Economics Bulletin

Volume 36, Issue 1

Consumption and Money Uncertainty at the Zero Lower Bound

Riyad Abubaker
University of California, Riverside

Abstract

With the recent financial crisis of 2008, the Federal Reserve (Fed) reduced the nominal interest rate to nearly zero. This paper examines the impact of the Zero Lower Bound (ZLB) on the uncertainty of personal consumption and money stock. To calculate the second conditional moments as a proxy for uncertainty, the paper implements a multivariate GARCH model on U.S. personal consumption and real money balance from January 1980 to December 2014. A dummy variable is added to the variance equation. Here, the dummy variable takes 1 after the Fed encounters the ZLB constraint. Our main findings demonstrate that consumption uncertainty declines; and real money uncertainty increases significantly when the economy is constrained by the zero lower bound.

Ph.D Candidate, Department of Economics, University of California, Riverside, 900 University Ave., Riverside, CA 92521, USA e-mail: rabub001@ucr.edu URL: http://www.Riyadabubaker.com

Citation: Riyad Abubaker, (2016) "Consumption and Money Uncertainty at the Zero Lower Bound", *Economics Bulletin*, Volume 36, Issue 1, pages 449-463

Contact: Riyad Abubaker - rabub001@ucr.edu.

Submitted: June 23, 2015. **Published:** March 17, 2016.

1. Introduction

This paper studies uncertainty measured by conditional volatility at the Zero Lower Bound. Uncertainty measured by conditional volatility is a negative feature of the U.S. economy through which the instability of the economy become transparent. Our interest focuses on consumption uncertainty. We examine if a zero interest rate regime affects the Fed's ability to fully offset shocks and achieve optimal policy. From a theoretical background, we demonstrate that both money uncertainty and consumption uncertainty are related to the nominal interest rate. To empirically illustrate this, the paper implements a multivariate GARCH model on U.S. personal consumption and real M1 from January 1980 to December 2014. Recent literature uses second conditional moments as a proxy of uncertainty. Engle's (1982) introduction of the GARCH model serves as a powerful tool in modeling economic uncertainty. Economists such as Chiriac and Voeb (2010), Fountas, Karanasos, and Kim (2006), Grier and Perry (2000), Grier, Henry, Olekalns, and Shields (2004) utilize this framework to model inflation and output volatility.

During the era of the Great Depression, economic uncertainty reached a record breaking high (Mathy, 2014). This triggered a reduction in employment, investment and output. With the recent financial crisis of 2008, the Federal Reserve reduced the federal fund rate to nearly zero. Even though the federal fund rate is constrained by the Zero Lower Bound, the Federal Reserve continuously aims to control inflation and output growth through unconventional policies. The Fed purchases governmental securities in order to keep it's policy rate low for an extended duration of time. Lowering the nominal interest rate reduces the opportunity cost of holding money. In Sidrausky (1967) model, the marginal rate of substitution between personal consumption and the quantity of money relies on the nominal interest rate. Hence, the opportunity cost of holding money reaches its lowest levels when the nominal interest rate is at the Zero Lower Bound. This can have a potential effect on the relationship between consumption and money.

The quantity of money, rather than the price of money can affect the economy if the Federal Reserve commits to a low interest rate for an extended period of time. This is due to the Federal Reserves reliance on open market operations during which money stock changes to maintain the federal fund rate at a very low level. From the perspective of the individual, the returns they get on their deposits made at commercial banks become less appealing when the short-term interest rate is lowered. To best illustrate, the Zero Lower Bound, Figure 1 provides series on the federal fund rate and the nominal interest rate given by the 3 month treasury bills. The remaining sections of this paper are organized as follows: Section 2 offers the related literature. Section 3 provides a theoretical background and introduces the empirical model. Section 4 describes the data and Section 5 yields the results of this paper. Section 6 offers the paper's conclusion.

2. Related literature

Most economist neglect to include a theoretical background for their GARCH empirical models; thus without properly establishing a connection between the empirics and theory, the results can become misleading. For this reason, this paper strives to use empirical methods directly driven by the money in the utility function (MIU) developed by Sidrauski (1967).

In Sidrauski's model, households gain utility from money services with the opportunity cost of holding money being given by the nominal interest rate. Thus, the Zero Lower Bound has its direct impact on the marginal rate of substitution between consumption and money services. Friedman (1969) argues that the optimal interest rate should equal zero. For Friedman, the marginal cost of creating additional money is zero; here the optimality condition implies that the opportunity cost of holding money has to be equal to the social cost of creating money. Nevertheless, it remains critical to examine the extent to which the zero optimal value of the nominal interest rate contributes to the volatility of economic variables.

According to Basu and Bundick (2014), the Zero Lower Bound creates risk and uncertainty that leads to precautionary savings by households. Complementary to these findings, Plante et al. (2014) suggests that the ZLB generates a strong correlation between macroeconomic uncertainty and real GDP growth. Others such as Canzoneri, Cumby, and Diba (2007); Hartzmark (2013); Crowder and Hoffman (1996) show that higher moments of consumption and output can be functions of the nominal interest rate. These researches don't include money stock in their models, even though money is assumed to be more sensitive to changes in the nominal interest rate.

