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The Feldstein-Horioka puzzle, the Frankel-Dooley-Mathieson puzzle, spurious ratio correlation and measurement errors

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Abstract

This paper extends the Feldstein-Horioka regression as a spurious ratio correlation due to a common deflator (Chu 2012) by incorporating measurement errors. The results explain not only the Feldstein-Horioka puzzle but also the Frankel-Dooley-Mathieson puzzle, namely the counter-intuitive finding that the Feldstein-Horioka coefficients for LDCs are lower than their OECD counterparts. The lower coefficients or downward biases arise from LDCs′ poorer data quality rather than higher international capital mobility. The global trend of growth in shadow economies, thus larger measurement errors, is also a factor contributing to the decline in the Feldstein-Horioka coefficients across countries over time.

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1. Introduction

Recently Chu (2012) shows that the high slope coefficient and R^2 of the Feldstein-Horioka (1980) regression are spurious because of a common deflator (Pearson 1896/7, Kuh and Meyer 1955). However, he did not address the empirical result in the literature that the regression coefficients for LDCs are lower than those for industrial countries (e.g. Dooley, et al. 1987). Subsequent studies like Wong (1990), Montiel (1994), Obstfeld and Rogoff (2000), Younas and Nandwa (2010), Chang and Smith (2014), Ma and Li (2016), among many others, find similar results despite using different empirical methods. More specifically, for example, the average regression coefficient for industrial countries is 0.81 over the years 1970-2010, compared with 0.56 for LDCs (Vegh 2013, Table 1.3). This robust finding, coined the Frankel-Dooley-Mathieson (1987) puzzle by Hamori (2007), is both counter-intuitive and counterfactual, because it suggests higher international capital mobility among LDCs than among OECD countries. Vegh (2013) attributes the finding to higher permanent shocks experienced by LDCs.

The literature has become so voluminous that it is not the place here to give a survey because of limited space. Instead, we refer the reader to recent surveys by Chu (2012) and Singh (2016). In particular, the latter offers a table listing the empirical techniques and findings of the major studies starting from the original Feldstein and Horioka (1980) study to those recently published in 2015.

By extending the framework of Chu (2012) to incorporate measurement errors, this paper offers explanations that go beyond the original Feldstein-Horioka puzzle into the Frankel-Dooley-Mathieson puzzle and the across-the-board declines in the coefficients over time. The next section provides the theoretical analysis to show the impacts of measurement errors on the Feldstein-Horioka coefficient, followed by Monte Carlo simulation results. The penultimate section applies the theoretical and simulation results to explain the main "stylized" facts in the literature before the paper concludes.

2. The Feldstein-Horioka Regression with Measurement Errors

Consider the Feldstein-Horioka original ratio regression:

$$\left(\frac{I}{Y}\right)_i = \beta_0 + \beta_1\left(\frac{S}{Y}\right)_i + \varepsilon_i \qquad \textbf{(1)}$$

where I, S and Y denote gross domestic investment, domestic savings, and GDP; β_0 and β_1 are parameters to be estimated; ε_i is a random disturbance term; and i stands for country i. Following Chu (2012), savings and investment are linear homogeneous functions of GDP and random disturbances u_i and v_i:

$$S_i = sY_i + \mu_i \qquad \textbf{(2)}$$

and

$$I_i = \alpha Y_i + v_i \qquad \textbf{(3)}$$

It is assumed $Y_i \sim (\bar{Y}, \sigma_y^2)$, $u_i \sim (0, \sigma_u^2)$, $v_i \sim (0, \sigma_v^2)$, $E(Y_i u_i) = E(Y_i v_i) = 0$, and $E(u_i v_i) = \sigma_{uv}$.

To incorporate measurement errors:

$$S_i = S_i^* + \xi_i \qquad \textbf{(4)}$$

$$I_i = I_i^* + \omega_i \qquad \textbf{(5)}$$

and

$$Y_i = Y_i^* + \eta_i \qquad \textbf{(6)}$$

where S_i, I_i and Y_i denote measured (observed) savings, investment and GDP; S_i^*, I_i^* and Y_i^* are the actual (unobserved) values; and measurement errors ξ_i, ω_i and η_i are all white-noise terms with zero means, finite variances --- σ^2_{ξ}, σ^2_{ω}, and σ^2_{η} --- but no contemporaneous correlation or autocorrelation.

