time, t and $t+\theta$, from the interval (T_1, T_2) and reallocate water in such a way that in moment t the more expensive substitute is replaced by the groundwater for user 1, i.e. $\Delta g_{1t} = -\Delta b_{1t} > 0$, then the stock of groundwater available at $t+\theta$ is reduced by $\delta_1 \Delta g_{1t}$. As only the second user consumes groundwater at $t+\theta$, then we should reduce groundwater consumption at $t+\theta$ by $\delta_1 \Delta g_{1t} / \delta_2$ so that $\Delta g_{2t+\theta} = -\Delta b_{2t+\theta} = \delta_1 \Delta g_{1t} / \delta_2$. Since the total resource consumption for each user is unaffected by this reallocation, the social welfare changes only due to cost change. We can easily see that the total cost would go up $e^{-rt}(c - c_b)\Delta g_{1t} - e^{-r(t+\theta)}(c - c_b)\delta_1 \Delta g_{2t} / \delta_2 = \Delta g_{1t} e^{-r(t+\theta)}(c - c_b)(\delta_1 / \delta_2 - e^{r\theta}) > 0$ as $\theta < T_2 - T_1$ and $e^{r\theta} < e^{r(T_2 - T_1)} = \delta_1 / \delta_2$ due to part (c) of proposition 1. Thus, postponing extraction of the least cost resource in the interval (T_1, T_2) reduces social welfare.

4. Conclusion

We analyzed the optimal extraction path for the resource with return flows using partial equilibrium framework. It was shown that if users differ in terms of return flow coefficients, then optimal extraction path has non-empty time interval characterized by simultaneous use of low-cost and high-cost resources, so that Herfindahl rule is violated at aggregate level. The reason comes from the differences in scarcity value of the exhaustible resource for the users with different return flow coefficients. As a result, the order of extraction is determined by marginal user costs rather than marginal extraction cost, where user cost includes, in addition to extraction cost, the user-specific scarcity value that is higher for the users with lower return flows.

One of the implications of the proposed analysis deals with decentralization of the optimal path that requires user-specific tariffs with higher per unit prices for the users with lower return flows.

Finally, it should be pointed out that we have provided purely theoretical example of the Herfindahl rule violation under return flows; further empirical research based on specific case studies would be highly valuable.

Appendix

Proof of claim 1.

First of all we will show that the exhaustible resource is scarce, i.e. its shadow value is positive. Condition (4) implies that for all moments with $S_i > 0$ the shadow value of exhaustible resource is constant: $\lambda_t = \lambda$. Let us show that $\lambda > 0$. Suppose that $\lambda = 0$, then (2) and (3) imply $u_i'(x_{it}) \le c < c_b$, that is, the backstop is never used. Note that groundwater consumption is positive at any t since diminishing marginal utility implies $u_i'(0) \ge u_i'(\bar{g}/\delta_i) > c_b > c$ and condition (2) is violated for zero consumption. Thus, consumption of user i at time t is given by $\bar{x}_i = u_i^{-1}(c) > 0$. This amount exceeds the one that could be provided by natural recharge combined with return flows as $u_i'(\bar{x}_i) = c < c_b < u_i'(\bar{g}/\delta_i)$, which implies $\bar{x}_i > \bar{g}/\delta_i$ due to diminishing marginal utility. Thus, the aggregate net instantaneous extraction exceeds groundwater recharge $\sum_i \delta_i \bar{x}_i > \bar{g}$, which implies that the stock will be exhausted in finite time given by $S_0 \Big/ \left(\sum_i \delta_i \bar{x}_i - \bar{g} \right)$, that makes this consumption path unsustainable over infinite horizon.

Suppose that at time t we have $g_{it} > 0$ and $b_{jt} > 0$ for some $j > i$. Then (2) and (3) imply $u_j'(x_j) = c_b \ge u_i'(x_{it}) = c + \delta_i \lambda e^{rt}$. As $j > i$ then $c + \delta_j \lambda e^{rt} < c + \delta_i \lambda e^{rt} \le u_j'(x_j)$, which contradicts to condition (3).

Proof of proposition 1.

As it follows from condition (2), when exhaustible resource is used by the user i, then price equals to the marginal user cost. Denoting by T_i the moment, when user i starts the use of backstop we have $p_{it} \equiv u_i'(x_{it}) = c + \delta_i \lambda e^{rt}$, $x_{it} = g_{it}$ if $t < T_i$. After T_i due to condition (3) the price is constant: $p_{it} \equiv u_i'(x_{it}) = c_b$.

The price path should be continuous so that at the switching moment T_i the user prices for both sources are the same, i.e. $c + \delta_i \lambda e^{rt} = u_i'(x_{it}) = c_b$. If this is not the case and there is a jump in marginal benefit, then the instantaneous net surplus could be increased if we reduce consumption at the moment with low marginal benefit and increase it at the moment with high marginal benefit.

If at some moment t stock is not exhausted $S_t > 0$ then its shadow cost λ is constant so that for any $t > T_i$ we have $u_i'(x_{it}) = c + \delta_i \lambda e^{rt} > c + \delta_i \lambda e^{rT_i} = c_b$, which due to conditions (2) implies $g_{it} = 0$ and $x_{it} = b_{it}$.

If we take two arbitrary users i and j with switching moments $T_i > 0$ and $T_j > 0$ then the following conditions take place: $c + \delta_i \lambda e^{rT_i} = c_b$ and $c + \delta_j \lambda e^{rT_j} = c_b$, which implies that $\delta_i e^{rT_i} = \delta_j e^{rTj}$. By taking logs we get that the distance between the two switching moments is determined by the ratio of return flow coefficients and social discount rate $T_i - T_j = \ln(\delta_j / \delta_i)/r$.

Due to condition (4) the shadow value of the resource is constant until full exhaustion, which allows restating the transversality condition (5) in the form $\lim_{t\to\infty} S_t = 0$. The equation for the evolution of the stock together with initial condition allows finding the stock at time t as the difference between initial stock adjusted for natural recharge and total net extraction, $S_t = S_0 - \int_0^t \left(\sum_{i=1}^N \delta_i g_{i\tau} - \bar{g} \right) d\tau$ and plugging this into transversality condition we get $\int_0^\infty \left(\sum_{i=1}^N \delta_i g_{it} - \bar{g} \right) dt = S_0$.

As $T_i - T_j = \ln(\delta_j / \delta_i)/r$, then for $i > j$ we have $\delta_j > \delta_i$, which brings $T_i > T_j$, that is, the user N is the last who switches to the backstop. First, let us show that at the moment when this user starts to exploit renewable substitute the groundwater stock is depleted, that is, $S_{T_N} = 0$. If this is not the case then the shadow value λ is constant so that for any $t > T_N$ we have $u'_N(x_{Nt}) = c + \delta_N \lambda e^{rt} > c + \delta_N \lambda e^{rT_N} = c_b$, which due to conditions (2) implies $g_{Nt} = 0$ and $x_{it} = b_{it}$. Then groundwater stock will be replenished by natural recharge which contradicts to full exhaustion condition derived above. As $S_{T_N} = 0$ then groundwater use is constrained by natural recharge combined with return flows so that $g_{Nt} = \bar{g}/\delta_N$. Since by assumption $u'_N(\bar{g}/\delta_N) > c_b$ and for any $t > T_N$ $u'_N(x_N) = c_b$ then $b_{Nt} = x_N - \bar{g}/\delta_N > 0$.

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