The Zero Lower Bound can affect the economy through different channels. On the one hand, Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011) suggest that the fiscal policy is more effective than the monetary policy at the Zero Lower Bound. In contrary, Swanson and Williams (2014) argue that the effectiveness of monetary and fiscal policy is the same -regardless of the economy being in or out of the ZLB. For Swanson and Williams (2014), the output gap can be written as a function of the entire future path of the nominal interest rate, instead of the current rate. That is −the current rate can hit zero; however, the future nominal rate is unconstrained with the ZLB. Furthermore, Wieland (2012) demonstrates that negative supply shocks can be expansionary at the ZLB.

What is the role of the Central Bank at the Zero Lower Bound? If the nominal interest rate reaches the zero lower bound, the Federal Reserve will face a challenge in stabilizing prices. Ireland (2001) suggests that real money balances can eliminate the impact of the ZLB which the central banks use to control prices. In addition, Ireland articulates that agents are worse off under a zero nominal interest rate. Strong empirical evidence suggests that the real money balance increases the marginal utility of consumption (Koenig, 1990). The role of the Central Bank revolves around manipulating the money stock in a low and consistent nominal interest rate environment. The Central Bank uses the Forward Guidance tool. In this tool, the Central Bank promises to commit to a low interest rate for an extended period of time. For Coenen and Warne (2014), forward guidance can possibly be a successful policy in downsizing the risks to price stability.

3. The model

3.1. Theoretical background

In closing the last section, we introduce related literature that enhances our stance on the impact of the Zero Lower Bound on the uncertainty of personal consumption and money. Now, we turn our analysis over to methodology; here we provide our main empirical model with a brief introduction into the theory behind it -this helps encourage our empirical analysis and provide a sense of insight for our results. The purpose of this section is to derive the uncertainty model GARCH from the optimal conditions of the household maximization problem. The research in this paper is empirical. Our use of a theoretical model demonstrates the correlation among variables from a theoretical angle. In our theoretical section, we implement the Money in the Utility Function (MIU) by

Sidrauski (1967), Walsh (2010); Bhattacharjee and Thoenissen (2007). In this function, households maximize the following future discounted utility:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(C_t, M_t/P_t, N_t)\right] \tag{1}$$

subject to budget constraints:

$$P_tC_t + M_t + Q_tB_t \leq B_{t-1} + W_tN_t + M_{t-1} \tag{2}$$

Where N_t represents households supply of labor $-$ with W_t being nominal wage and M_t stands for nominal money balance. β is a discounting factor between zero and one. C_t is consumption, and P_t denotes the aggregate price index. B_t represents risk-less discounted bonds purchased at time t and maturing at time $t+1$. Each bond yields one unit of money at maturity with its price being Q_t where $Q_t = exp(-i_t)$. We use a non-separable utility function with respect to money and consumption. In this model, we follow Benchimol and Fourcans (2012)[1]:

$$U_t \equiv \frac{1}{1-\sigma}\left((1-b)C_t^{1-v} + b\left(\frac{M_t}{P_t}\right)^{1-v}\right)^{\frac{1-\sigma}{1-v}} - \frac{N_t^{1-\phi}}{1-\phi} \tag{3}$$

Households gain positive utility from consumption C_t and real money services M_t/P_t. However, they get negative utility from work N_t. σ is households coefficient of risk-aversion or the inverse of the intertemporal elasticity of substitution. v is the inverse of the elasticity of money holdings with respect to the interest rate. b is a positive scaler and ϕ^{-1} is the elasticity of work with respect to real wage.
The first order conditions are given by equation (4):

$$\begin{gathered} Q_t = \beta E_t\left[\frac{U_c'\Big(C_{t+1}, M_{t+1}/P_{t+1}, N_{t+1}\Big)}{U_c'\Big(C_t, M_t/P_t, N_t\Big)}\Pi_{t+1}^{-1}\right] \\ \frac{U_m'\Big(C_t, M_t/P_t, N_t\Big)}{U_c'\Big(C_t, M_t/P_t, N_t\Big)} = 1 - Q_t \end{gathered} \tag{4}$$

Where the top part of equation (4) represents the Euler equation. This is the marginal rate of substitution between consumption at time t and t+1. The bottom part of equation (4) equals the opportunity cost of holding money. Plugging the marginal utilities in

[1]Preference, money, and hours worked shocks are included in the authors' model.

equations (4) yields:

$$e^{-i_t} = \beta E_t \left[\frac{C_{t+1}^{-v}}{C_t^{-v}} \frac{\left((1-b)C_{t+1}^{1-v} + b\left(\frac{M_{t+1}}{P_{t+1}}\right)^{1-v}\right)^{\frac{1-\sigma}{1-v}-1}}{\left((1-b)C_t^{1-v} + b\left(\frac{M_t}{P_t}\right)^{1-v}\right)^{\frac{1-\sigma}{1-v}-1}} \Pi_{t+1}^{-1} \right]$$
$$\frac{b(M_t/P_t)^{-v}}{(1-b)C_t^{-v}} = 1 - Q_t \tag{5}$$

