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In search of causality between debt and growth: a graph theoretic approach

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Abstract

This paper detects the direction of causality between government debt and economic growth in a multivariate framework using graph theoretic approach for six major OECD countries. The results show that growth causes debt in most countries. A comparison between the full sample and a reduced sample (leaving out the periods of recent financial crisis) indicates that the causal direction from growth to debt is a more recent phenomenon.

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1. Introduction

The relationship between government debt and economic growth has been a topic of recurrent importance in the recent past. After the recent financial crisis, in their influential (and controversial) paper Reinhart and Rogoff (2010) concluded that growth is hampered greatly after debt crosses the threshold value of 90% of GDP. Soon after, there was a stream of papers which concentrated on detecting the threshold value of debt which is harmful for growth (e.g. Kumar and Woo (2010), Cecchetti et al. (2011), Minea and Parent (2012), Checherita and Rother (2012)). They used different methods and came up with different answers. More recently, Panizza and Presbitero (2013) conducted a detailed study of the methodologies used in these papers and concluded that the evidence of a common threshold value of debt after which growth collapses is not robust. They also argued that the direction of causality between debt and growth is not clear and it needs to be detected before searching for a threshold value. Many of these studies simply assumed that debt causes growth and did not take other important macroeconomic variables into consideration. In this complex economic environment it is very likely that other macroeconomic variables affect debt and growth and ignoring them might give wrong results. Recent studies on the detection of causality between debt and growth (e.g. Lof and Malinen (2014) and Ajovin and Navarro (2015), Panizza and Presbitero (2014)) also refrained from considering other related macroeconomic variables in estimating equations.

This article searches for the direction of causality by using a comparatively new method known as the graph theoretic approach which can detect causality in the multivariate case in a straightforward manner. This paper considers six major OECD countries (Canada, France, Germany, Japan, UK and USA) and controls for five other macroeconomic variables which can theoretically affect both debt and growth. The rest of the article is organized as follows: Section 2 describes the methodology, section 3 explains the data and results and the final section concludes.

2. Methodology

This section describes the methodology of the graph theoretic approach which is used in this article to detect the causal relationship between debt and growth. Departing from the traditional approaches, this methodology uses an algorithm to reduce the set of possible causal structures by examining the correlations and conditional correlations in the data in a multivariate framework. This method was first suggested by Spirtes et al. (1993) and its use was initially limited to psychology, biometrics and medicine. More recently its application has also spread in Economics (e.g. Awokuse and Bessler (2003), Hoover et al. (2009), Saghaian (2010), Jinjarak and Sheffrin (2011)). To detect the causal structure from the conditional correlations, a special software 'TETRAD V' has been used. This software was first developed by Spirtes, Glymour and Scheines at Carnegie Mellon University. The algorithm that is used from this software is known as the 'PC algorithm'.

The advocates of graph theory emphasize that, in traditional regression equations where endogeneity and multicollinearity can be a problem, graph theory is better able to detect the causal structures between variables. In regression equations, the econometricians often suggest to increase the number of variables to control for the effects of other relevant variables. Spirtes et al. (2000) say that this can give misleading results even if the coefficients of the variables are stable. They argue the strategy of regressing on a larger set of variables and checking stability may compound rather than remedy problems, because if the typical empirical data sets to which multiple regression methods are applied have some correlated regressors, in uncontrolled studies, it is rare to know that unmeasured common causes are not acting on both the outcome variable and the regressors.

Panizza and Presbitero (2013) argue that many of the recent causality studies of government debt and growth failed to address this endogeneity problem properly. So in these kind of multivariate models where endogeneity can be an issue, the graph theoretic approach provides an alternative to detect the causal direction between variables. The graph theoretic approach considers all possible correlations and partial correlations necessary to come to a conclusion about causal relationships among variables and it does not rely on a restricted set of structural parameterizations. Moreover, Demiralp and Hoover (2003) have showed that the PC algorithm appears to have well behaved statistical properties and it rarely identifies a false causal link.

