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Complementarity and inefficient renegotiation: an incomplete contract approach

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Abstract

In business, wage renegotiations often result in dead weight losses (e.g, a delay in production or labor strikes). We present a model in which one boss and two employees sign a wage contract before knowing what the future revenue will be. Because of the inefficiency of renegotiation, the optimal wage contract minimizes the probability of renegotiation. The boss will prefer to renegotiate when the contracted bonus is high compared to the realized revenue, whereas the employees will when it is relatively low. We show that the probability of renegotiation under the optimal contract and the expected efficiency loss from renegotiation increase as technology becomes more complementary.

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1 Introduction

The standard incomplete contract theory and that of property right theory (Grossman and Hart, 1986; Hart, 1995) consider contract renegotiation to be inefficient, not because renegotiation itself is costly but rather because it results in a so-called hold-up problem that can lead to underinvestment. In the business world, however, the renegotiation process is indeed inefficient even in the absence of such underinvestment problems (e.g., it can delay production or even trigger a labor strikes). Our work is a part of the growing literature on incomplete contracts that treats renegotiation itself as inefficient (see Segal and Whinston (2013) for a survey of this literature).

In particular, our work is most related to that of Hart (2009). Hart discusses the structure of optimal contracts that minimizes the expected loss from inefficient renegotiation. Although Hart focuses on trades between a single seller and a single buyer and models the renegotiation between the two by using Nash's bargaining solution, Hart's idea is applicable to many economic situations. In this paper, we consider one such situation: wage bargaining between one boss and two employees. Our proposal contains two features: (i) an explicit consideration of complementarity between the employees; and (ii) the adoption of Shapley's value as a solution to the renegotiation stage when treating a case with three participants. Furthermore, we discuss how these features affect the shape of the optimal contract and the welfare (note that these features are relevant in our three-participant model, not in the two-participant model in Hart (2009)). We also differ from Hart (2009) in the way we interpret the inefficiency of renegotiation. Whereas Hart (2009) attributes the source of renegotiation inefficiency to behavioral reasons, we interpret it as a delay in production.

We show that when the employees are more complementary, it is more difficult to prevent costly renegotiations. Our result predicts that costly renegotiations that can results in strikes are more likely to occur in an industry where production technology exhibits more complementarity, as in the automobile industry.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the factors that lead to renegotiation and those that do not given the existence of a contract. Section 4 derives the optimal wage contract(s) offered by the boss, the ex ante utilities of the participants, and the social surplus under the contract. Section 5 describes how the welfare under the optimal contract changes as the complementarity increases. In Section 6, we present our conclusion.

2 Model

Consider a boss B who is hiring two employees E_1 and E_2 for a future project. Let $\mathcal{N} \equiv \{B, E_1, E_2\}$ denote the set of participants. The revenue from the project depends on a state R, which is non-contractible and ex ante uncertain, and the state R distributes according to a probability density function f with unbounded support $[\underline{R}, +\infty)$, for some $\underline{R} \in \mathbb{R}$.

First, B makes a take-it-or-leave-it offer of a grand wage contract $w = (\alpha_1, \alpha_2, \beta_1, \beta_2)$ to both employees. In this contract, α_i is an ex ante payment which B pays to employee i immediately following the wage contract w. Moreover, β_i is an ex post payment that B pays to employee i after the state R is realized. Because R is non-contractible, both α_i and β_i are constants that do not depend on R. After B offers the grand contract w, both employees decide whether to accept the contract or not simultaneously. If at least one of them rejects the contract, the game ends and all participants receive their reservation utilities. We assume that the employees' reservation utilities are sufficiently small that B can offer a wage contract which gives the employees more expected utility than their reservation utilities.

After the wage contract is signed and the ex ante payments are paid, the state R is realized and observed by all three participants. Then, all three simultaneously decide whether to make a renegotiation offer or not. If no one offers to renegotiate, the participants continue to work on the project until it is complete. At this point, the project yields revenue equal to R. Then, B pays β_1 and β_2 to E_1 and E_2 respectively, and collects the realized revenue R. Thus, the gain from completing the project for B is $R - \beta_1 - \beta_2$ and the gain for the employees E_1 and E_2 is β_1 and β_2, respectively.