The gross inflation rate is given by Π_t, where $\Pi_t = Pt/P_{t+1} = 1 + \pi_t$. Equation (5) relates consumption, price level, and money stock to the nominal interest rate. The log-linearized Euler equation and opportunity cost of holding money can be written as[2]:

$$i_t = \theta_0 + \theta_1(E_t c_{t+1} - c_t) + \theta_2(E_t m_{t+1} - m_t) + \theta_3 E_t \pi_{t+1}$$
$$\lambda_0 m_t + \lambda_1 c_t = \lambda_3 i_t \tag{6}$$

where $c_t = log\left(\frac{C_t}{C}\right)$ and $m_t = log\left(\frac{M_t/P_t}{M/P}\right)$. These variables represent the deviations of their logs from steady state[3]. For simplicity, the parameters of the households optimality conditions are reduced to a set of θ and λ coefficients . We follow Canzoneri, Cumby, and Diba (2007); Fuhrer (2000), and Hartzmark (2013) by assuming that the Euler equation follows a conditional log-normality. For this reason, we include uncertainty measures and log-linearize the Euler equation in (5) as follows:

$$\begin{aligned} i_t = \theta_0 + \theta_1(E_t c_{t+1} - c_t) + \frac{\theta_1^2}{2} V_t c_{t+1} + \theta_2(E_t m_{t+1} - m_t) + \frac{\theta_2^2}{2} V_t m_{t+1} + \theta_3 E_t \pi_{t+1} \\ + \frac{\theta_3^2}{2} V_t \pi_{t+1} + \theta_1\theta_2 Cov_t(c_{t+1}, m_{t+1}) + \theta_1\theta_3 Cov_t(c_{t+1}, \pi_{t+1}) + \theta_2\theta_3 Cov_t(m_{t+1}, \pi_{t+1}) \end{aligned} \tag{7}$$

The above equation is estimated by the moment generating function[4]. V_t is the conditional variance, and Cov_t is the conditional covariance. We use this equation to show that consumption and money uncertainty are both functions of the nominal interest rate. Alternatively, the interest rate drives the relationship between consumption, real money balance, and their uncertainty.

3.2. Empirical model

This section examines whether the second moments of consumption and money are affected by the level of the nominal interest rate. In addition, we focus on whether or not these moments behave asymmetrically within a money regime that is constrained by a remarkably low interest rate. Our empirical model is motivated by Fuhrer (2000) and Canzoneri (2007) who show that the Euler equation can be written as a vector autoregressive. We use a multivariate GARCH model that links consumption, real money, and their uncertainty to the nominal interest rate. Our adaptation of the GARCH model complements the work of Engle and Sheppard (2001) and Engle (2002). The mean equations of the multivariate GARCH model are presented in the form of Vector autoregression VAR

[2] The reader is encouraged to see Benchimol and Fourcans (2012) for more details on how to derive equation (6) from (5).

[3] Values without time subscript refer to the steady state

[4] The moment-generating function for a normal random variable $X : E[e^{bX}] = b\mu + \frac{1}{2}b^2\sigma$

(P) as follows:

$$Y_t = \Lambda + \sum_{s=1}^{P} \Psi_s Y_{t-s} + \Xi_t$$
$$\Xi_t | \Omega_{t-1} \sim N(0, H_t) \tag{8}$$

where 0 is a null vector, and Ω_{t-1} is a past information set. H_t is a time varying variance−covariance matrix. Y_t is 2×1 a vector of dependent variables, and Ξ_t is 2×1 a vector of error terms

$$Y_t = \begin{bmatrix} c_t \\ m_t \end{bmatrix}, \quad \Lambda = \begin{bmatrix} a_c \\ a_m \end{bmatrix}, \quad \Psi_s = \begin{bmatrix} \psi_{cc,s} & \psi_{cm,s} \\ \psi_{mc,s} & \psi_{mm,s} \end{bmatrix}, \quad \Xi_t = \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{m,t} \end{bmatrix}$$

We implement a multivariate GARCH model with a dynamic conditional correlation DCC-GARCH as proposed by Engle and Sheppard (2001). In this model, the variance−covariance matrix H_t can be written as:

$$H_t = D_t R_t D_t \tag{9}$$

where D_t is the 2×2 diagonal matrix of time varying standard deviations from GARCH models with $\sqrt{h_{kk,t}}$ on the $k^{th.}$ diagonal. R_t is a dynamic correlation matrix. In the DCC-GARCH model[5],

$$h_{kk,t} = s_k + \alpha_k \varepsilon^2_{k,t-1} + \beta_k h_{kk,t-1}$$
$$k = c, m \tag{10}$$

