Economics Bulletin

Volume 35, Issue 4

Welfare criteria for quasi-hyperbolic time preferences

Minwook Kang
Nanyang Technological University

Abstract

The most prevalent welfare criteria for quasi-hyperbolic discounting models are the Pareto criterion that takes into account all periods' intertemporal utilities and the long-run perspective criterion that considers the intertemporal utility in fictitious period 0. This paper shows that efficiency by the Pareto criterion implies efficiency by the long-run criterion.

I gratefully acknowledges AcRF Tier-1 Grant (RG171/14) for research support. I would like to thank associate editor Kangoh Lee and the anonymous reviewer for their constructive comments and suggestions.
Citation: Minwook Kang, (2015) "Welfare criteria for quasi-hyperbolic time preferences", *Economics Bulletin*, Volume 35, Issue 4, pages 2506-2511
Contact: Minwook Kang - mwkang@ntu.edu.sg.
Submitted: October 25, 2015. **Published:** December 13, 2015.

1 Introduction

Economists have tried to incorporate present-biased preferences into economic models. The most successful model of present-bias is quasi-hyperbolic discounting time preferences, which have been developed by Strotz (1956), Phelps and Pollak (1968) and Pollak (1968).[1] Laibson (1996, 1997) showed that the economy with quasi-hyperbolic preferences results in suboptimal undersaving problems.[2] The suboptimality of present-biased agents' decisions provides justifications for government interventions. After Laibson's work, there have been plenty of policy suggestions and implications, such as consumption taxation, encouraging savings, retirement and pension system, and accumulation of illiquid assets.[3]

These economic policies cannot be effectively evaluated without well-defined welfare functions. With present-biased preferences, there is no single dominating welfare criterion because there are multiple preferences one can use. Among several welfare criteria, the major two are the *Pareto* criterion and the *long-run perspective* criterion.[4] The Pareto criterion takes into account intertemporal utilities from all perspectives, while the "long-run perspective" welfare criterion considers intertemporal utility from a prior perspective where the person weights all future periods equally.

Even though the two welfare criteria are commonly used in research of policy implications,[5] to my knowledge none of these works compare these two criteria. It is likely that the two welfare criteria are quite distinct because they are based on different intertemporal utilities. However, this paper shows that if a strategy is superior to another in the Pareto sense, it would be also superior based on the long-run perspective criterion. Many research papers assume that the outside authority, such as the government, evaluates individual's welfare based on long-run utility rather than the biased intertemporal

[1] See Frederick, Loewenstein and O'Donoghue (2002) and Loewenstein and Prelec (1992) for broad discussion on time-inconsistent preferences.

[2] The quasi-hyperbolic preferences are applied in various economic models such as a repreated games model by Chade, Prokopovych and Smith (2008), a contract design model by Dellavigna and Malmendier (2004), mechanish design problems by Gilpatric (2008) and principal-agent problems with moral hazard by Yilmaz (2013).

[3] See O'Donoghue and Rabin (2006), Laibson (1996, 1998), Diamond and Koszegi (2003), Thaler and Benartzi (2004) and many others.

[4] For detailed discussion of the two welfare criteria, see O'Donoghue and Rabin (2015) on page 276 and O'Donoghue and Rabin (1999a) on pages 112-113.

[5] For policy evaluations based on the "long-run" welfare criterion, see O'Donoghue and Rabin (1999b, 2001, 2003), Krusell and Smith (2002), Diamond and Koszegi (2003) and Guo and Krause (2015). For policy implications based on the Pareto criterion, see Goldman (1979), Laibson (1996) and Bisin, Lizzeri and Yariv (2015).

utilities. The main result of this paper implies that Pareto-improving policies in terms of all the biased intertemporal utilities is also welfare-improving from the unbiased long-run perspective.