An approximation for the regression coefficient with measurement errors is derived:[1]

$$\hat{b}_1 = \frac{\alpha s(1+\sigma_\eta^2)+\sigma_{\mu\nu}}{s^2(1+\sigma_\eta^2)+\sigma_\xi^2+\sigma_\mu^2} \qquad (7)$$

It differs from the original Feldstein-Horioka regression coefficient b_1 by

$$b_1 - \hat{b}_1 = \frac{\alpha s(\sigma_\xi^2 - \sigma_\mu^2\sigma_\eta^2)+\sigma_{\mu\nu}(s^2\sigma_\eta^2+\sigma_\xi^2)}{(s^2+\sigma_\mu^2)[s^2(1+\sigma_\eta^2)+\sigma_\xi^2+\sigma_\mu^2]} \gtreqless 0. \qquad (8)$$

As α and s are fractions, their own products or cross products are negligible in magnitude relative to the variances of the disturbance terms. Equation (8) is simplified to:

$$b_1 - \hat{b}_1 = \frac{\sigma_{\mu\nu}\sigma_\xi^2}{\sigma_\mu^2(\sigma_\xi^2+\sigma_\mu^2)} \geq 0. \qquad (9)$$

Therefore, measurement errors bias the slope downwards. As in the traditional case of OLS with measurement errors, the extent of bias will be insignificant if the variability of measurement errors is small by itself (i.e., σ^2_ξ is close to zero) or it is small relative to the variability of the explanatory variable, (i.e. σ_μ^2 is considerably large).

In the general case when measurement errors are contemporaneously correlated, we have

$$\tilde{b}_1 = \frac{\alpha s(1+\sigma_\eta^2)+\sigma_{\mu\nu}+\sigma_{\xi\omega}-\alpha\sigma_{\xi\eta}-s\sigma_{\eta\omega}}{s^2(1+\sigma_\eta^2)+\sigma_\xi^2+\sigma_\mu^2-2s\sigma_{\xi\eta}} \qquad (10)$$

and

$$b_1 - \tilde{b}_1 = \frac{\sigma_{\mu\nu}}{\sigma_\mu^2} - \frac{\sigma_{\mu\nu}+\sigma_{\xi\omega}}{\sigma_\mu^2+\sigma_\xi^2} = \frac{\sigma_{\mu\nu}\sigma_\xi^2 - \sigma_\mu^2\sigma_{\xi\omega}}{\sigma_\mu^2(\sigma_\xi^2+\sigma_\mu^2)} \gtreqless 0. \qquad (11)$$

The last equation demonstrates the well-recognized difficulty in generalizing the biases due to measurement errors (e.g. Bollen and Lennox 1991). Nevertheless, this difference is positive if either (i) $\sigma_{\xi\omega} = 0$, i.e., no correlation between measurement errors in investment and in savings, or (ii) $(\sigma_{\mu\nu}/\sigma_\mu^2) > (\sigma_{\xi\omega}/\sigma_\xi^2)$, or roughly speaking, the correlation between shocks to investment and to savings is larger than the correlation between measurement errors in investment and in savings.

The first condition, a standard assumption in the econometrics literature, is unlikely to hold in reality.

The second condition, however, is likely to hold because of conventional national income accounting methods and aggregate activities in the regular and underground or shadow economies. In national income accounting, we have

$$C + I + G + EX - IM \equiv Y \equiv YD + (TA - TR) \equiv C + S + (TA - TR) \qquad (12)$$

On the income side, in practice personal savings (S) is obtained as a residual by subtracting consumption (C) from disposable income (YD). For the fiscal budget, measurement errors in government expenditure (G), taxes (TA) and transfer payments (TR) are mainly omission and adding-up errors that are negligible relative to measurement errors in other national income components

[1] All derivations are shown in the Appendix.

arising from underground economic activities.[2] Furthermore, consumption is relatively stable both theoretically and empirically. So measurement errors in *Y*, i.e., mainly underground economic activities, are largely captured by measurement errors in savings.

On the expenditure side, with the above assumptions about *C* and *G*, measurement errors in *Y* are mainly captured by measurement errors in investment (*I*), exports (*EX*) and imports (*IM*). In practice *I* includes expenditures on residential and nonresidential construction, machinery and equipment, and additions to business inventories. For profit and tax purposes, firms have incentives to deliberately over- or under-state changes in inventories and it is costly for tax authorities to audit firms' inventories. Nonetheless, changes in inventories are only a small share of total investment. In contrast, the other investment items are more difficult to be significantly unreported or misrepresented by firms. On the other hand, measurement errors in exports and imports exist because firms have incentives to understate export revenues, overstate import costs, get around exchange control, engage in money laundering, etc. For a typical trade sector, these measurement errors are considerable.[3] Variations in measurement errors in investment are associated with variations in the official and unreported national income, but the extent of association should be less than the counterparts in savings because of measurement errors in imports and exports.