The conditional covariance between consumption and money is written as:

$$h_{cm,t} = \rho_{cm,t} \sqrt{h_{cc,t} h_{mm,t}} \tag{11}$$

where the diagonal elements $h_{cc,t}$ and $h_{mm,t}$ follow univariate GARCH processes and $\rho_{cm,t}$ follows the dynamic process specified in Engle (2002).
Bollerslev (1990) proposed a constant conditional correlation model CCC-GARCH in which the matrix R_t is assumed to be time-invariant which implies that:

$$H_t = D_t R D_t \tag{12}$$

and

$$h_{cm,t} = \rho_{cm} \sqrt{h_{cc,t} h_{mm,t}} \tag{13}$$

To avoid unidentified parameters in the DCC-GARCH, the paper tests the constancy of conditional correlation by performing two tests. The first test is derived from the seminal work of Engle and Sheppard(2001). Here, the null hypothesis is written as:

$$H_0 : R_t = R \quad \forall t \in T \tag{14}$$

The null hypothesis is tested against H_a where $H_a : vech(R_t) = vech(R) + \beta_1 vech(R_{t-1}) + \beta_2 vech(R_{t-2}) + ...\beta_p vech(R_{t-p})$ and p is the number of lags.

The second constant correlation test is the Lagrange multiplier statistic, LMC, as introduced by Tse (2000). We implement these tests to support the use of appropriate

[5]The paper places more emphasis on the money and consumption uncertainty(conditional variance), not on the correlation between these variables

multivariate GARCH models. In rejecting the null hypothesis, we demonstrate that the correlations among variables are subject to time-varying[6]. If we reject the null hypothesis; then the use of dynamic conditional correlation GARCH is seen as more suitable for the analysis of our paper. In equation (6) of the theoretical background section of our paper, we indicate that a correlation between money and consumption exists. Hence, as the nominal interest rate varies, the marginal rate of substitution between consumption and money stock will change. When the Federal Reserve adjusts its policy rate against prices and real activity fluctuations to achieve maximum employment and stabilize prices, consumption and money change accordingly. The demand for money service increases due to the U.S. monetary authority reduction of the nominal interest rate. Maintaining the nominal interest rate at low target levels requires the Fed to apply expansionary monetary policy. This policy is often known as Quantitative Easing where the Fed purchases financial assets from commercial banks and other financial institutions. . We motive the DCC-GARCH model by testing the constancy of the conditional correlation −which reveals that consumption and money are driven by the dynamics of the nominal interest rate. Our paper investigates the potential impact of the nominal interest rate level on consumption and money stock uncertainty. The lagged nominal interest rate is added i_{t-1} to equation (10) as follows[7]:

$$\begin{aligned} h_{kk,t} = s_k + \alpha_k \varepsilon^2_{k,t-1} + \beta_k h_{kk,t-1} + \delta_k i_{t-1} \\ k = c, m \end{aligned} \tag{15}$$

The paper additionally examines the uncertainty surrounding consumption and real money stock in a zero nominal interest rate regime. We do this by including a dummy variable $\mathbb{I}_t$. This dummy variable takes 1 after the Fed encounters the ZLB constraint[8]. We include another dummy variable $\mathcal{T}_t$ which controls the effects on uncertainty generated by recessions. The latter dummy variable takes 1 whenever the National Bureau of Economic Research (NBER) dates time t as a recession,

$$h_{kk,t} = s_k + \alpha_k \varepsilon^2_{k,t-1} + \beta_k h_{kk,t-1} + \gamma_k \mathbb{I}_{t(t \geq 01.2009)} + \eta_k \mathcal{T}_t$$

$$\mathbb{I}_t = \begin{cases} 1 & \text{if} \quad t \geq \text{Jan.2009} \\ 0 & \text{if} \quad t < \text{Jan.2009} \end{cases} \qquad \mathcal{T}_t = \begin{cases} 1 & \text{if} \quad \text{NBER recession} \\ 0 & \text{if} \quad \text{Otherwise} \end{cases} \tag{16}$$

Without including the recessionary dummy variable $\mathcal{T}_t$, the coefficient γ may be incorrectly interpreted. For example, during the Great Recession of 2008-2009, consumption recorded an unusually sharp contraction. Subsequently, as a result of larger recessionary fluctuations, −a statistically significant γ may result in the event of an omitted variable $\mathcal{T}_t$. NBER dates the duration of the Great Recession from January 2008 to June 2009. During the time span within the economic recession of 2008-2009, we distinguish the behavior of consumption and money uncertainty within a nearly zero interest regime by including this recessionary dummy variable. Finally, an estimation for the multivariate

[6]The paper examines the constancy of conditional correlation to verify which multivariate GARCH model fits the data so that model parameters are not unidentified.

[7]Previous literature uses the level of the interest rate as an explanatory variable in the conditional variance equations. [Henry and Olekalns (2005); Gruber and Vigfusson (2012)]

[8]Lamoureux and Lastrapes (1990) apply similar models in which a dummy variable is added to the variance equation of the GARCH model.

GARCH model is provided by applying the Maximum Likelihood[9].