2 The main result

An individual lives $T \geq 3$ periods. Let $u_\tau \in \mathbb{R}$ be the individual's instantaneous utility experienced in period $\tau \in \{1, 2, ..., T\}$. Given $\{u_\tau\}_{\tau=1}^T \in \mathbb{R}^T$, the intertemporal preferences from the perspective of period t is

$$U_t\left(\{u_\tau\}_{\tau=t}^T\right) = u_t + \beta \sum_{i=1}^{T-t} \delta^i u_{t+i}. \tag{1}$$

With this functional form in (1), $\beta = 1$ corresponds to exponential discounting while $\beta \in (0, 1)$ reflects present bias.

The Pareto criterion ranks different strategies based on all periods' intertemporal utilities. With two different sets of instantaneous utilities, $\{u_\tau\}_{\tau=1}^T$ and $\{u'_\tau\}_{\tau=1}^T$, the Pareto criterion is defined as follows:

Definition 1 *The strategy resulting in $\{u_\tau\}_{\tau=1}^T$ is strictly preferred in the Pareto sense to another resulting in $\{u'_\tau\}_{\tau=1}^T$, that is $\{u_\tau\}_{\tau=1}^T \succ_P \{u_\tau\}_{\tau=1}^T$, if*

$$U_t\left(\{u_\tau\}_{\tau=t}^T\right) \geq U_t\left(\{u'_\tau\}_{\tau=t}^T\right) \text{ for all } t \in \{1, ..., T\}, \tag{2}$$

and

$$U_t\left(\{u_\tau\}_{\tau=t}^T\right) > U_t\left(\{u'_\tau\}_{\tau=t}^T\right) \text{ for some } t \in \{1, ..., T\}. \tag{3}$$

O'Donoghue and Rabin (1999) proposed a different welfare criterion of a "long-run perspective" utility in which the intertemporal utility function U_t is evaluated from period 0. Assuming that $u_0 = 0$, the intertemporal utility in period 0 is defined as

$$U_0\left(\{u_\tau\}_{\tau=1}^T\right) = \beta \sum_{i=1}^{T} \delta^i u_i, \tag{4}$$

which is affinely equivalent to $U_1\left(\{u_\tau\}_{\tau=1}^T\right)$ using $\beta = 1$.

From (4), the long-run welfare criterion is defined as the following:

Definition 2 *The strategy resulting in $\{u_\tau\}_{\tau=1}^T$ is strictly preferred in the long-run perspective to another resulting in $\{u'_\tau\}_{\tau=1}^T$, that is $\{u_\tau\}_{\tau=1}^T \succ_L \{u'_\tau\}_{\tau=1}^T$, if*

$$U_0\left(\{u_\tau\}_{\tau=1}^T\right) > U_0\left(\{u'_\tau\}_{\tau=1}^T\right). \tag{5}$$

The following proposition shows that welfare-improvement in the Pareto sense is sufficient for welfare-improvement in the long-run perspective sense.

Proposition 1 *For any given two streams of instantaneous utilities* $\{u_\tau\}_{\tau=1}^T$ *and* $\{u'_\tau\}_{\tau=1}^T$*, if* $\{u_\tau\}_{\tau=1}^T \succ_P \{u'_\tau\}_{\tau=1}^T$*, then* $\{u_\tau\}_{\tau=1}^T \succ_L \{u'_\tau\}_{\tau=1}^T$*.*

Proof of Proposition 1: Defining a function $V_t : \mathbb{R}^t \to \mathbb{R}$ as

$$V_t\left(\{u_\tau\}_{\tau=t}^T\right) = u_t + \delta u_{t+1} + \delta^2 u_{t+2} + ... + \delta^{T-t} u_T = \sum_{i=0}^{T-t} \delta^i u_{t+i}, \tag{6}$$

we have

$$U_0\left(\{u_\tau\}_{\tau=1}^T\right) = \beta\delta V_1\left(\{u_\tau\}_{\tau=1}^T\right). \tag{7}$$

We use the following simplified notations:

$$U_t = U_t\left(\{u_\tau\}_{\tau=t}^T\right), \quad U'_t = U_t\left(\{u'_\tau\}_{\tau=t}^T\right),$$

$$V_t = V^t\left(\{u_\tau\}_{\tau=t}^T\right) \quad \text{and} \quad V'_t = V_t\left(\{u'_\tau\}_{\tau=t}^T\right).$$

From the definition of V_t in (6), we have the following lemma:

Lemma 1: If $U_t > U'_t$ and $V_{t+1} \geq V'_{t+1}$, then $V_t > V'_t$ for all $t \in \{1, ..., T-1\}$. If $U_t \geq U'_t$ and $V_{t+1} > V'_{t+1}$, then $V_t > V'_t$ for all $t \in \{1, ..., T-1\}$.