Put differently, from an accounting perspective and assuming negligible measurement errors in *G*, *TA* and *TR,* measurement errors in *Y* approximately equal to the sum of measurement errors in *C*, *I*, *EX* and *IM* on the expenditure side, which in turn approximately equals to the sum of measurement errors in *C* and *S* on the income side. After equating the two sides, the sum of measurement errors in *I*, *EX* and *IM* equals measurement errors in *S.* With random measurement errors, the sum of co-variations of measurement errors in *I*, *EX* and *IM* with measurement errors in savings equals the variance of measurement errors in savings. Hence $\sigma_{\xi\omega} \leq \sigma_{\xi}^{2}$ unless measurement errors in trade statistics are negligible or roughly sum to zero.[4]

On the other hand, $\sigma_{\mu v}$ tends to be high because aggregate shocks, such as technological and demographic changes, affect investment and savings alike.

Summing up, as long as $(\sigma_{\xi\omega}/\sigma_{\xi}^{2}) < (\sigma_{\mu v}/\sigma_{\mu}^{2})$,

$$b_1 - \tilde{b}_1 = \frac{\sigma_{\mu v}}{\sigma_{\mu}^{2}} - \frac{\sigma_{\mu v} + \sigma_{\xi\omega}}{\sigma_{\mu}^{2} + \sigma_{\xi}^{2}} = \frac{\sigma_{\mu v}\sigma_{\xi}^{2} - \sigma_{\mu}^{2}\sigma_{\xi\omega}}{\sigma_{\mu}^{2}(\sigma_{\xi}^{2} + \sigma_{\mu}^{2})} \geq 0 \qquad \textbf{(13)}$$

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[2] Here we refer to measurement errors in compiling the reported statistics, not the discrepancies between the reported figures and the "actual" figures on taxes and transfer payments had economic agents honestly reported their true incomes.

[3] For example, US exports, imports and investment are in the proportions 0.7:0.9:1 in 2012 (World Bank 2016).

[4] Based on the proportions in Footnote 3 and assuming measurement errors equally proportional to the official figures, a guesstimate is $\sigma_{\xi\omega}/\sigma_{\xi}^{2} < 0.38$.

Or measurement errors bias the slope downwards.[5] As in the case of Equation (9), the extent of bias will be insignificant if the variances of measurement errors are small themselves (i.e., both σ_ξ^2 and $\sigma_{\xi\omega}$ are close to zero) or they are small relative to the variance of the explanatory variable, σ_μ^2.

3. Monte Carlo Study

Properties of the Feldstein-Horioka regression with measurement errors are analyzed with simulations. Random numbers are generated for Y, S and I such that s and α mimic the OECD data. A saving-investment correlation is specified. Each random sample consists of 20 observations and a regression is run. This is repeated 1,000 times. These procedures are repeated with different saving-investment correlations and measurement errors ξ_i, ω_i and η_i, which are randomly generated as 10% (small-errors cases) and 40% (large-errors cases) in magnitude of the generated regression data.[6]

The first column of Table I shows the simulation results for the case of zero saving-investment correlation without measurement errors. The slopes range from 0.3079 to 1.3543, with a mean of 0.7567 and a standard deviation of 0.1581. The corresponding figures for R^2 are also reported. Apparently these regression results tend to erroneously reject the hypothesis of zero saving-investment correlation. The next two columns report the results for the small- and large-errors cases, whereas other columns report results for other saving-investment correlations.

The results clearly indicate that the slope and R^2 increase with the saving-investment correlation. But the slope is unbiased only in the case of unit saving-investment correlation without measurement errors. Results in the last three columns reveal that measurement errors bias the slope downwards. For other cases, however, the slopes are biased upwards and overstate the actual saving-investment correlations, though the biases are smaller for higher saving-investment correlations.

Further simulations are conducted to assess the impact of measurement errors of each variable in the Feldstein-Horioka regression. Results for measurement errors in GDP only (Table II) clearly indicate that the slopes overstate the actual saving-investment correlations – unless the correlation is unity – and larger measurement errors induce larger upward biases.

However, the slopes remain virtually unaffected by measurement errors in investment only (Table III) for their absence in Equation (9). This confirms the OLS property that the slope remains unbiased if the dependent variable only is measured with errors (e.g. Greene 2012). Nevertheless, the slopes in general overstate the true saving-investment correlations because of the common deflator.

By contrast, larger measurement errors in savings induce smaller slopes, which can be verified by differentiating Equation (9) with respect to σ^2_ξ. This is consistent with the OLS property that the slope is downward biased if the independent variable is measured with errors. In this case, the deflator induces upward biases whereas the measurement errors induce downward biases. Although these two biases offset each other, the slopes in general still overstate the underlying

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[5] It is impossible to have data on measurement errors to verify if this inequality holds in reality. To illustrate, suppose Feldstein and Horioka's regression for OECD is without measurement errors. Their slope estimate of 0.89 approximately equals $\sigma_{\mu\nu}/\sigma_\mu^2$ in theory. This together with Footnote 4 suggests $\sigma_{\xi\omega}/\sigma_\xi^2 < \sigma_{\mu\nu}/\sigma_\mu^2$.