To use the graph theoretic approach, I follow Jinjarak and Sheffrin (2011). They use the covariance matrix of the residual of an unrestricted VAR as an input. However, the non-stationary nature of the macroeconomic variables can cause problems in a VAR framework and first differencing the data to achieve stationarity can make the data lose long run information. Moreover, detection of stationarity can vary and may depend on which particular stationarity test is chosen. To avoid these problems, unlike Jinjarak and Sheffrin (2011), I estimate an augmented VAR model for causality testing suggested by Toda and Yamamoto (1995) which is of the following form:

$$\mathbf{X_t} = \mu + \sum_{i=1}^{p} \mathbf{A_i X_{t-i}} + \mathbf{A}_{p+d_{\max}} \mathbf{X}_{\mathbf{t}-p-d_{\max}} + \varepsilon_t$$

where μ and $\mathbf{X}_t$ are the vector of constant terms and endogenous variables respectively, p is the lag length, **A** represents the coefficient matrices and $\varepsilon_t \sim (0,\Omega)$ is the error term. Here the lag length of the original VAR model, p, is augmented by the maximum order of integration ($d_{\max}$) of the variables in the system, resulting in a VAR order of $k = p + d_{\max}$. The VAR model is then estimated with k lags, but the coefficients of the last $d_{\max}$ lagged vectors are ignored in conducting the Wald test. Toda and Yamamoto (1995) showed that the Wald statistic of the VAR (p+$d_{\max}$) has an asymptotic χ^2 distribution even if the time series are non-stationary. So the main advantage of this process is that the data can be used in their level form without worrying about their stationarity properties. The covariance matrix of the estimated ε_t is then used as an input in the software TETRAD V. This software can use the PC algorithm of the graph theoretic approach to detect causal structures.

Since the details of how the PC algorithm of graph theory works are described in detail in many other papers, only some of the terms related to the process are described here.[1] In graph theory the causal structures are expressed as directed graphs. A directed graph is basically a picture which shows the direction of causation between variables known as nodes. If two nodes have some relation, they are connected with a line which may or may not have arrowheads. These lines are also known as edges. The arrowheads indicate the direction of causation. The map which only has lines but no arrowheads is called the skeleton of the graph. The graph is called acyclical if no causal chains come back to the same variable. If, between two variables a and b, at least one of them is a direct cause of another, then they are called adjacent. If a and b both cause c but a and b are not adjacent, then c is called an

[1] For more explanation, see the appendix (section A.1)

unshielded collider. This can be expressed as: *a* → *c* ← *b*. Instead if *a* and *b* are adjacent then *c* will become a *shielded collider*.

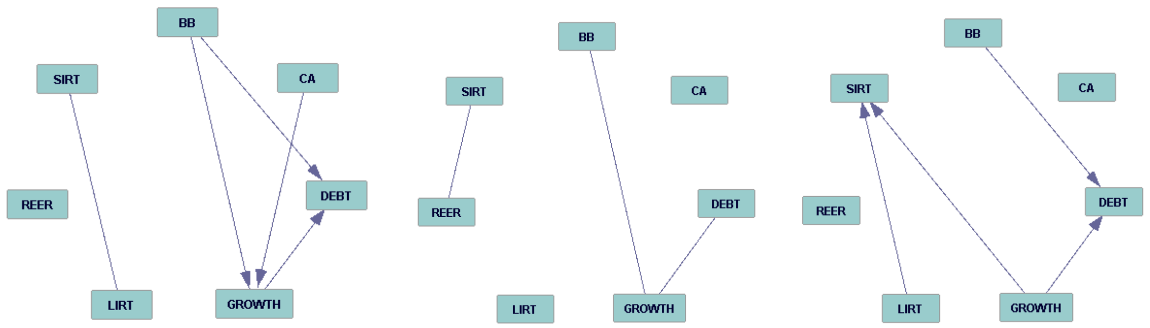
TETRAD's PC algorithm starts with a complete graph where all nodes (variables) are connected to each other with an edge. Next it tries to detect whether any two nodes are independent or conditionally independent. It then sequentially removes adjacent edges based on zero correlation or partial correlation (conditional correlation) and determines causal flow directions for the remaining edges based on the partial correlations of the residuals. In applications, Fisher's z is used to test whether conditional correlations are significantly different from zero. The remaining edges form a graph with the direction of causality indicated by an arrow. If two variables are connected with an edge but there are no arrowheads, it indicates a common latent cause of those two variables, or some combination of them, but the direction of causality is not known given the nature of the residuals.