Proof of Lemma 1: From (1) and (6), U_t can be written as

$$\begin{aligned} U_t &= u_t + \beta \sum_{i=1}^{T-t} \delta^i u_{t+i} = u_t + \beta\delta V_{t+1} \\ &= u_t + \delta V_{t+1} - (1-\beta)\,\delta V_{t+1} \end{aligned} \tag{8}$$

From (6) and (8), we have

$$U_t = V_t - (1-\beta)\,\delta V_{t+1} \tag{9}$$

From (9), we know that if $U_t > U'_t$ and $V_{t+1} \geq V'_{t+1}$, we have $V_t > V'_t$ because $\beta < 1$. We also know that if $U_t \geq U'_t$ and $V_{t+1} > V'_{t+1}$, we have $V_t > V'_t$ because $\beta < 1$.

End of Lemma 1 proof.

Where $t = T$, we have $V_T = U_T$. Therefore, from Lemmas 1, (2) and (3) we have

$$V_{T-1} \geq V'_{T-1} \quad \text{or } V_{T-1} > V'_{T-1}. \tag{10}$$

By Lemma 1 and inequality (10), we have

$$\begin{array}{ll} V_{T-2} \geq V'_{T-2} & \text{if} \quad U_{T-1} = U'_{T-1}, \\ V_{T-2} > V'_{T-2} & \text{if} \quad U_{T-1} > U'_{T-1}. \end{array} \tag{11}$$

Given inequalities in (11), for $t \in \{1, 2, ..., T-2\}$, we have $V_t \geq V'_t$ if $U_{t+1} = U'_{t+1}$, and $V_t > V'_t$ if $U_{t+1} > U'_{t+1}$. Thus, we have $V_t \geq V'_t$ for $t \in \{1, 2, ..., T-2\}$. As noted, there is at least one $\tau \in \{1, 2, ..., T\}$ with $U_\tau > U'_\tau$ by Pareto improvement property. If $U_t = U'_t$ for $t = 1, 2, ..., \tau - 1$ and $U_\tau > U'_\tau$, then $V_\tau - (1-\beta)\delta V_{\tau+1} > V'_\tau - (1-\beta)\delta V'_{\tau+1}$ from equation (9). Since we have $V_{\tau+1} > V'_{\tau+1}$, $V_\tau > V'_\tau$. Then, $U_{\tau-1} \geq U'_{\tau-1}$ together with $V_\tau > V'_\tau$ implies $V_{\tau-1} > V'_{\tau-1}$. Recursively, we get $V_1 > V'_1$, which implies that $U_0 > U'_0$ by (7).

End of Proposition 1 Proof.

The main proof of Proposition 1 starts from equation (9), which shows the relationship between present-biased intertemporal utility (U_t) and unbiased intertemporal utilities (V_t and V_{t+1}). From (9), the unbiased intertemporal utility in period t can be expressed as

$$V_t = U_t + (1-\beta)\,\delta V_{t+1}, \tag{12}$$

which implies that the strategy resulting in higher unbiased utility in period $t+1$ and higher biased utility in period t is also resulting in higher unbiased utility in period t. Proposition 1 is proven in a recursive way from $t = T-1$ to $t = 1$. From equation (12), we know that the main result does not hold if $\beta > 1$.