[6] For analytical tractability, these measurement errors are uncorrelated in the simulations.

saving-investment correlations (Table IV).

4. Explaining the Puzzles

Irrespective of measurement errors, the regression results are "spurious" due to a common deflator and overstate the true saving-investment correlations in virtually all cases. This finding explains the original Feldstein-Horioka puzzle.

More importantly, our findings also offer an explanation for the Frankel-Dooley-Mathieson puzzle: lower Feldstein-Horioka coefficients for LDCs can be due to larger measurement errors in their data rather than higher international capital mobility. Take the results in Table IV for illustration. The mean value of the Feldstein-Horioka coefficient is 0.4774 when the saving-investment correlation is 0.75 (low international capital mobility according to the Feldstein-Horioka hypothesis) and measurement errors are large. This mean value is lower than its counterparts when the saving-investment correlations are low but measurement errors are zero or low, e.g. 0.7486 for the case with perfect capital mobility and no measurement errors. Put differently, when measurement errors in savings are sufficiently large, they can bias the slope coefficient downwards and distort the true picture of international capital mobility. Therefore it is empirically possible and not counterfactual to obtain a coefficient of 0.89 for OECD countries (Feldstein and Horioka 1980), higher than the 0.6 for LDCs (Montiel 1994) when OECD countries have both higher capital mobility and better quality data than LDCs.

There are several reasons for poorer data quality in LDCs. First, historically the LDCs lag industrial countries in developing national income accounts and economic statistics.

Second, the extensiveness of the underground economy distorts the official data and their quality (Reuter 1982). Efficiency of the tax system and share of the primary sector in GDP are two major determinants. Rising tax burden and social security contributions are important causes driving growth of shadow economies (Schneider and Enste 2000, Kirchgässner 2011). Typically LDCs are more agricultural or resource-based. Absent efficient tax systems, their primary sectors have incentives to under-report incomes for tax avoidance or evasion. Consequently, GDP and savings are measured inaccurately.

Third, socioeconomic factors like higher crime rates, social instability, corruption, etc, in LDCs tend to produce larger underground economies, whereas improvement in institutional quality undermines corruption and reduces the shadow economies (Dreher, et al. 2011).

Finally, financial development induces less tax evasion and a smaller underground economy (Capasso and Jappelli 2013) but LDCs in general have lower levels of financial development (Levine 1997).

The above factors are certainly non-exhaustive. For more details, see Schneider and Enste (2000) and Schneider (2011). Schneider and Enste (Table 3, p. 81) find across-the-board increases in the shadow economies over 1960-95 and estimate the average size to be 12% for OECD countries, 23% for transition economies and 39% for LDCs in the early 1990s. Latest figures for 2006 are 19% for OECD countries, 38% for Eastern European and Central Asian countries and 39% for LDCs (Schneider, et al. 2011).

These figures offer explanations for not only the Frankel-Dooley-Mathieson puzzle but also another stylized fact – declines in the Feldstein-Horioka coefficients across all countries over time (Bayoumi 1990, Feldstein 1983, Feldstein and Bacchetta 1991, Tesar 1991), e.g., from Feldstein and Horioka's 0.89 to 0.60 for the OECD countries for 1990-97 (Obstfeld and Rogoff 2000). This "stylized fact" can be attributed to both increasing international capital mobility and growth of shadow economies over time. According to Schneider and Enste (Table 3, p. 81, 2000), the shadow economies of selected OECD countries had increases across the board over 1960-1995, ranging from the lowest increase of 5.7% (as a percentage of the official GDP) for Switzerland to the highest 16.5% for Norway. The estimates of Schneider and Enste (2000) and Schneider, et al. (2011) also reveal that both OECD countries' and LDCs' shadow economies have grown considerably over time.

5. Conclusion

Measurement errors can be a source, or at least partially responsible, for certain economic paradoxes that cannot be satisfactorily explained, if not inexplicable, by theories (Reuter 1982). Two notable examples are decline in US productivity (Denison 1979) and decreased responsiveness of wages to changes in unemployment rate (Hall 1980).

Based on "spurious" ratio regressions and measurement errors, we are able to use one single analytical framework to offer at the same time coherent explanations for three prominent "stylized facts" in the Feldstein-Horioka literature: (i) the original Feldstein-Horioka, i.e., robustly high regression coefficients despite international capital mobility, (ii) the Frankel-Dooley-Mathieson puzzle, i.e., lower regression coefficients for LDCs than for OECD countries, and (iii) declines in the coefficients across countries and over time.