4. Data

Our variables are taken from the Federal Reserve Economic Data (FRED). These variables include: money stock (M1), monthly nominal personal consumption expenditure, Consumer Price Index (CPI), NBER recessions indicators, the federal fund rate, and the 3-month Treasury bill. M1 and consumption are in billions of U.S dollars. Real money balance is the nominal money stock divided by the Consumer Price Index. We apply the Hodrick-Prescott (HP) filter to take the cyclical component of consumption c_t and real money stock m_t. By de-trending the variables of consumption and real money stock, the data is transformed to become stationary. The nominal interest rate i_t is the 3-month Treasury bill. We implement an Augmented Dickey Fuller test to examine the unit root up to 10 lags. This test produces a p-value of 0.0000 for consumption and real money. Thus this implies that the unit root is rejected for both variables. Table 1 provides descriptive statistics for the main variables used in this research.

Our data is based on monthly frequency which ranges from January 1980-December 2014. Additionally, we implement a White diagnostic test to examine whether consumption and real M1 residuals display heteroskedasticity. From this, we conclude that the null hypothesis of homoscedasticity is rejected with a p-value=0.000 for each variable. Figure 2 in our Tables and Figures section plots the HP cyclical components of consumption and real money balances. In our section to follow, we turn our discussion over to our main findings.

5. Main Empirical Results

The constant correlation test results are shown in Table 2. Under the null hypothesis, the conditional correlation between consumption and money stock is constant. However, in Table 2, the constant correlation test suggests that we can safely reject the null hypothesis in favor of a multivariate GARCH with dynamic conditional correlation (DCC-GARCH). In selecting the number of VAR lags used, we rely on the Akaike Information Criterion; the mean equation (8) is estimated with 4 lags[10].

Table 3 reports the impact of the nominal interest rate on consumption and money conditional volatility. The lagged nominal interest rate is added to the variance equations. In addition, in this table, we use the 3 month treasury bill as a short term risk-less nominal rate.

Consumption uncertainty is positively affected by the level of the nominal interest rate with a coefficient of 0.113. Personal consumption expenditures is less volatile and smoother when the economy experiences a lower nominal interest rate. On the other hand, real money stock exhibits a negative relationship with a coefficient of -0.234 between the nominal interest rate and its uncertainty $-$with more magnitude in comparison to consumption uncertainty. From the results mentioned above, one is left to ponder whether consumption uncertainty increases when the nominal interest rate reaches its lowest levels at the ZLB? To adequately answer the question we raised above, we add a dummy variable to the variance equations as it appears in Table 4.

This table aims to show whether the zero nominal interest rate environment contributes to consumption and money uncertainty. It appears from this table that both

[9] See Engle and Sheppard (2001) for more details on the estimation methods.

[10] AIC is described by Ivanov and Kilian (2005) to be the best method for monthly data lag order selection.

consumption and money uncertainty increases significantly during recessions. With regards to consumption uncertainty at the ZLB, our results reveal a negative significant coefficient of the dummy variable $\mathbb{I}_t$ that equals -0.79. However, a significant increase with respect to money uncertainty is seen throughout the period in which the nominal interest rate is at or near zero. These results indicate the Fed's promise to maintain the nominal rate consistently low for an extended period of time resulted in lower fluctuations in personal consumption. However, when the nominal interest rate is pushed to the zero, a higher volatility in real money stock is noted. In Figure 3, the conditional standard errors of money stock display notable spikes within the Zero Lower Bound −especially in the Quantitative Easing periods. In targeting the nominal interest rate, The Fed can accommodate all shocks to money demand with equivalent shocks to money supply. Hence, when the Fed follows a nearly zero interest rate policy; consumption volatility decreases at the cost of a significantly increased volatility of money shock.

6. Conclusion

This paper studies the implications of the ZLB on uncertainty of consumption and money stock. We employ a dynamic conditional correlation type of multivariate GARCH model (DCC-GARCH). In this model, the nominal interest rate and dummy variables are added to the variance equations. The model is implemented on monthly data within the U.S. that ranges from January 1980 to December 2014. Our empirical findings suggest that consumption uncertainty displays a notable decrease in the months following January 2009. Within the same period of time, our results indicate that money uncertainty increases. The primary focus of this study lies in assessing the extent at which a monetary policy regime within the ZLB may influence the uncertainty that surrounds personal consumption. In conclusion, this paper proposes that the Fed's attempt to keep the federal fund rate at a low target results in lower consumption volatility at the expense of higher volatility in money stock.

References

[1] Attanasio, O. P. and H. Low (2004).“Estimating Euler Equations.”, *Review of Economic Dynamics*, 7(2): 406−435.

