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# Unbiased Adaptive Expectation Schemes

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## Abstract

There are situations in which the old-fashioned adaptive expectation process seems to provide a good description of agents' behavior (Chow, 2011). Unfortunately, this expectation scheme may not satisfy the necessary rationality condition (unconditional mean-zero error). This paper shows how to simply fix the problem introducing a bias correction term.

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# 1 Introduction

In 1906 the statistician Francis Galton discovered - during a competition to guess the weight of an ox - that people are capable (collectively) of guessing averages of unknown quantities (see Surowiecki (2005)). In fact, the average guess was extremely close to the actual weight of the ox. This result was the foundation of the Muth's rational expectation hypothesis Muth (1961) assuming that errors made by agents are not systematic.

Using the law of iterated expectations, it is simple to show that Galton's result is consistent with this notion of rationality[1]. In a sense, we may say that it is a necessary condition for the Lucas' rationality requirements (Lucas Jr, 1976).

But it is not a sufficient condition since Galton's finding only suggests that agents (at least collectively) are able to guess unconditional moments and says nothing about conditional ones[2].

Anyway, Galton's result is important because it gives a restriction to individual behaviors and rational expectations are also important in macroeconomic since they represent a fixed point of the *expectations-actions feedback* and so one of the possible solutions.

Rational expectations are, as said above, a subset of mean-zero expectation schemes. On the contrary, even though *adaptive expectation schemes* often seem to be a good representation of actual agents' behaviors in empirical analysis (see Chow (2011)), in many economic situations the adaptive scheme does not seem to satisfy the unconditional mean-zero requirements; *i.e.*, the necessary condition for rationality.

Those situations are relevant, as shown in the next section, representing cases in which economic variables are non-stationary (*e.g.*, a unit root with drift).

Section 2 shows that if economic theory requires that expectations are on average zero, then the well known *adaptive expectation process* may be simply modified to fulfill the requirement adding an appropriate bias correction term.

Section 3 applies the section 2 result to the Cagan's model of inflation. Finally, section 4 concludes.

[1] Simply because $E[E[\varepsilon_{t+1}|I_t]] = E[\varepsilon_{t+1}] = 0$. Where $\varepsilon_{t+1}$ is error made at time $t+1$ and $I_t$ is the information set at time $t$.

[2] Note, also, that the economic and psychological literature following Galton's result found the existence of an expectation bias (see Tversky and Kahneman (1974); Ariely and Jones (2008)).

## 2 Adaptive expectations with a bias correction term

The simplest process in modeling agent forecast of future variables is the static or naive one which means that the expectation of an economic variable, let's say $x_t$, is equal to the observation at time $t-1$, $x_t^e = x_{t-1}$.

In the 1960s macroeconomic models tried to incorporate more sophisticated expectation processes like the adaptive one[3]

$$x_{t+1}^e = x_t^e + \lambda(x_t - x_t^e) = \lambda x_t + (1-\lambda)x_t^e \tag{1}$$

in which the expectation is revised according to the error made in the previous period multiplied by a correction parameter $\lambda$ between zero and one[4].

This modeling strategy was abandoned during the 1970s because agents may make non mean-zero systematic errors[5].

As discussed in the introduction, Galton's discovery suggests that agents *collectively* are able to estimate *unconditional* means.

This weaker assumption of a mean-zero error may be a problem in the *adaptive learning scheme* since it could produce agent's expectation that over or underestimates economics future variables; *i.e.*, with a non-zero bias.

To see the reason consider the time process of the error

$$\Xi_t = x_t^e - x_t = \lambda x_{t-1} + (1-\lambda)x_{t-1}^e - x_t \tag{2}$$

that, adding and subtracting $x_{t-1}$ from the RHS and rearranging terms, may be written as

$$\Xi_t = x_t^e - x_t = (1-\lambda)[x_{t-1}^e - x_{t-1}] - \Delta x_t \tag{3}$$

with the following recursive structure

$$\Xi_t = (1-\lambda)\Xi_{t-1} - \Delta x_t. \tag{4}$$

Now it is easy to see that even in situations in which the $x_t$ variable follows a very simple deterministic process, like $\Delta x_t = d$, the error process

$$\Xi_t = (1-\lambda)\Xi_{t-1} - d \tag{5}$$

[3] Of which the static expectation is a particular case ($\lambda = 1$).