Therefore, this paper contributes to our understanding of the Feldstein-Horioka and related puzzles from an econometric perspective. Admittedly, further knowledge about the measurement errors, disturbances and also the underlying economics such as more information about shadow economies and other potential contributing factors, like strong home bias of investment, before we have a better understanding and can completely resolve these puzzles. Nonetheless, we should have by now realized the pitfall in using the Feldstein-Horioka coefficient as an indicator of international capital mobility.

Empirical or country studies indicate that international capital flows can benefit both investors and recipient countries by bringing substantial gains in the form of augmentation to capital and saving, improvement in incentives, management and technology, etc, on the one hand, but they can also bring risk like currency and financial crises on the other (Feldstein 1999). In response to the potential gains and problems associated with capital flows, some countries have encouraged financial and trade liberalization whereas some have imposed capital controls to limit capital flows (e.g. BIS 2008). Countries which use the Feldstein-Horioka coefficient as an indicator of international capital mobility are highly unlikely to come up with the right policy responses to capital flows simply because of the biases in the coefficient arising from spurious ratio regressions with measurement errors.

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Appendix

This appendix shows the details of the derivations of the theoretical results in the text. The notation and assumptions follow the text and will not be repeated here. To begin with, take the case without measurement errors as the benchmark for comparison. The slope coefficient of the Feldstein and Horioka's regression equation is

$$b_1 = \frac{\alpha s + \sigma_{\mu v}}{s^2 + \sigma_\mu^2} \tag{A1}$$

and the coefficient of determination is

$$R^2 = \frac{(\alpha s + \sigma_{\mu v})^2}{(s^2 + \sigma_\mu^2)(\alpha^2 + \sigma_v^2)} \tag{A2}$$

The derivations of these two equations will not be repeated here as they have already been given in Chu (2012). Now we extend the analysis for the case of measurement errors based on these two equations. In the presence of measurement errors in national income data, we have

$$S_i = S_i^* + \xi_i \tag{A3}$$

$$I_i = I_i^* + \omega_i \tag{A4}$$

and

$$Y_i = Y_i^* + \eta_i \tag{A5}$$

Taking into account of these measurement errors, the actual saving and investment functions (i.e., Equations (2) and (3) in the text) are now

$$S_i^* = sY_i^* + \mu_i \tag{A6}$$

and

$$I_i^* = \alpha Y_i^* + v_i \tag{A7}$$

After substituting Equations (A3)-(A5) into the above two equations, the observed saving and investment functions used in the Feldstein-Horioka regression become

$$S_i = s(Y_i - \eta_i) + \xi_i + \mu_i \tag{A8}$$

and

$$I_i = \alpha(Y_i - \eta_i) + \omega_i + v_i \tag{A9}$$

Following the same procedures in Chu (2012), we can derive accordingly the slope coefficient of the Feldstein-Horioka regression with measurement errors, denoted as $\hat{b}_1$. Alternatively, a simpler and quicker derivation is as follows. Note that (A8) and (A9) can be re-written as

$$S_i = sY_i - s\eta_i + \xi_i + \mu_i \tag{A10}$$

and

$$I_i = \alpha Y_i - \alpha\eta_i + \omega_i + v_i \tag{A11}$$

Contrasting with Equations (2) and (3) with disturbance terms u_i and v_i respectively, the disturbance terms of (A10) and (A11) can be expressed as

$$\hat{\mu}_i = \xi_i + \mu_i - s\eta_i \tag{A12}$$

and

$$\hat{v}_i = \omega_i + v_i - \alpha\eta_i \tag{A13}$$

Hence

$$\sigma_{\hat{\mu}\hat{v}} = E(\hat{\mu}\hat{v}) = \sigma_{\mu v} + \alpha s \sigma_\eta^2 \tag{A14}$$

by expanding the cross product terms of (A12) and (A13) and imposing the standard assumptions about the random disturbance terms. By the same token, we can obtain

$$\sigma_{\hat{\mu}}{}^2 = E(\hat{\mu}^2) = \sigma_\xi^2 + \sigma_\mu^2 + s^2\sigma_\eta^2 \tag{A15}$$

and $$\sigma_{\hat{v}}^2 = E(\hat{v}^2) = \sigma_{\omega}^2 + \sigma_{\upsilon}^2 + \alpha^2\sigma_{\eta}^2 \qquad \text{(A16)}$$

