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Impact of Network Externality on End-User Piracy: Revisited

Sougata Poddar
University of Redlands, CA, USA

Yuanzhu Lu
Central University of Finance and Economics, Beijing

Abstract

We revisit the issue whether a strong presence of network externality in the digital products market could be a reason for the copyright holders to allow piracy. We find that except under a limited circumstance this is not true in a framework that involves IPR protection and copyright holder's costly effort to prevent piracy. We further show that as the degree of network externality increases, the strategic piracy deterrence level of the copyright holder increases and the actual rate of piracy decreases.

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Contact: Sougata Poddar - sougata_poddar@redlands.edu, Yuanzhu Lu - yuanzhulu@cufe.edu.cn.
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1. Introduction

A strand of literature (Conner and Rumelt 1991, Takeyama 1994, Slive and Bernhardt 1998 among others) on digital piracy advocates that tolerance of piracy may lead to higher profit to the copyright holder when the effect of network externality is strong. In spite losing sales due to piracy, the unauthorized reproduction of copies increases the installed base and network size of the digital product significantly, which translates into higher valuation of the original product. As a result, the copyright holder can charge a higher price of its product and earn higher profit compared to the case when piracy is not allowed. King and Lampe (2003) explained this phenomenon from a different angle and concluded that it is the interaction between price discrimination and piracy that drives this result. They argue against the finding and show that allowing piracy may not be profitable to the copyright holder, if the copyright holder can actually price discriminate between potential-pirates and non-pirates. In this paper, we try to shed a light from a different angle on the connection between network effect and piracy in a more general framework; and show that only under limited circumstances allowing piracy can be profitable to the copyright holder, however network effect may not play a significant role in driving the result. On the contrary, we find that the possibility to allow piracy actually decreases as the strength of network externality increases.

Copyright violations or piracy for digital products is a very real phenomenon in today's world. It not only affects the revenue stream of the copyrighted product, but also impacts innovation incentive in the digital industry. As a result, a lot of private investment from the copyright holders goes to limit the extent of piracy. At the same time, the IPR protection policies from public authorities are also tuned to encourage innovation and stop copyright violations. Given this, we consider a model of piracy which has these private and public anti-piracy policies in place. In our model, there is a monopoly copyright holder of the product who faces numerous end-use pirates. The copyright holder chooses a profit maximizing price and profit maximizing piracy deterrence level. No price discrimination is allowed.

We focus to specifically see the impact of network externality on the extent of piracy, the rate of piracy and the original firm's piracy deterrence strategy. We find that when consumers' taste variety is sufficiently large compared to the degree of network externality and the IPR protection is weak, it is profitable for the original producer to allow piracy. This is where we find some support of allowing limited piracy in the presence of network externality. In all the other cases, it is profitable for the copyright holder to deter the pirate irrespective of the strength or degree of network externality.

In the comparative analysis, we interestingly find that the possibility to tolerate piracy actually decreases as the degree of network externality increases. We also find that the profit maximizing piracy deterrence level chosen by the copyright holder increases with the degree of network externality. Further when the original producer allows piracy, the rate of piracy also decreases as the strength of network externality increases. Therefore, our findings mostly argue that impact of strong network effect may not be a good reason always to allow piracy in general, actually it may work otherwise. However, piracy can still happen in a limited way.

2. The Framework

There is one original product developer (the monopoly copyright holder) and a group of heterogeneous consumers who are also the potential pirates. The original product is of high quality and denoted by H while the pirated product is of low quality and denoted by L. We will interchangeably call the original product as the high quality product and the pirated product as the low quality product. The products exhibit the feature of positive network externality. However, the impact of the network effect or externality is *asymmetric* between the users of the original product and the pirated product. The original product has all latest features and applications to absorb all the network effect along with the support service from the producer, while the pirated product which is not licensed from the original producer has limited functionality and absorption capacity of the network effect. Therefore, the original quality differential influences the extent to which the network effect extends between H and L.

In terms of utility, the consumer who buys original product, first of all, gets all the intrinsic benefit from the product due to its high quality; secondly, she also enjoys the full extent of the network externality generated by those users who also buy the original product, plus the (limited) network externality generated by the pirated product users. The buyers of the pirated product can enjoy all the value of the product (intrinsic as well as network) subject to limitation that the lower quality can permit. We normalize the quality of the original firm's product to one. The quality of the pirated product is indexed by $q \in (0,1)$, where q captures the quality depreciation. Consumers with different valuations for the product are indexed by X which is uniformly distributed over the interval $[0, \theta]$ with density $\frac{1}{\theta}$. Consumers have the choice to buy the original product from the product developer or they can pirate themselves.

The copyright holder undertakes costly investment to deter or limit piracy. It targets the end user pirates to stop or limit piracy as it stands to lose its potential market share because of them. It tries to make the piracy costly to the end-users by increasing the cost of copying by an amount x ($x \geq 0$). We assume the cost of investment of the original product developer to set a level of deterrence, x to the end-user pirate, is given by $c_o(x) = x^2/2$. Thus, the higher the investment made by the original product developer, the higher would be the cost of piracy to the end-user pirate. This is private protection.[1]

There also exists a public protection to stop copyright violations in the form of IPR to reduce piracy in the economy which is denoted by c ($c \geq 0$). Thus, when the level of private deterrence is x and the level of public deterrence is c, we assume the total deterrence or the cost of copying to an end-user is ($c + x$). We assumed an additive form between c and x since both the original firm's private effort (investment) and the legal protection and enforcement of copyright legislations (public protection) contribute to the deterrence of piracy. We would like to interpret c as the degree or the strength of IPR protection and it is exogenous to the model.[2]

[1] The deterrence level x could be technical or non-technical deterrence to the end-user pirate. If it is a technical deterrence, think of technical protection imbedded in the product for making copies difficult. The non-technical deterrence could be private monitoring and informing the authority for penalty.