Spirtes et al. (1993) have explored several versions of the PC algorithm on simulated data with respect to errors on both edge inclusion (yes or no) and direction. They conclude that there is little chance of the algorithm including an edge that is not in the "true" model.

3. Data and results

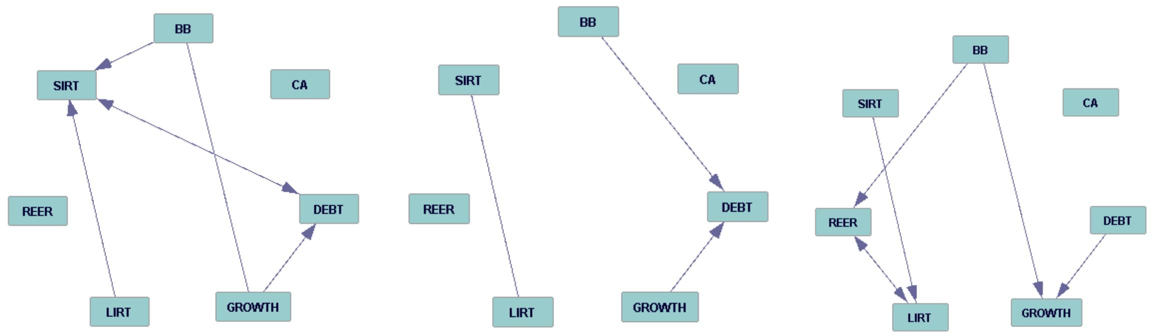
The causality between debt and growth is investigated for six OECD countries using quarterly data from 1980Q1 to 2013Q4. In addition to government debt as % of GDP and the growth rate, five additional macroeconomic variables are used. These are the government budget balance as % of GDP, the current account, the real effective exchange rate, a long run interest rate and a short run interest rate. The optimal lag length in the augmented VAR is selected using the Schwarz information criterion. Two different samples of data are used. Beside the full sample of data, a reduced sample running from 1980Q1 to 2007Q4 is used which leaves out the time period of the Great Recession and its aftermath. This way it becomes possible to indirectly assess the impact of the financial crisis on the outcome.[2]

The results for the full sample are shown in Fig. 1:



Canada *France* *Germany*

[2] Due to the short sample size it was not possible to use the period of the financial crisis only.

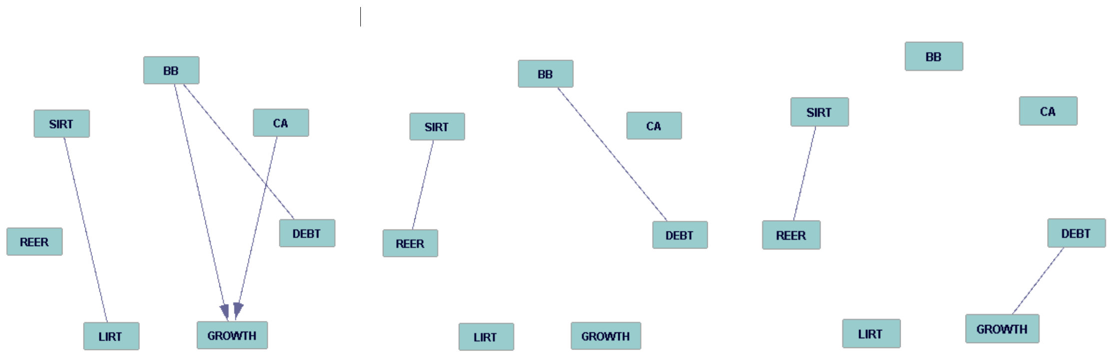


Japan *UK* *USA*

Figure 1: Causality detected by graph theory for the full sample.
Notes: DEBT: government debt as % of GDP; GROWTH: GDP growth rate; BB: government budget balance as % of GDP; CA: current account; REER: real effective exchange rate; LIRT: long run interest rate; SIRT: short run interest rate.