If at least one of the participants offer to renegotiate, a costly renegotiation occurs. From this renegotiation, it is determined who will continue to work on the project and how the ex post surplus will be allocated. The ex post surplus equals the revenue generated by the members who continue to work minus renegotiation costs $\lambda > 0$. For any nonempty subset of the participants $S \subseteq \mathcal{N}$, the ex post surplus generated when only the members of S continue to work on the project is denoted by $v(S)$. We assume that, for some $\theta \in [\frac{1}{2}, 1]$, $v(S)$ is as follows.

$$v(\{B\}) = v(\{E_1\}) = v(\{E_2\}) = v(\{E_1, E_2\}) = -\lambda, \quad (1)$$

$$v(\{B, E_1\}) = v(\{B, E_2\}) = (1-\theta)R - \lambda, \quad (2)$$

$$v(\mathcal{N}) = R - \lambda. \quad (3)$$

We can interpret (1)–(3) as follows. Here, (1) means that B has an essential asset, and that both the asset and at least one unit of labor are necessary for the project to yield a positive surplus. Moreover, (2) and (3) mean that, given that B continues to work on the project, $(1-\theta)R$ is the additional revenue from the first employee's labor and θR is the additional revenue from the second employee's labor.

In the above, θ measures the degree of technological complementarity between the employees in the project. When $\theta = 1$, the employees are perfect complements. In this case, if either employee chooses to leave the project, the project will earn nothing. As such, B and both employee are pivotal to success the project. When $\theta = 1/2$, the employees are perfect substitutes. In this case, given that B continues to work on the project, the first and second employees will produce the same additional revenue.

During a renegotiation, the ex post surplus is allocated to the participants through bargaining. We follow Hart and Moore (1990) in adopting the Shapley value as our solution to the bargaining game.[1] The characteristic function in this bargaining game is the function v described above. We assume that $\underline{R} - \lambda > 0$. This guarantees that given that all participants continue to work on the project, the project will yield a positive ex post surplus even when the worst state $\underline{R}$ is realized.

We denote the Shapley values of B, E_1, and E_2 by ϕ_B, ϕ_{E_1}, and ϕ_{E_2}, respectively. ϕ_B, ϕ_{E_1}, ϕ_{E_2} can be computed as follows (see Appendix A for a detail).

$$\phi_B = \frac{1}{6}\{-2\lambda + (4 - 2\theta R)\}. \quad (4)$$

$$\phi_{E_1} = \phi_{E_2} = \phi_E = \frac{1}{6}\{-2\lambda + (1+\theta)R\}. \quad (5)$$

Because $\phi_{E_1} = \phi_{E_2}$, we drop the subscript i on ϕ_{E_i} hereafter.

Finally, we should remark the solution to the renegotiation stage in this paper and Hart (2009). Whereas our solution adopts Shapley value, Hart's adopts Nash

[1]For a noncooperative justification of the use of Shapley value, see Gul (1989).

bargaining solution. Nevertheless, this difference is superficial. When the utilities are transferable and the number of participants is two, Shapley value and Nash bargaining solution have the same value. Thus, these two solutions are equal in the setting described by Hart (2009) (See Appendix A for a detail).

3 When Renegotiation Occurs

To understand when a contract $w \equiv (\alpha_1, \alpha_2, \beta_1, \beta_2)$ is optimal for B, this section investigates under what state R a renegotiation occurs and under what state R no renegotiation occurs given w.

According to (4) and (5), given a contract $w = (\alpha_1, \alpha_2, \beta_1, \beta_2)$ and R, all participants' payoffs without renegotiation are equal to or higher than their respective payoffs after renegotiation when

$$R - \beta_1 - \beta_2 \geq \frac{1}{6}\{-2\lambda + (4 - 2\theta)R\} \quad \text{and} \quad \min\{\beta_1, \beta_2\} \geq \frac{1}{6}\{-2\lambda + (1 + \theta)R\}. \tag{6}$$

If (6) does not hold, the contract is renegotiated. We assume that none of the participants chooses a weakly dominated action at this stage. Thus, when both inequalities in (6) hold with strict inequality, no participant makes an offer to renegotiate. When either one of the inequalities holds with equality, a renegotiation may or may not occur in an equilibrium. However, because the ex ante probability of this event is zero, this multiplicity does not affect the ex ante payoff or the optimal contract.