Then replace σ_{uv}, σ_u^2 and σ_v^2 in Equations (A1) and (A2) by their corresponding parts in Equations (A14)-(A16) to get the slope coefficient

$$\hat{b}_1 = \frac{\alpha s(1+\sigma_{\eta}^2) + \sigma_{\mu\nu}}{s^2(1+\sigma_{\eta}^2) + \sigma_{\xi}^2 + \sigma_{\mu}^2} \qquad \text{(A17)}$$

and the coefficient of determination

$$\hat{R}^2 = \frac{[\alpha s(1+\sigma_{\eta}^2) + \sigma_{\mu\nu}]^2}{(s^2\sigma_{\eta}^2 + \sigma_{\xi}^2 + \sigma_{\mu}^2)(\alpha^2\sigma_{\eta}^2 + \sigma_{\omega}^2 + \sigma_{\nu}^2)} \qquad \text{(A18)}$$

where (A17) is Equation (7) in the text. It is then straightforward to show

$$b_1 - \hat{b}_1 = \frac{\alpha s(\sigma_{\xi}^2 - \sigma_{\mu}^2\sigma_{\eta}^2) + \sigma_{\mu\nu}(s^2\sigma_{\eta}^2 + \sigma_{\xi}^2)}{(s^2 + \sigma_{\mu}^2)[s^2(1+\sigma_{\eta}^2) + \sigma_{\xi}^2 + \sigma_{\mu}^2]} \qquad \text{(A19)}$$

Ignoring the product and cross product terms of α and s, as they are negligible in magnitude relative to the other terms, the difference between the two slopes can be approximately simplified to

$$b_1 - \hat{b}_1 = \frac{\sigma_{\mu\nu}}{\sigma_{\mu}^2} - \frac{\sigma_{\mu\nu}}{\sigma_{\mu}^2 + \sigma_{\xi}^2} = \frac{\sigma_{\mu\nu}\sigma_{\xi}^2}{\sigma_{\mu}^2(\sigma_{\xi}^2 + \sigma_{\mu}^2)} \geq 0. \qquad \text{(A20)}$$

which is Equation (9) in the text. As in the traditional case of OLS with measurement errors, the extent of bias will be insignificant if the variability of measurement errors is small by itself (i.e., σ^2_{ξ} is close to zero) or it is small relative to the variability of the explanatory variable, (i.e. σ_{μ}^2 is considerably large). For in either case, the difference between the two slopes will tend to zero.

Now turn to the more realistic case in which there are correlations among the measurement errors. Let $\tilde{\mu}$ and $\tilde{v}$ denote the disturbance terms in the general case and replace $\hat{\mu}$ and $\hat{v}$ respectively in the above derivation. In this case we have

$$E(\tilde{\mu}\tilde{\nu}) = \sigma_{\xi\omega} + \sigma_{\xi\nu} - \alpha\sigma_{\xi\eta} + \sigma_{\mu\omega} + \sigma_{\mu\nu} - \alpha\sigma_{\mu\eta} - s\,\sigma_{\eta\nu} - s\,\sigma_{\eta\omega} + \alpha s\sigma_{\eta}^2 \qquad \text{(A21)}$$

and $$E(\tilde{\mu}^2) = \sigma_{\xi}^2 + \sigma_{\xi\mu} - s\,\sigma_{\xi\eta} + \sigma_{\mu\xi} + \sigma_{\mu}^2 - s\,\sigma_{\mu\eta} - s\,\sigma_{\eta\xi} - s\,\sigma_{\eta\nu} + s^2\sigma_{\eta}^2 \qquad \text{(A22)}$$

With the assumption that there are correlations between measurement errors but not between shocks and measurement errors, the above two equations can be respectively reduced to

$$E(\tilde{\mu}\tilde{\nu}) = \sigma_{\mu\nu} + \alpha s\sigma_{\eta}^2 + \sigma_{\xi\omega} - \alpha\sigma_{\xi\eta} - s\sigma_{\eta\omega} \qquad \text{(A23)}$$

and $$E(\tilde{\mu}^2) = \sigma_{\mu}^2 + \sigma_{\xi}^2 + s^2\sigma_{\eta}^2 - 2s\sigma_{\xi\eta} \qquad \text{(A24)}$$

Substitute the above two findings into the slope coefficient equation to get

$$\tilde{b}_1 = \frac{\alpha s(1+\sigma_{\eta}^2) + \sigma_{\mu\nu} + \sigma_{\xi\omega} - \alpha\sigma_{\xi\eta} - s\sigma_{\eta\omega}}{s^2(1+\sigma_{\eta}^2) + \sigma_{\xi}^2 + \sigma_{\mu}^2 - 2s\sigma_{\xi\eta}} \qquad \text{(A25)}$$