The results find unidirectional causality from growth to debt for Canada, Germany, Japan and UK. For the USA, causality runs from debt to growth. Only in the case of France, the graph theoretic approach cannot detect the direction of causality between debt and growth, though it shows that they are closely linked (indicated by the straight line). As for the other variables, the budget balance causes debt for Canada, Germany and UK. For Canada and the USA, there is also a unidirectional causality from the budget balance to growth, and for the UK causality runs from the budget balance to debt. The causality results for debt and growth are broadly in line with recent causality studies (e.g. Panizza and Presbitero (2014), Lof and Malinen (2014), Puente-Ajovin and Sanso-Navarro (2015) and Kempa and Khan (2016)), which find either no causal link or causality running from growth to debt.

The results of the reduced sample are shown in Fig. 2:



Canada *France* *Germany*

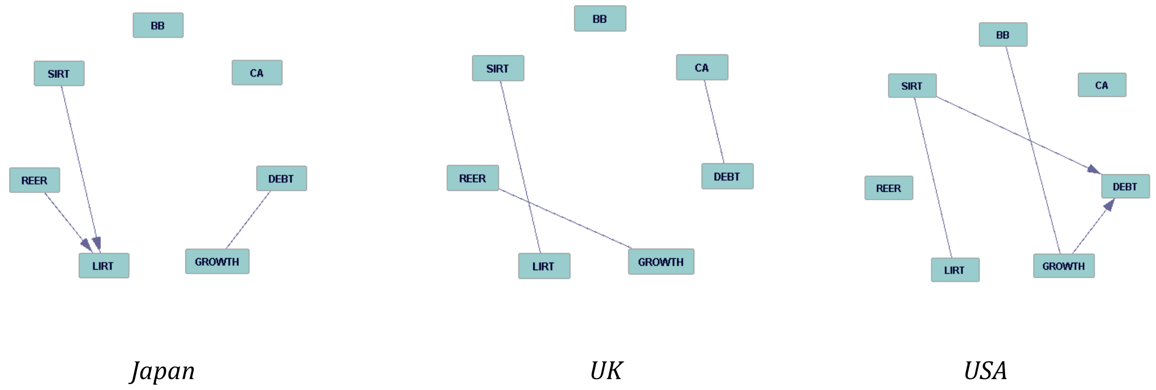


Figure 2: Causality detected by graph theory for the reduced sample. See the Notes to Fig. 1 for a definition of the variables.

When the financial crisis period is left out, except for the USA where the causality direction reverses, the causal link between government debt and growth vanishes for all countries. This might be the result of relatively stable government debt and GDP growth in these countries prior to the financial crisis. Following the financial crisis, for most countries GDP growth rates dropped substantially while debt ratios increased dramatically. It thus appears that this time period is responsible for the measured causality of growth on debt in the full sample.

4. Conclusion

This paper uses a graph theoretic approach to investigate the direction of causality between government debt and economic growth in a sample of quarterly data from 1980Q1 to 2013Q4 for the six OECD countries Canada, France, Germany, Japan, UK and USA. In particular, the graph-theoretic PC algorithm developed by Spirtes et al. (1993) is used to determine the causal structures in data on government debt, economic growth plus a number of control variables. For most countries, the results provide strong evidence of reverse causation, i.e. growth causing debt rather than the other way around. A comparison between the full sample and a reduced sample that leaves out the recent financial crisis indicates that the causal direction from growth to debt appears to be a post crisis phenomenon.

The causality results between debt and growth detected by the graph theoretic approach are broadly in line with recent causality studies (e.g. Panizza and Presbitero (2014), Lof and Malinen (2014), Puente-Ajovin and Sanso-Navarro (2015) and Kempa and Khan (2016)), which find either no causal link or causality running from growth to debt.

References

Awokuse, T.O. and Bessler, D.A. (2003) "Vector autoregressions, policy analysis, and directed acyclic graphs: an application to the U.S. economy" *Journal of Applied Economics* **6,** 1–24.

Cecchetti, S.G., Mohanty, M.S. and Zampolli, F. (2011) "The real effects of debt" BIS Working Paper number 352.

Checherita-Westphal, C., Hallett, A.H. and Rother, P. (2012) "Fiscal Sustainability using Growth-maximising Debt Targets". European Central Bank Working Paper number 1472.

Demiralp, S. and Hoover, K.D. (2003) "Searching for the causal structure of a vector autoregression" *Oxford Bulletin of Economics and Statistics* **65 (Suppl.)**, 745–767.