Note that the ex ante payments α_1 and α_2 do not appear in this condition because they have already been paid and sunk when the participants decide whether or not to make a renegotiation offer. (6) can be rewritten as follows.

$$R_L(\beta_1, \beta_2) \equiv \frac{3\beta_1 + 3\beta_2 - \lambda}{1 + \theta} \leq R \leq \frac{6\min\{\beta_1, \beta_2\} + 2\lambda}{1 + \theta} \equiv R_H(\beta_1, \beta_2). \tag{7}$$

In (7), the left and right inequalities correspond to the left and right inequalities in (6), respectively. When the ex post payments are symmetric, $\beta = \beta_1 = \beta_2$ for some β, (7) is represented as

$$R_L(\beta, \beta) \equiv \frac{6\beta - \lambda}{1 + \theta} \leq R \leq \frac{6\beta + 2\lambda}{1 + \theta} = R_L(\beta, \beta) + \frac{3\lambda}{1 + \theta}. \tag{8}$$

When $R \notin [R_L(\beta_1, \beta_2), R_H(\beta_1, \beta_2)]$, at least one participant offers to renegotiate and a renegotiation occurs. Thus, the probability of renegotiation is $\int_{R \notin [R_L(\beta_1,\beta_2), R_H(\beta_1,\beta_2)]} f(R)dR$, which is always positive because f has unbounded support.

The social surplus under a pair of ex post payments (β_1, β_2) is

$$W(\beta_1, \beta_2) = E(R) - \lambda \int_{R \notin [R_L(\beta_1,\beta_2), R_H(\beta_1,\beta_2)]} f(R)dR. \tag{9}$$

4 Optimal Wage Contract

In this section, we describe the optimal contract offered by B at the beginning of the game. This optimal contract maximizes B's ex ante utility. Let U_B, U_{E_1}, and U_{E_2} be the ex ante utilities for B, E_1, and E_2, respectively. Under an arbitrary contract $w \equiv (\alpha_1, \alpha_2, \beta_1, \beta_2)$, the ex ante utilities are computed as follows.

$$U_B = W(\beta_1, \beta_2) - U_{E_1} - U_{E_2},$$
$$U_{E_i} = \alpha_i + \beta_i \int_{R_L(\beta_1,\beta_2)}^{R_H(\beta_1,\beta_2)} f(R)\ dR\ + \int_{R \notin [R_L(\beta_1,\beta_2), R_H(\beta_1,\beta_2)]} \phi_E(R) f(R)\ dR \geq \overline{U}_{E_i} \ \text{ for } i = 1, 2.$$

Note that the social surplus $W(\beta_1, \beta_2)$ is the sum of the ex ante utilities U_B, U_{E_1}, and U_{E_2}. U_{E_i} is the sum of three terms: the ex ante payment α_i, which employee i receives ex ante; the ex post payment β_i multiplied by the probability that the contract will not be renegotiated–i.e., the probability of $R \in [R_L(\beta_1, \beta_2), R_H(\beta_1, \beta_2)]$; and $\phi_E(R)$ multiplied by the probability of renegotiation–i.e., the probability of $R \notin [R_L(\beta_1, \beta_2), R_H(\beta_1, \beta_2)]$.

Thus, the optimal wage contract $w^* \equiv (\alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*)$ solves the following:

$$\max_{(\alpha_1,\alpha_2,\beta_1,\beta_2)\in\mathbb{R}^4} U_B \equiv W(\beta_1, \beta_2) - U_{E_1} - U_{E_2}$$

subject to

$$U_{E_i} \equiv \alpha_i + \beta_i \int_{R_L(\beta_1,\beta_2)}^{R_H(\beta_1,\beta_2)} f(R)\ dR\ + \int_{R \notin [R_L(\beta_1,\beta_2), R_H(\beta_1,\beta_2)]} \phi_E(R) f(R)\ dR \geq \overline{U}_{E_i} \ \text{ for } i = 1, 2.$$

The constraints $U_i \geq \overline{U}_{E_i}$ for $i = 1, 2$ guarantee both employees' participation by maintaining their ex ante utility levels not less than their reservation utility levels $\overline{U}_{E_1}$ and $\overline{U}_{E_2}$.

Theorem 4.1. *Let $w^* \equiv (\alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*)$ be the optimal contract, offered by B at the beginning of the game given the complementarity between the employees θ. Furthermore, we denote the ex ante utility levels for B, E_1, and E_2 and the social welfare level attained by the contract w^* by U_B^*, $U_{E_1}^*$, $U_{E_2}^*$, and W^*, respectively. Then, w^* always exists and*