[2] Basu, S. and B. Bundick (2014). “ Downside Risk at the Zero Lower Bound.”, Boston College, NBER, and Federal Reserve Bank of Kansas City mimeo

[3] Benchimol, J. and , A. Fourcans (2012).“ Money and risk in a DSGE framework: a Bayesian application to the Eurozone .”, *Journal of Macroeconomics*, 34(1): 95−111

[4] Bhattacharjee A. and Ch. Thoenissen (2007).“ Money and Monetary Policy in Dynamic Stochastic General Equilibrium Models.”, *Manchester School*, 75 (s1): 88122

[5] Bilbiie,F. O., T. Monacelli, and R. Perotti (2014).“Is Government Spending at the Zero Lower Bound Desirable?.”, Working Paper No. 20687, NBER.

[6] Bloom, N. (2009).“The Impact of Uncertainty Shocks.”, *Econometrica*, 77(3): 623−685.

[7] Bloom, N. (2014). “Fluctuations in Uncertainty.”, *Journal of Economic Perspectives*, 28(2): 153−176.

[8] Bodenstein, M., J. Hebden, and R. Nunes (2012). "Imperfect Credibility and the Zero Lower Bound.", *Journal of Monetary Economics*, 59(2012): 135−149.

[9] Bodenstein, M., L. Guerrieri, and Ch. Gust (2013). "Oil Shocks and the Zero Bound on Nominal Interest Rates.", *Journal of International Money and Finance*, 32(2013): 941−967.

[10] Bollerslev, T. (1990). " Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Model.", *The Review of Economics and Statistics*, 72(3): 498−505

[11] Bundick, B. (2013). "Forward Guidance Under Uncertainty.", Boston College mimeo

[12] Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2007). "Euler Equations and Money Market Interest Rates: A challenge for Monetary Policy Models.", *Journal of Monetary Economics*, 54(2007): 1863−1881.

[13] Chattopadhyay, S. and B. C. Daniel (2014). " The Ination Target at the Zero Lower Bound.", IIT Kharagpur and University at Albany SUNY mimeo

[14] Christiano, L., M. Eichenbaum, and S. Rebelo (2011). "When Is the Government Spending Multiplier Large?.", *Journal of Political Economy*, 119(1): 78−121.

[15] Coenen, G. and A. Warne (2014). "Risks to Price Stability, the Zero Lower Bound, and Forward Guidance: A Real-Time Assessment.", *International Journal of Central Banking*, 10(2): 7−54.

[16] Crowder, W. J. and D. L. Hoffman (1996). "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited.", *Journal of Money, Credit and Banking*, 28(1): 102−118.

[17] Engle, R.(2002). " Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models.", *Journal of Business and Economic Statistics*, 20(3): 339−350

[18] Engle, R.(1982). "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.", *Econometrica*, 50(4): 987−1007.

[19] Engle, R. and K. Sheppard (2001). "Theoretical and Empirical properties of Dynamic Conditional Correlation Multivariate GARCH.", Working Paper 8554, NBER

[20] Fountas, S., M. Karanasos, and J. Kim (2006). " Inflation Uncertainty, Output Growth Uncertainty and Macroeconomic Performance.", *Oxford Bulletin of Economics and Statistics*, 68(3): 0305−9049

[21] Friedman, M. (1969). " The optimum quantity of money and other essays.", Chicago Aldine Pub. Co.

[22] Gambacorta, L., B. Hofmann, and G. Peersman (2014). "The Effectiveness of Unconventional Monetary Policy at the Zero Lower Bound: A Cross-Country Analysis.", *Journal of Money, Credit and Banking*, 46(4): 615−642.

[23] Gavin, W. T., Keen, B. D., Richter, A. W., and N. A. Throckmorton (2014). "The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics.", Working Paper No. 2013-007E, Federal Reserve Bank of St. Louis

[24] Gilchrist, S., D. Lopez−Salido, and E. Zakrajek (2014). "Monetary Policy and Real Borrowing Costs at the Zero Lower Bound.", Working Paper No. 20094, NBER.

[25] Gomes, S., P. Jacquinot, R. Mestre, and J. Sousa (2010). "Global policy at the zero lower bound in a large-scale DSGE model, Working Paper Series 1254, European Central Bank.

[26] Grier, K. and M. Perry (2000). "The Effects of Real and Nominal Uncertainty on Inflation and Output Growth: Some GARCH-M Evidence.", *Journal of Applied Econometrics*, 15(2000): 45−58.

[27] Grier, K., O. Henry, N. Olekalns, and K. Shields (2004). " The Asymmetric Effects of Uncertainty on Inflation and Output Growth.", *Journal of Applied Econometrics*, 19(5): 551−565

[28] Gruber, J. and R. Vigfusson (2012) " Interest Rate and the Volatility and Correlation of Commodity Prices.", International Finance Discussion Papers No. 1065, Board of Governors of the Federal Reserve System.

[29] Hartzmark, S. M. (2013). " Economic Uncertainty and Interest Rates.", University of Southern California mimeo

[30] Henry, O. and N. Olekalns (2005). " Equity Return and Short-Term Interest Rate Volatility: Level Effects and Asymmetric Dynamics.", MRG Discussion Paper Series No. 0206, School of Economics, University of Queensland

[31] Ireland, P. N. (2001). "The Real Balance Effects.", Working Paper No. 8136, NBER.