Similarly, ignoring the product and cross product terms of α and s, the difference between the two slopes can be approximately simplified to

$$b_1 - \tilde{b}_1 = \frac{\sigma_{\mu\nu}}{\sigma_{\mu}^2} - \frac{\sigma_{\mu\nu} + \sigma_{\xi\omega}}{\sigma_{\mu}^2 + \sigma_{\xi}^2} = \frac{\sigma_{\mu\nu}\sigma_{\xi}^2 - \sigma_{\mu}^2\sigma_{\xi\omega}}{\sigma_{\mu}^2(\sigma_{\xi}^2 + \sigma_{\mu}^2)} \qquad \text{(A26)}$$

which can be positive, negative, or zero depending on the magnitude of the numerator. In can be seen from Equation (A26) that there are two conditions under which the difference will be positive if either one of them is satisfied. The first condition is $\sigma_{\xi\omega} = 0$ or in words there is no correlation between the measurement errors in investment and those in savings. The second condition is $(\sigma_{\mu\nu}/\sigma_{\mu}^{2}) > (\sigma_{\xi\omega}/\sigma_{\xi}^{2})$ or

$$\frac{\sigma_{\mu\nu}}{\sigma_{\mu}^{2}} > \frac{\sigma_{\mu\nu}+\sigma_{\xi\omega}}{\sigma_{\mu}^{2}+\sigma_{\xi}^{2}} > \frac{\sigma_{\xi\omega}}{\sigma_{\xi}^{2}} \quad \text{(A27)}$$

which follows from the mathematical property that a/b > (a+c)/(b+d) > c/d for any numbers a, b, c and d. Roughly speaking, this condition can be interpreted as the correlation between the shocks to investment and those to savings is larger than the correlation between the measurement errors in investment and measurement errors in savings. Note that the first terms in Equation (A27), i.e., $\sigma_{\mu\nu}/\sigma_{\mu}^{2}$ is approximately equal to b_1, or the slope coefficient of the Feldstein-Horioka regression without measurement errors. The last term $\sigma_{\xi\omega}/\sigma_{\xi}^{2}$ is the slope coefficient of a regression of measurement errors in investment on measurement errors in savings, if we could actually observe such measurement errors!

Table I: Monte Carlo results indicating downward biases in Feldstein-Horioka regressions with measurement errors in savings, investment and GDP

Number of Iterations =1,000; Size of each sample = 20 observations

	Specified Correlation between Savings and Investment														
	0			0.25			0.5			0.75			1		
	Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors		
	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large
Results for the Slope Coefficient:															
Mean	0.7567	0.7650	0.8778	0.8205	0.8158	0.8769	0.8751	0.8572	0.8796	0.9384	0.9061	0.8960	1.0000	0.9546	0.8864
S.D.	0.1581	0.1829	0.3744	0.1182	0.2565	0.2840	0.0807	0.1058	0.2680	0.0434	0.0889	0.2877	0.0000	0.0870	0.2913
Max.	1.3543	1.3493	5.3784	1.2672	1.3377	2.3851	1.1574	1.2097	2.6672	1.1481	1.2699	2.8166	1.0000	1.3295	3.0426
Min.	0.3079	0.0675	0.0254	0.4956	0.4333	0.0747	0.6252	0.5451	0.0468	0.7974	0.6020	0.0267	1.0000	0.6244	0.1041
Results for the Coefficient of Determination:															
Mean	0.5572	0.5716	0.7235	0.7257	0.7150	0.7792	0.8686	0.8239	0.7977	0.9687	0.8917	0.7978	1.0000	0.9035	0.7656
S.D.	0.1357	0.1590	0.2439	0.0951	0.1202	0.2163	0.0556	0.0840	0.1969	0.0145	0.0587	0.1978	0.0000	0.0520	0.2069
Max.	0.8626	0.9197	1.0000	0.9415	0.9658	1.0000	0.9637	0.9719	1.0000	0.9933	0.9893	1.0000	1.0000	0.9859	1.0000
Min.	0.1499	0.0043	0.0009	0.3421	0.2989	0.0078	0.6263	0.3681	0.0056	0.8771	0.5051	0.0011	1.0000	0.5996	0.0185

Table II: Monte Carlo results indicating downward biases in Feldstein-Horioka regressions with measurement errors in GDP only

Number of Iterations =1,000; Size of each sample = 20 observations

	Specified Correlation between Savings and Investment														
	0			0.25			0.5			0.75			1		
	Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors		
	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large
Results for the Slope Coefficient:															
Mean	0.7610	0.8070	0.9742	0.8186	0.8574	0.9766	0.8811	0.9013	0.9930	0.9366	0.9481	0.9929	1.0000	1.0000	1.0000
S.D.	0.1625	0.1673	0.1468	0.1233	0.1244	0.1111	0.0807	0.0833	0.0746	0.0388	0.0391	0.0379	0.0000	0.0000	0.0000
Max.	1.3586	1.3767	1.4496	1.3702	1.3318	1.3554	1.2981	1.2226	1.2361	1.0666	1.1270	1.1208	1.0000	1.0000	1.0000
Min.	0.1372	0.1217	0.6090	0.4069	0.4359	0.6418	0.6420	0.6640	0.8164	0.8216	0.8450	0.8996	1.0000	1.0000	1.0000
Results for the Coefficient of Determination:															
Mean	0.5616	0.6312	0.9256	0.7198	0.7697	0.9565	0.8705	0.8924	0.9818	0.9682	0.9735	0.9952	1.0000	1.0000	1.0000
S.D.	0.1345	0.1449	0.0775	0.1015	0.0974	0.0507	0.0548	0.0524	0.0212	0.0153	0.0142	0.0056	0.0000	0.0000	0.0000
Max.	0.9073	0.9440	1.0000	0.9310	0.9637	1.0000	0.9704	0.9791	1.0000	0.9941	0.9962	1.0000	1.0000	1.0000	1.0000
Min.	0.0204	0.0151	0.5700	0.2214	0.3356	0.3242	0.5952	0.6342	0.8574	0.8470	0.9055	0.9518	1.0000	1.0000	1.0000