Although a welfare-improvement in the sense of the Pareto criterion is sufficient for a welfare-improvement in the long-run criterion by Proposition 1, the converse is not true.[6]

3 Concluding remark

This paper shows that welfare-improving policies for the intertemporal utilities of all periods $t \in \{1, ..., T\}$ is necessarily also welfare-improving for the intertemporal utility from fictitious period 0. Economists conceived that the Pareto efficiency criterion is strong compared to the long-run criterion,

[6] A simple example is the case where $T = 3$, $\beta = 1/2, \delta = 0.9, \{u_1, u_2, u_3\} = \{3, 0, 0\}$ and $\{u'_1, u'_2, u'_3\} = \{0, 0, 4\}$. In the example, we have $U_0 < U'_0$, $U_1 > U'_1$, $U_2 < U'_2$ and $U_3 < U'_3$; The inequality $U_0 < U'_0$ does not imply that $U_t < U'_t$ for all $t \in \{1, ..., T\}$.

but they did not realize that improvement by the Pareto-efficiency criterion implies the long-run utility improvement, which is shown in this paper.

References

Bisin, A., A. Lizzeri, and L. Yariv (2015). Government policy with time inconsistent voters. *American Economic Review 105*(6), 1711–1737.

Chade, H., P. Prokopovych, and L. Smith (2008). Repeated games with present-biased preferences. *Journal of Economic Theory 139*(1), 157–175.

DellaVigna and Malmendier (2004). Contract design and self-control: Theory and evidence. *Quarterly Journal of Economics 119*(2), 353–402.

Diamond, P. and B. Koszegi (2003). Quasi-hyperbolic discounting and retirement. *Journal of Public Economics 87*(9), 1839–1872.

Frederick, S., G. Loewenstein, and T. O'Donoghue (2002). Time discounting and time preference: A critical review. *Journal of economic literature 20*(2), 351–401.

Gilpatric, S. M. (2008). Present-biased preferences, self-awareness and shirking. *Journal of Economic Behavior & Organization 67*(3), 735–754.

Goldman, S. M. (1979). Intertemporally inconsistent preferences and the rate of consumption. *Econometrica 47*(3), 621–626.

Guo, J.-T. and A. Krause (2015). Dynamic nonlinear income taxation with quasi-hyperbolic discounting and no commitment. *Journal of Economic Behavior & Organization 109*, 101–119.

Krusell, P. and A. A. Smith, Jr (2002). Equilibrium welfare and government policy with quasi-geometric discounting. *Journal of Economic Theory 105*(1), 42–72.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics 112*(2), 443–477.

Laibson, D. (1998). Life-cycle consumption and hyperbolic discount functions. *European Economic Review 42*(3), 861–871.

Laibson, D. I. (1996). Hyperbolic discount functions, undersaving, and savings policy. Technical report, National bureau of economic research.

Loewenstein, G. and D. Prelec (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics 107*(2), 573–597.

O'Donoghue, T. and M. Rabin (1999a). Doing it now or later. *American Economic Review 89*(1), 103–124.

O'Donoghue, T. and M. Rabin (1999b). Incentives for procrastinators. *Quarterly Journal of Economics 114*(3), 769–816.

O'Donoghue, T. and M. Rabin (2001). Choice and procrastination. *Quarterly Journal of Economics 116*(1), 121–160.

O'Donoghue, T. and M. Rabin (2003). Studying optimal paternalism, illustrated by a model of sin taxes. *American Economic Review*, 186–191.

O'Donoghue, T. and M. Rabin (2006). Optimal sin taxes. *Journal of Public Economics 90*(10), 1825–1849.

O'Donoghue, T. and M. Rabin (2015). Present bias: Lessons learned and to be learned. *American Economic Review 105*(5), 273–79.

Phelps, E. S. and R. A. Pollak (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies 35*(2), 185–199.

Pollak, R. A. (1968). Consistent planning. *Review of Economic Studies 35*(2), 201–208.

Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies 23*(3), 165–180.

Thaler, R. H. and S. Benartzi (2004). Save more tomorrow: Using behavioral economics to increase employee saving tomorrow. *Journal of Political Economy 112*(S1), S164–S187.

Yılmaz, M. (2013). Repeated moral hazard with a time-inconsistent agent. *Journal of Economic Behavior & Organization 95*, 70–89.