[32] Ivanov, V. and L. Kilian (2005). " A Practitioner's Guide to Lag Order Selection For VAR Impulse Response Analysis.", *Studies in Nonlinear Dynamics and Econometrics."*, 9(1): Article 2

[33] Kiley, M.(2014). "The Response of Equity Prices to Movements in Long-Term Interest Rates Associated with Monetary Policy Statements: Before and After the Zero Lower Bound.", *Journal of Money, Credit and Banking*, 46(5): 1057−1071.

[34] Koeda, J. (2011). " Japanese Yield Curves In and Out of a Zero Rate Environment: A Macro-Finance Perspective.", Working Paper No. 254, CARF

[35] Koenig, E. F. (1990). "Real Money Balances and the Timing of Consumption: An Empirical Investigation.", *The Quarterly Journal of Economics*, 105(2): 399−425.

[36] Lamoureux, Ch. and W. Lastrapes (1990). " Persistence in Variance, Structural Change, and the GARCH Model.", *Journal of Business and Economic Statistics*, 8(2): 225−34

[37] Mathy, G. (2014). " How Much Did Uncertainty Shocks Matter in the Great Depression?.", American University mimeo

[38] Parker , J. and B. Preston (2005). " Precautionary Saving and Consumption Fluctuations.", *American Economic Review*, 95(4): 1119−1143

[39] Plante, M., Richter, A. W., and N. A. Throckmorton (2014). "The Zero Lower Bound and Endogenous Uncertainty.", Working Paper No. 1405, Federal Reserve Bank of Dallas

[40] Rahman, S. (2011). "The Asymmetric Effects Of Oil Price Shocks.", *Macroeconomic Dynamics*, 15(S3): 437−471.

[41] Sidrauski, M. (1967). "Rational Choice and Patterns of Growth in a Monetary Economy.", *American Economic Review*, 57(2): 534−544.

[42] Swanson, E. T. and J. C. Williams (2014). "Measuring the Effect of the Zero Lower Bound on Yields and Exchange Rates in the U.K. and Germany.", *Journal of International Economics*, 92(1): 2−21.

[43] Swanson, E. T. and J. C. Williams (2014). "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates.", *American Economic Review*, 104(10): 3154−3185.

[44] Taylor, J. B. (1993). "Discretion Versus Policy Rules in Practice.", *Carnegie-Rochester Conference Series on Public Policy*, 39(1993): 195−214.

[45] Tse Y. K. (2000). " A Test for Constant Correlations in a Multivariate GARCH Model.", *Journal of Econometrics*, 98: 107−127

[46] Tse, Y. K. and A. K. Tsui (2002). "A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations.", *Journal of Business & Economic Statistics*, 20(3): 351−362.

[47] Walsh, C. E. (2010). "Monetary Theory and Policy.", *MIT Press, Cambridge, MA.*, Third Edition: 33− 75

[48] Wieland, J. (2012). "Are Negative Supply Shocks Expansionary at the Zero Lower Bound?.", University of California at San Diego mimeo

[49] Woodford, M. (2011). "Simple Analytics of the Government Expenditure Multiplier.", Working Paper No. 15714, NBER

[50] Woodford, M. (2012). "Methods of Policy Accommodation at the Interest-Rate Lower Bound.", presented at the Jackson Hole symposium

[51] Wu,J. C. and F. D. Xia (2014). "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound.", Working Paper No. 20117, NBER.

Tables and Figures

Table 1: Summary statistics (1980.01−2014.12)

Variable	Mean	Standard Deviation	Min	Max	N
c_t	1.71×10^{-11}	0.0081	−0.025742	0.026441	420
m_t	$-\ 5.78\times10^{-11}$	0.020455	−0.076967	0.070487	420
Fed_t(%)	5.179071	4.070646	0.07	19.1	420
i_t(%)	4.686667	3.571948	0.01	16.3	420

Fed_t is the federal fund rate and i_t is 3-Month Treasury Bill.

Table 2: Constant correlation test

Test	Stat	P-value
Tse (2000)		
LMC	75.5812	0.0000000
Engle and Sheppard (2001)		
5 lags	144.903	0.0000000
10 lags	148.392	0.0000000

Stat is the test statistic of the non-constant correlation.
P-value is calculated for the null hypothesis H0: $R_t = R$

Table 3: Dynamic conditional correlation multivariate GARCH including the First Lag of the Nominal Interest Rate.

	Variance equations (15)			
	Consumption variance, $h_{c,t}$		Money stock variance , $h_{m,t}$	
Variable	Coefficient	Standard Error	Coefficient	Standard Error
constant	−13.58521 ***	0.4236789	−11.54428 ***	0.5949923
$\varepsilon^2_{c,t-1}$	0.1650034 ***	0.0412326	——	——
$\varepsilon^2_{m,t-1}$	——	——	0.3132255***	0.0746904
$h_{c,t-1}$	0.7182717 ***	0.0683992	——	——
$h_{m,t-1}$	——	——	0.65544***	0.0791658
i_{t-1}	0.1132174 ***	0.035852	−0.2342142***	0.0706924

***significant at 1%; **significant at 5%; *significant at 10%. .

Table 4: Dynamic conditional correlation multivariate GARCH within the Zero Lower Bound.