$$\alpha_i^* = \overline{U}_{E_i} - \beta_i^* \text{Prob}\left(R \in \left[\frac{6\beta_1^* - \lambda}{1+\theta}, \frac{6\beta_1^* + 2\lambda}{1+\theta}\right]\right)$$
$$- \int_{R \notin [R_L(\beta_1^*,\beta_2^*), R_H(\beta_1^*,\beta_2^*)]} \phi_E(R) f(R)\ dR \quad \textit{for } i = 1, 2,$$
$$\beta_1^* = \beta_2^* = \frac{(1+\theta)R_L^* + \lambda}{6},$$
$$R_L^* \in \arg\max_x \ \text{Prob}\left(R \in \left[x, x + \frac{3\lambda}{1+\theta}\right]\right),$$
$$U_{E_1}^* = \overline{U}_{E_1}, \qquad U_{E_2}^* = \overline{U}_{E_2}, \qquad U_B^* = W^* - \overline{U}_{E_1} - \overline{U}_{E_2}, \textit{ and}$$
$$W^* = E(R) - \lambda \cdot \text{Prob}\left(R \notin \left[\frac{6\beta_1^* - \lambda}{1+\theta}, \frac{6\beta_1^* + 2\lambda}{1+\theta}\right]\right).$$

If f is single-peaked, R_L^* is unique and hence w^* is unique too. Especially, if f is decreasing in its support, $R_L^* = \underline{R}$.

Proof. See Appendix A. □

Despite of the asymmetry of the employees' reservation utilities, the optimal ex post payments are symmetric. This is because the difference in the reservation utilities exclusively affects the optimal ex ante payments. In fact, B chooses the interval without any renegotiation $[x, x + (3\lambda/(1+\theta))]$ through ex post payments in order to maximize the probability that the contract will not be renegotiated. Therefore, W^* and U_B^* decrease as the probability distribution of the state R flattens. Conversely, they increase with less variance to the probability distribution.

5 Complementarity and Welfare

In this section, we investigate the relation between the complementarity θ and the social welfare under the optimal contract.

Given arbitrary symmetric ex post payments $(\beta_1, \beta_2) = (\beta, \beta)$, the probability that the contract will not be renegotiated $\int_{R_L(\beta,\beta)}^{R_L(\beta,\beta)+\{3\lambda/(1+\theta)\}} f(R)dR$ strictly decreases with θ unless β is so small that $R_L(\beta,\beta) + \{3\lambda/(1+\theta)\} < \underline{R}$. When $R_L(\beta,\beta) + \{3\lambda/(1+\theta)\} < \underline{R}$, β is not optimal because the probability that the contract will not be renegotiated increases when setting β as $R_L(\beta,\beta) = \underline{R}$. Thus, under the optimal contract, the probability that the contract will not be renegotiated and the welfare level under the optimal contract both decrease with θ.

As a result, we have the next theorem.

Theorem 5.1. *Let $W^*(\theta)$ be the social surplus for complementarity between the employees θ. Then, $W^*(\theta)$ strictly decreases with θ.*

Why does $W^*(\theta)$ decrease with the degree of complementarity θ? We must first note that B's Shapley value ϕ_B decreases with θ, because B's marginal contribution depends on θ only when B is the second participant with a marginal contribution that is decreasing with θ at that time. Thus, the ratio of B's Shapley value to the ex post surplus during renegotiation $R - \lambda$ decreases with θ and hence that of the employees increases with θ. Therefore, the employee's Shapley value ϕ_E is more correlated with R for a large θ than for a small θ. According to (6), when both the employees' ex post payments are β, the probability that the contract will not be renegotiated equals the probability that the following inequality holds:

$$\beta \leq \phi_E \leq \beta + \frac{\lambda}{2}.$$

When θ is large, ϕ_E varies considerably as R changes. Hence, the above inequality is unlikely to hold.[2]

These results suggest that the boss should decrease their employees' ex post bargaining power by adopting less complementary technology when possible. By adopting less complementary technology, the boss is more likely to avoid haggling over wages. It is worth noting that the boss benefits not from exploiting the employees but rather by making renegotiation less likely. In practice, making technology less complementary risks decreasing productivity. In such cases, the boss faces a trade-off between productivity and the expected renegotiation costs.

Finally, it is worth noting that it is not straightforward how welfare is affected when renegotiation costs λ increase. As λ increases, the probability of a renegotiation decreases, but the loss in efficiency in the event of a renegotiation increases. Thus, an increment of λ can either improve or worsen the welfare. With the bounded support of R, the first best welfare would be achieved both when $\lambda = 0$ and when λ is sufficiently high. Muramoto (2013) discussed how participants' payoffs and the welfare level change as renegotiation costs increase in a two-participant model with a binary state.