Table III: Monte Carlo results indicating downward biases in Feldstein-Horioka regressions with measurement errors in investment only

Number of Iterations =1,000; Size of each sample = 20 observations

	Specified Correlation between Savings and Investment														
	0			0.25			0.5			0.75			1		
	Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors		
	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large
Results for the Slope Coefficient:															
Mean	0.7525	0.7511	0.7476	0.8137	0.8114	0.8181	0.8779	0.8776	0.8814	0.9376	0.9384	0.9331	1.0000	0.9990	0.9958
S.D.	0.1546	0.1646	0.2978	0.1192	0.1264	0.1658	0.0837	0.0945	0.2095	0.0397	0.0640	0.2095	0.0000	0.0641	0.2626
Max.	1.2322	1.3463	1.8765	1.3068	1.2901	1.5382	1.2117	1.3410	1.6660	1.0925	1.1254	1.7377	1.0000	1.2159	1.9161
Min.	0.2158	0.3008	-0.266	0.4478	0.3197	-0.007	0.6306	0.6144	0.2407	0.8321	0.7461	0.2937	1.0000	0.7648	-0.091
Results for the Coefficient of Determination:															
Mean	0.5644	0.5349	0.3067	0.7250	0.6934	0.4416	0.8721	0.8425	0.5670	0.9684	0.9283	0.5753	1.0000	0.9397	0.5065
S.D.	0.1325	0.1417	0.1633	0.0997	0.1091	0.1658	0.0539	0.0628	0.1562	0.0146	0.0319	0.1550	0.0000	0.0307	0.1721
Max.	0.8853	0.8799	0.7802	0.9440	0.9572	0.8423	0.9770	0.9592	0.9049	0.9931	0.9835	0.8894	1.0000	0.9897	0.8957
Min.	0.0873	0.1014	0.0000	0.2601	0.1426	0.0000	0.4159	0.3791	0.0263	0.8837	0.7642	0.0934	1.0000	0.6894	0.0031

Table IV: Monte Carlo results indicating downward biases in Feldstein-Horioka regressions with measurement errors in savings only

Number of Iterations =1,000; Size of each sample = 20 observations

	Specified Correlation between Savings and Investment														
	0			0.25			0.5			0.75			1		
	Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors			Measurement Errors		
	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large	No	Small	Large
Results for the Slope Coefficient:															
Mean	0.7486	0.7040	0.3779	0.8071	0.7574	0.4108	0.8765	0.8273	0.4448	0.9393	0.8866	0.4774	1.0000	0.9483	0.5097
S.D.	0.1582	0.1612	0.1588	0.1018	0.1195	0.1341	0.0784	0.0947	0.1267	0.0413	0.0697	0.1268	0.0000	0.0631	0.1286
Max.	1.7158	1.5443	1.1302	1.2366	1.2210	0.9539	1.1436	1.1486	0.9261	1.0975	1.0919	1.0483	1.0000	1.2081	1.0331
Min.	0.2690	0.2571	-0.167	0.4055	0.3432	-0.031	0.6455	0.4867	-0.079	0.8293	0.6452	-0.007	1.0000	0.7703	-0.027
Results for the Coefficient of Determination:															
Mean	0.5586	0.5262	0.2995	0.7574	0.6784	0.3787	0.8722	0.8191	0.4568	0.9685	0.9087	0.4982	1.0000	0.9393	0.5102
S.D.	0.1360	0.1453	0.1682	0.1018	0.1195	0.1700	0.0531	0.0766	0.1744	0.0140	0.0444	0.1691	0.0000	0.0303	0.1637
Max.	0.8433	0.8463	0.8081	0.9294	0.9333	0.8458	0.9730	0.9637	0.8731	0.9946	0.9817	0.9103	1.0000	0.9932	0.8958
Min.	0.1359	0.0760	0.0000	0.2247	0.2199	0.0027	0.5382	0.2838	0.0004	0.8894	0.6877	0.0001	1.0000	0.7230	0.0012