	Variance equations (16)			
	Consumption variance, $h_{c,t}$		Money stock variance , $h_{m,t}$	
Variable	Coefficient	Standard Error	Coefficient	Standard Error
constant	-13.17557 ***	0.3619935	-12.87203 ***	0.4051186
$\varepsilon^2_{c,t-1}$	0.1213627 ***	0.0332584	——	——
$\varepsilon^2_{m,t-1}$	——	——	0.2401187***	0.0626496
$h_{c,t-1}$	0.7450905 ***	0.0568198	——	——
$h_{m,t-1}$	——	——	0.6718309***	0.0657819
$\mathbb{I}_{t(t\geq 01.2009)}$	-0.7942867 ***	0.3141533	1.486595 ***	0.3856828
$\mathcal{T}_t$	1.345503 ***	0.3333198	1.946544 ***	0.3891582

***significant at 1%; **significant at 5%; *significant at 10%. .

Figure 1: The Federal Fund Rate and 3-Month Treasury Bill before and after the ZLB.

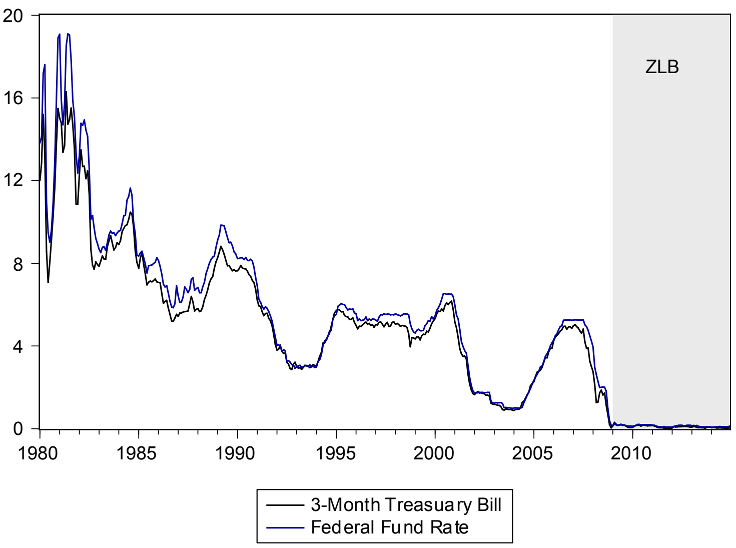

Figure 2: Consumption and real M1 from 1980.01 to 2014.12. The area shaded green represents NBER recessions.

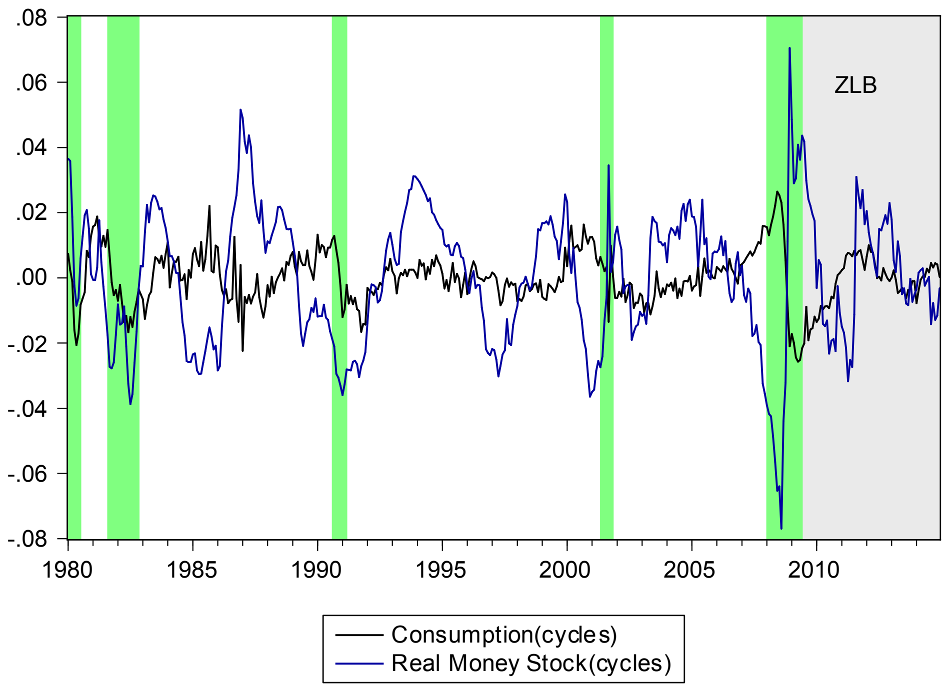

Figure 3: The conditional standard errors of consumption and real M1. The area shaded green represents NBER recessions.