Hoover, K.D., Demiralp, S. and Perez, S.J. (2009) "Empirical identification of the vector autoregression: the causes and effects of US M2". In: Castle, J.L., Shephard, N. (Eds.), The Methodology and Practice of Econometrics. *Oxford University Press*, 37–58.

Jinjarak, Y. and Sheffrin, S.M (2011) "Causality, real estate prices, and the current account" *Journal of Macroeconomics* **33**, 233-246.

Kempa, B. and Khan, N. (2015) "Government debt and economic growth in the G7 countries: are there any causal linkages?" *Applied Economics Letters* **23,** 440-443.

Kumar, M.S. and Woo, J. (2010) "Public debt and growth" *Economica* **82(328)**, 705-739.

Lof, M. and Malinen, T. (2014) "Does sovereign debt weaken economic growth? A panel VAR analysis" *Economic Letters* **122,** 403-407.

Minea, A. and Parent, A. (2012) "Is High Public Debt always Harmful to Economic Growth? Reinhart and Rogoff and Some Complex Nonlinearities" Association Francaise de Cliometrie. Working Paper number 8.

Panizza, U. and Presbitero, A.F. (2013) "Public debt and economic growth in advanced economies: a survey" *Swiss Journal of Economics and Statistics* **149**, 175–204.

Panizza, U. and Presbitero, A.F. (2014) "Public debt and economic growth: Is there a causal effect?" *Journal of Macroeconomics* **41,** 21-41.

Puente-Ajovin, M. and Sanso-Navarro, M. (2015) "Granger causality between debt and growth: Evidence from OECD countries" *International Review of Economics and Finance* **35**, 66-77.

Reinhart, C.M. and Rogoff, K.S., (2010) "Growth in a time of debt". *American Economic Review Papers and Proceedings* **100,** 573–578.

Saghaian, S.H., (2010) "The Impact of the oil sector on commodity prices: correlation or causation?" *Journal of Agricultural and Applied Economics* **42**, 477–485.

Spirtes, P., Glymour, C., and Scheines, R. (1993) *Causation, Prediction, and Search,* Springer-Verlag: New York.

Spirtes, P., Glymour, C. and Scheines, R., (2000) *Causation, Prediction, and Search,* MIT Press.

Toda, H. Y and T. Yamamoto (1995),"Statistical inferences in vector autoregressions with possibly integrated processes" *Journal of Econometrics* **66,** 225-250.