[4] Making a simple back substitution it is simple to show that the expectation process follows a distributed lag process with weights declining exponentially. Expectations have the following representation, $x_t^e = \sum_{i=1}^{\infty} \lambda(1-\lambda)^{i-1} x_{t-i}$; see Evans-Honkapohja "Economics of Expectations" in Smelser *et al.* (2001).

[5] In other words, not satisfying necessary and sufficient conditions for rationality.

does not go to zero but converges to $\Xi = -d/\lambda$.

Furthermore, many econometric studies show that even in situations in which the adaptive expectation process seems a reasonable representation of agents' behavior (Chow, 2011), the parameter $\lambda$ may be time variant.

In the following we analyze the more general adaptive expectation process

$$x^e_{t+1} = \lambda_t x_t + (1 - \lambda_t)x^e_t + \zeta_t. \tag{6}$$

Equation (6) generalizes the standard adaptive scheme in two respects: 1) It has a time variant learning parameter $\lambda_t$ following an *i.i.d.* random process (between 0 and 1) with mean $1-\lambda$, and 2) there is an bias correction parameter, $\zeta$.

As before, we can compute the error process

$$\Xi_{t+1} = x^e_{t+1} - x_{t+1} = \lambda_t x_t + (1 - \lambda_t)x^e_t + \zeta_t - x_{t+1} \tag{7}$$

and add and subtract $x_t$ to the RHS,

$$\Xi_{t+1} = -(1 - \lambda_t)x_t + (1 - \lambda_t)x^e_t + \zeta_t - \Delta x_{t+1} \tag{8}$$

that simplifies to[6]

$$\Xi_{t+1} = (1 - \lambda_t)\Xi_t + \zeta_t - \Delta x_{t+1}. \tag{9}$$

Equation (9) is a stochastic random difference process that has a stationary solution provided that the stability conditions are met[7].

If unconditional expectation exists we can take the unconditional expectation operator in both sites and equate to zero searching for a solution with $E[\Xi_{t+1}] = 0$, that is

$$E[\Xi_{t+1}] = 0 = (1 - \lambda)0 + \zeta_t - E[\Delta x_{t+1}]. \tag{10}$$

Solving for $\zeta$ we get

$$\zeta_t = E[\Delta x_{t+1}], \tag{11}$$

showing that the adaptive expectation scheme may be simply corrected estimating the drift (trend) of $x_t$.

[6]In case in which $\lambda_t = 1$ (static expectation), the error process is, obviously, $\Xi_{t+1} = \zeta_t - \Delta x_{t+1}$.

[7]See Babillot *et al.* (1997), and Bhattacharya and Majumdar (2007) pg. 304.

# 3 An example using the *Cagan's Model*

In this section we apply the above correction analysis to the *Cagan's model of inflation*[8] in which the dynamic motion for the log of the price index ($p_t$) depends on the log of future expected price and the log of the money supply at time $t$ ($m_t$),

$$p_t = \alpha p^e_{t+1} + \beta m_t + \varepsilon_t \tag{12}$$

where $\alpha$ and $\beta$ are parameters depending on the absolute value of the elasticity ($\epsilon$) of money demand to expected inflation and $\varepsilon_t$ is an *i.i.d.* shock with zero mean[9].

Assume that the log of money follows an exogenous unit-root stochastic process

$$m_t = a + m_{t-1} + \eta_t \tag{13}$$

Now suppose that agents use simple static expectations $p^e_{t+1} = p_t$ ($\lambda = 1$)[10], by substituting agent's expectation in equation (12) we obtain the following dynamic motion of price

$$p_t = m_t + (1-\alpha)^{-1}\varepsilon_t. \tag{14}$$

Applying the static expectation scheme to equation (14) we observe the bias in the error term shown in figure 1 (graph below, dashed line)[11]. We compute 120 iterates of the static expectation scheme dropping the first 20. The error process fluctuates around a negative term ($\bar{\Xi}_t = 100^{-1}\sum_{t=1}^{100}\Xi_t = -0.9850$) as we expected from the solution of equation (9) with $\zeta = 0$[12].