6 Concluding remarks

We developed an incomplete contract model in which a boss offers a non-contingent wage contract to two employees, where renegotiations of this contract are costly. We

[2] The discussion here is closely similar to that concerning the "self-enforcing" price range in Hart (2009).

characterized the shape of the wage contract, the participants' utility levels, and the social surplus in equilibrium. We also showed that the social surplus decreases in proportion to the complementarity between the employees, because the stronger the employees' bargaining powers are, the more difficult it becomes to prevent renegotiation.

Unlike in a standard dynamic principal-agent model, the participants cannot sign a so-called "renegotiation-proof" contract: a contract that prevents renegotiation even if any possible state R is realized. This is because the contracts are incomplete. If contracts were complete–and, as such contingent on R–the renegotiation can be prevented by changing the wage according to R. However, in our model, the participants need to renegotiate in order to change the wages according to R owing to the incompleteness of the contracts.

Our results are consistent with those in Hart (2009). Indeed, the sensitivity of the bargaining position to non-contractible and uncertain variables affects the contracting parties' ability to prevent costly renegotiations. This, in turn, affects the social surplus. A primary difference between this work and Hart's, however, is that we investigate the relation between the social surplus and the complementarity, which affects bargaining position indirectly through the characteristic function. By contrast, Hart investigated the relation between the social surplus and asset allocations, which was assumed to affect the sensitivity of the bargaining positions directly.

Appendix A

The Shapley Value: In order to compute the Shapley value, note that there are six possible coalition formation orders.

$$(i)BE_1E_2,\quad (ii)BE_2E_1,\quad (iii)E_1BE_2,\quad (iv)E_1E_2B,\quad (v)E_2BE_1,\quad (vi)E_2E_1B.$$

Each player's Shapley value is computed as the average of that player's marginal contributions to the six possible grand coalition formation orders.

First, we compute ϕ_B. In (i) and (ii), B is the first participant: hence, B's marginal contribution is $v(B) = -\lambda$. In (iii) and (v), B is the second participant; hence, B's marginal contribution is $v(\{B, E_i\}) - v(\{E_i\}) = (1-\theta)R$. In (iv) and (vi), B is the final participant; hence, B's marginal contribution is $v(\mathcal{N}) - v(\{E_1, E_2\}) = R$. Thus, we have (4) as follows:

$$\phi_B = \frac{1}{6}\{\ 2(-\lambda) + 2(1-\theta)R + 2R\} = \frac{1}{6}\{-2\lambda + (4 - 2\theta R)\}.$$

Second, we compute ϕ_{E_1}. In (iii) and (iv), E_1 is the first participant with a marginal contribution of $v(\{E_1\}) = -\lambda$. In (vi), E_1 is the second participant and E_2 is the first participant. Thus, E_1's marginal contribution is $v(\{E_1, E_2\}) - v(\{E_2\}) = 0$. In (i), E_1 is the second participant and B is the first participant. Thus, E_1's marginal contribution is $v(\{B, E_1\}) - v(\{B\}) = (1-\theta)R$. In (ii) and (v), E_1 is the final participant with a marginal contribution of $v(\mathcal{N}) - v(\mathcal{N} \setminus \{E_1\}) = \theta R$. Finally, $\phi_{E_2} = \phi_{E_1}$, because of the symmetry between E_1 and E_2. Hereafter, we drop the subscript i on ϕ_{E_i}. Thus, we have (5) as follows:

$$\phi_E = \frac{1}{6}\{2(-\lambda) + 0 + (1-\theta)R + 2\theta R\} = \frac{1}{6}\{-2\lambda + (1+\theta)R\}.$$

□

Equivalence of Nash bargaining solution and Shapley value: Now, we show Nash bargaining solution and Shapley value are equivalent when the number of players are two and their utilities are transferable. Suppose that there are two players 1 and 2 and utilities are transferable. Let v be the characteristic function. With Nash bargaining, each player i's payoff is $1/2[v(\{1,2\}) - v(\{1\}) - v(\{2\})] + v(\{i\})$. On the other hand, the Shapley value is computed as $1/2[v(\{1,2\}) - v(\{j\})] + 1/2v(\{i\}) = 1/2[v(\{1,2\}) - v(\{1\}) - v(\{2\})] + v(\{i\})$ for $j \neq i$. The equivalence is thus demonstrated. □