Appendix

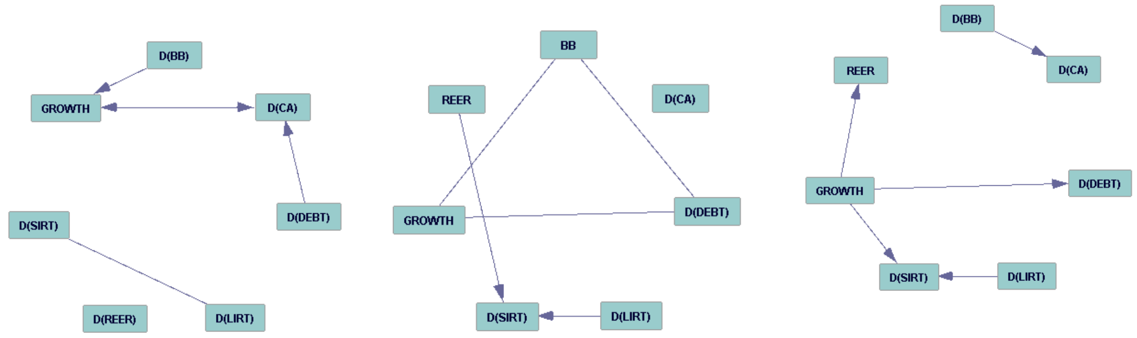
A.1 Graph theory and TETRAD

This section describes how the PC algorithm in TETRAD software detects the causal connection between variables[1]. It assumes that graphs are acyclical, i.e, loops like $a \rightarrow b \rightarrow c \rightarrow a$ are ruled out. Hoover (2005) describes the following steps of detecting causality:
(1) The algorithm starts with the complete set of variables in which all variables are connected by undirected edges, i.e. a line without an arrow head.
(2) It then tests for unconditional correlation among each pair of variables, and removes any edges whenever absence of correlation cannot be rejected.
(3) The procedure then tests for correlation among pairs of variables conditional on one other variable and again removes any edge for which conditional correlation vanishes. It then tests for conditioning on two, three variables and so forth until all variables are incorporated. This results in a skeleton. The orientation of edges is carried out in the next three steps.
(4) The algorithm starts orienting edges by seeking triples of linked (a—b—c) variables. For each conditionally uncorrelated pair of variables (i.e ones without a direct link) that are connected through third variables (a—b—c) the algorithm tests whether they become correlated conditional on that third variable (b) or not. In this case variable (b) is referred to as an unshielded collider. The algorithm orients the edges pointing toward the unshielded collider ($a \rightarrow b \leftarrow c$).
(5) If two variables (a and b) are not directly connected, but are connected through a third variable (c), so that one link points to the third variable (say, $a \rightarrow c$) and the other link is undirected (c--b), then the undirected link is pointed away from third variable ($c \rightarrow b$).
(6) Some edges may be oriented logically (rather than statistically) based on maintaining the assumption of acyclicality and avoiding implying the existence of unshielded colliders not identified statistically.
Finally, Fisher's z statistics is used to test whether the conditional correlations are significantly different from zero.

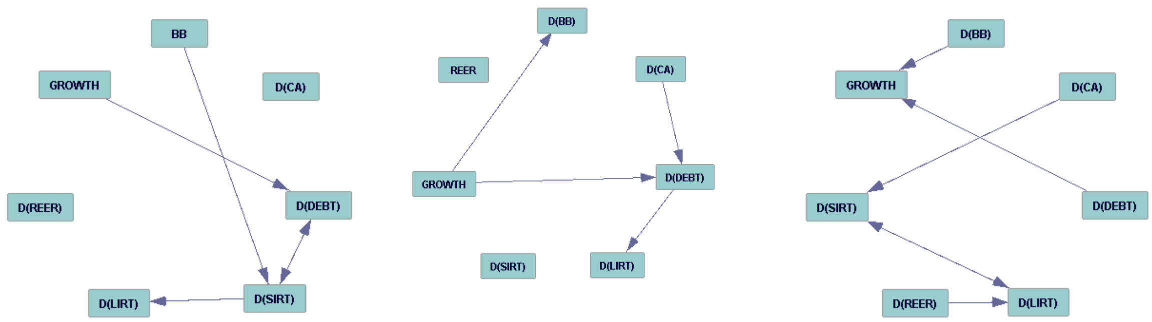
A.2 Robustness checks

This section checks the robustness of the results mentioned above in two different ways. Firstly, instead of using the level data in the Toda and Yamamoto (1995)'s augmented VAR framework, a traditional VAR model is utilized on differenced data whenever an ADF test indicates non-stationarity at the 5% level. The optimal lag length was selected using the Schwarz information criterion. Then the covariance matrix of the residuals is used as an input in the software TETRAD V. The results for the full sample and the reduced sample are shown in Figs. 3 and 4.

[1] For more details see Spirtes et al. (2000)

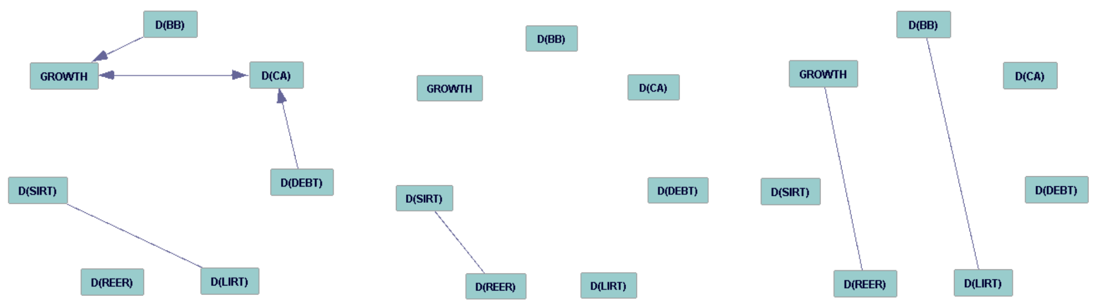


Canada *France* *Germany*



Japan *UK* *USA*

Figure 3: Causality detected by graph theory for the full sample with differenced data.
See the notes to Fig. 1 for a definition of the variables. Here the letter D indicates differenced data.