Inserting, in the expectation, the empirical counterpart of the error correction term of equation (11)[13], $\zeta_t = k^{-1}\sum_{i=1}^{k}\Delta p_{t-i}$ we obtain the graph of figure 1 (above, dashed line)[14] with an average error close to zero ($\bar{\Xi}_t =$

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[8] We follow the presentation of the model in Smelser *et al.* (2001), pg. 5062.

[9] In particular $\alpha = \epsilon(1+\epsilon)^{-1}$ and $\beta = 1-\alpha$.

[10] The original Cagan' work used adaptive expectations. Anyway, since $\alpha$ is between 0 and 1, the rational expectation solution of this model also exists and is equal to $p_t = \varepsilon_t + \beta\sum_{i=0}^{\infty}\alpha^i E[m_{t+i}|\Omega_t]$; a discounted sum of the mathematical expectation of money conditional to the information set at time $t$.

[11] In the numerical analysis parameters were set to $a = 1$, $\alpha = 0.4$ and $\sigma(\varepsilon_t) = \sigma(\eta_t) = 0.1$.

[12] And also equation (5) in a deterministic setting.

[13] An interesting possibility is to estimate the drift/trend using a filter like the *short-time Fourier trasform.*

[14] The parameter $k$ was set to 4. Note that using the modified static expectation process $x^e_{t+1} = x_t + \zeta_t$, the implied price dynamics is $p_t = \alpha(1-\alpha)^{-1}\zeta_t + m_t + (1-\alpha)^{-1}\varepsilon_t$. In the simulation $\alpha = 0.4$.

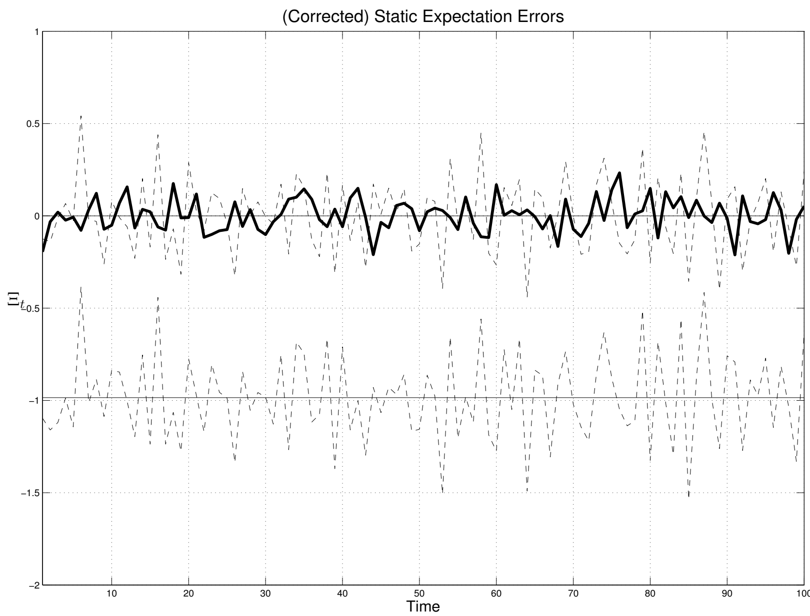


Figure 1: The error process for the simple static expectation case (graph below) and the modified process inserting an error correction term $\zeta_t = 4^{-1}\sum_{i=1}^{4}\Delta p_{t-i}$ (graph above, dashed line). The solid line (graph above) shows the rational expectation error process.

$-0.00038$). Finally, note that the standard deviation of the modified scheme ($\sigma(\Xi_t) = 0.2083$) is higher compared to the rational expectation error standard deviation ($\sigma(\varepsilon_t) = 0.1$) as shown in figure 1 (graph above, solid line).

# 4 Conclusions

In this paper we argue that, to be consistent with the Galton's result, the adaptive expectation scheme has to be adjusted in cases where the underling macrovariable is non-stationary. We read the unconditional mean error equal to zero as a necessary condition for the Lucas' rationality requirement and we show how to insert an error correction term to the standard adaptive expectation process. This procedure may have important applications in situations in which the adaptive scheme seems to be a good description of agents' behavior (Chow, 2011). Furthermore, this analysis may be of some value in situations in which the rational expectation solution is beyond our computational possibilities like in the *agent-based framework* (Tesfatsion and Judd,

2006) .

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