Proof of Theorem 4.1: Let $U_B^*, U_{E_1}^*$, $U_{E_2}^*$, and W^* denote the ex ante utility levels and social welfare attained under the optimal contract w^*. Because W is independent of α_1 and α_2, they are selected such that $U_{E_i} = \overline{U}_{E_i}$ for each i:

$$\alpha_i^* = \overline{U}_{E_i} - \beta_i^* \int_{R_L(\beta_1^*,\beta_2^*)}^{R_H(\beta_1^*,\beta_2^*)} f(R)\, dR - \int_{R \notin [R_L(\beta_1^*,\beta_2^*), R_H(\beta_1^*,\beta_2^*)]} \phi_E(R) f(R)\, dR \quad \text{for } i = 1, 2.$$

Next, β_1^* and β_2^* are selected to maximize $W(\beta_1, \beta_2)$, or equivalently the probability that the contract will not be renegotiated $\int_{R_L(\beta_1,\beta_2)}^{R_H(\beta_1,\beta_2)} f(R)\, dR$.

We prove $\beta_1^* = \beta_2^*$ by contradiction. Suppose that $\beta_1^* \neq \beta_2^*$. Let $\overline{\beta} \equiv (\beta_1^* + \beta_2^*)/2$. According to (7), $R_H(\overline{\beta}, \overline{\beta}) > R_H(\beta_1^*, \beta_2^*)$ and $R_L(\beta_1^*, \beta_2^*) = R_L(\overline{\beta}, \overline{\beta})$. According to the assumption on the support of f, $\int_{R_L(\overline{\beta},\overline{\beta})}^{R_H(\overline{\beta},\overline{\beta})} f(R)dR > \int_{R_L(\beta_1^*,\beta_2^*)}^{R_H(\beta_1^*,\beta_2^*)} f(R)dR$. Thus, a contradiction arises.

According to (8), β_1^* and β_2^* are selected such that

$$\beta_1^* = \beta_2^* = \frac{(1+\theta)R_L^* + \lambda}{6} \quad \text{for some } \ R_L^* \in \arg\max_x \int_x^{x+\frac{3\lambda}{1+\theta}} f(R)\ dR. \tag{10}$$

Finally, we show that there always exists some R_L^* that satisfies (10), but its uniqueness depends on the shape of the probability density function f. We can show the existence of R_L^* as follows. Let $A \equiv \int_{\underline{R}}^{\underline{R}+\{3\lambda/(1+\theta)\}} f(R)dR$. Then, there exists a K such that $\int_{R_L}^{R_L+\{3\lambda/(1+\theta)\}} f(R)dR < A$ for any $R_L > K$. If not, we can choose an infinite sequence $R_1, R_2, \ldots$ such that $R_{i+1} - R_i > 3\lambda/(1+\theta)$ and $\int_{R_i}^{R_i+\{3\lambda/(1+\theta)\}} f(R)dR > A$ for all i. For $n > 1/A$, $\int_{\underline{R}}^{R_n} f(R)dR > \sum_{i=1}^n \int_{R_i}^{R_i+\{3\lambda/(1+\theta)\}} f(R)dR > nA > 1$. It is therefore a contradiction.

The function $\int_{R_L}^{R_L+3\{\lambda/(1+\theta)\}}$ is continuous and hence has its maximum in the closed interval $[\underline{R}, K]$. We denote the maximum by B. Then, $B \geq A \geq \int_{R_L}^{R_L+\{3\lambda/(1+\theta)\}} f(R)dR$ for any $R_L > K$. Then, B is the global maximum. The existence has thus been shown.

R_L^* is unique when f is single peaked: for some $M \geq \underline{R}$ and for any $R, R' \geq \underline{R}$ (i) if $R < R' < M$, $f(R) < f(R') < f(M)$ and (ii) if $M < R < R'$, $f(M) > f(R) > f(R')$. Furthermore, when $M = \underline{R}$, or equivalently, when f is decreasing in its support such as with an exponential function and Pareto distribution, $R_L^* = \underline{R}$ and $\beta^* = (1+\theta)\underline{R} + \lambda/6$. For some non-single peaked density function f, R_L^* is not unique. Suppose that for some $\Delta > 3\lambda/(1+\theta)$, $f(R) = r$ for $R \in [\underline{R},\ \underline{R} + \Delta]$ and $f(R) < r$ for $R > \underline{R} + \Delta$. In this case, $\arg\max_x \int_x^{x+\{3\lambda/(1+\theta)\}} f(R)dR = [\underline{R},\ \underline{R}+\Delta-\{3\lambda/(1+\theta)\}]$ and the corresponding optimal symmetric ex post payments β^* exist infinitely in the continuum. □

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