Canada France Germany

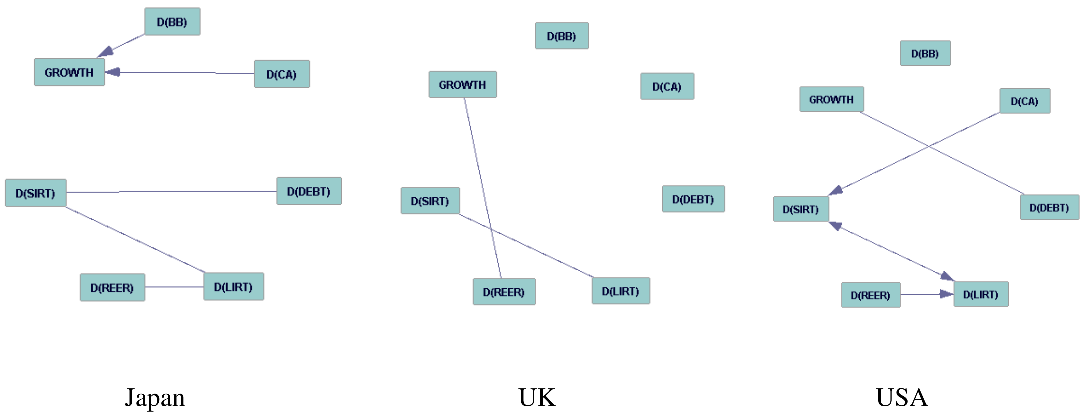


Figure 4: Causality detected by graph theory for the reduced sample with differenced data. See the Notes to Figs. 1 and 3 for a definition of the variables.

As far as the direction of causality between debt and growth is concerned, the results are not very different from the original results from Fig. 1. The only exceptions are Canada in the full sample and the USA in the reduced sample. For these two countries no causality was found in this case.

Secondly, a panel VAR was conducted for the full and the reduced sample by pooling the data of all the countries, where the covariance matrix of this panel VAR was used as an input in TETRAD V. First differenced data was used when the data was nonstationary. The optimal lag length was selected using the Schwarz information criterion. The results for the full and the reduced sample are shown in Figs. 5 and 6.

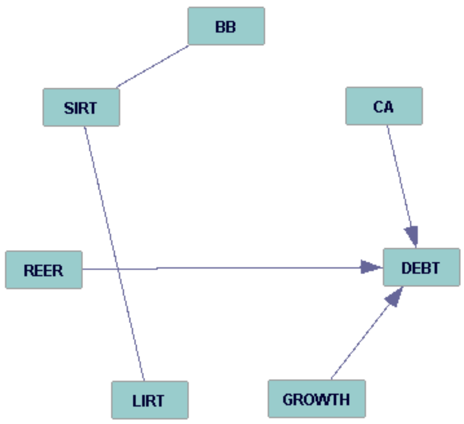


Figure 5: Causality detected by graph theory for the full sample using the covariance matrix of a panel VAR as input. See the Notes to Figs. 1 and 3 for a definition of the variables.

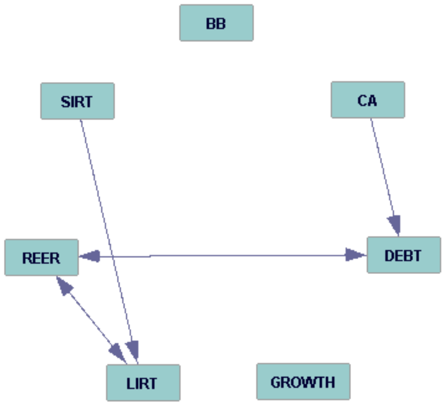


Figure 6: Causality detected by graph theory for the reduced sample using the covariance matrix of a panel VAR as input. See the Notes to Figs. 1 and 3 for a definition of the variables.

The results from the panel VAR also supports the general findings of the paper. In the full sample causality runs from growth to debt but in the reduced sample this causality is absent.