[2] It is generally understood that the government or the regulatory authority can influence c. Different countries have different levels of IPR protections; it can be weak or strong. More importantly, for a country it takes a

Given this environment, a typical consumer X's utility function can be written as follows.

$$U = \begin{cases} X + \gamma\left(D_o + qD_p\right) - p & \text{if buys the original product,} \\ q\left[X + \gamma\left(D_o + D_p\right)\right] - (c + x) & \text{if pirates the original product,} \\ 0 & \text{if neither buys nor pirates,} \end{cases}$$

where D_o and D_p stand for the demand for the original product and the pirated product respectively and p is the price of the original product. $\gamma \geq 0$ is the coefficient which measures the level or strength of network externalities. For example, higher γ implies stronger effect of network externality, whereas when γ is close to zero, it implies almost no effect of network externality.[3] Note that the above utility framework captures the feature of asymmetric absorption capacities of the network externalities of the respective products. To ensure that D_o and D_p are nonnegative, we assume $\theta > \gamma$.

2.1 Deriving Demand for the Original and Pirated Products

The demand for the original product and for the pirated product, D_o and D_p, can be derived from the distribution of buyers as follows.

Recall that consumers are heterogeneous with respect to their values towards the product. Thus, the marginal consumer, A, who is indifferent between buying the original product and the pirated version, is given by $A + \gamma\left(D_o + qD_p\right) - p = q\left[A + \gamma\left(D_o + D_p\right)\right] - (c + x)$, or $A = \frac{p-(c+x)}{1-q} - \gamma D_o$. The marginal consumer, B, who is indifferent between buying the pirated product and not buying any product, is given by $q\left[B + \gamma\left(D_o + D_p\right)\right] - (c + x) = 0$, or $B = \frac{c+x}{q} - \gamma\left(D_o + D_p\right)$. The marginal consumer, C, who is indifferent between buying the original product and not buying at all, is given by $C + \gamma\left(D_o + qD_p\right) - p = 0$, or $C = p - \gamma\left(D_o + qD_p\right)$. Define $\hat{X} = max\{A, C\}$ and $\hat{Y} = min\{B, C, 0\}$. Then the demand for the original product is $D_o = \frac{1}{\theta}\int_{\hat{X}}^{\theta} dx$ and the demand for the pirated product is $D_p = \frac{1}{\theta}\int_{\hat{Y}}^{\hat{X}} dx$. It turns out that the demand functions can be written as the following:

$$D_o = \begin{cases} \frac{(1-q)\theta - (p-(c+x))}{(1-q)(\theta-\gamma)} & if\ qp \geq c + x\ and\ \gamma \leq \frac{p-(c+x)}{1-q}, \\ \frac{\theta - p}{\theta - \gamma} & if\ qp \leq c + x\ and\ p \geq \gamma, \\ 1 & otherwise, \end{cases} \tag{1}$$

and

long time to adjust its IPR policy (more so if the government of that country is not very pro-active to reform IPR related policies), hence we assume it to be exogenous in our model.

[3] See also Banerjee (2003) for a similar utility function.

$$D_p = \begin{cases} \frac{qp-(c+x)}{q(1-q)(\theta-\gamma)} & if\ qp \geq c+x\ and\ \gamma \leq \frac{c+x}{q}, \\ \frac{p-(c+x)-(1-q)\gamma}{(1-q)(\theta-\gamma)} & if\ qp \geq c+x\ and\ \frac{c+x}{q} \leq \gamma \leq \frac{p-(c+x)}{1-q}, \\ 0 & otherwise. \end{cases} \tag{2}$$

2.2 Choice of Profit Maximizing Price and Level of Deterrence by the Product Developer

Note that as long as the high quality firm chooses *p* and *x* such that it obtains all the demand, it will choose a boundary solution since it has no incentive to choose a lower price for given *x*. However, the boundary solution is already included as a possibility into the first two scenarios specified by the demand function (1) given the continuity of the original firm's profit function. Thus, we need to compare the high quality firm's profits in the first two scenarios only, namely, (i) $qp \geq c+x$ and $\gamma \leq \frac{p-(c+x)}{1-q}$and (ii) $qp \leq c+x$ and $p \geq \gamma$.

When the developer chooses *p* and *x* such that$qp \geq c+x$ and $\gamma \leq \frac{p-(c+x)}{1-q}$, the firm's profit maximization problem is

$$\max_{p\geq 0, x\geq 0} \pi_o = pD_o - c_o(x) = p\left(\frac{(1-q)\theta - p + (c+x)}{(1-q)(\theta-\gamma)}\right) - \frac{1}{2}x^2,$$

$$s.t.\quad qp \geq c+x \text{ and } \gamma \leq \frac{p-(c+x)}{1-q}$$

which is labeled Problem I.

When the developer chooses *p* and *x* such that $qp \leq c+x$ and $p \geq \gamma$, the firm's profit maximization problem is

$$\max_{p\geq 0, x\geq 0} \pi_o = pD_o - c_o(x) = p\left(\frac{\theta - p}{\theta-\gamma}\right) - \frac{1}{2}x^2,$$

$$s.t.\quad qp \leq c+x \text{ and } p \geq \gamma$$

which is labeled Problem II.

We summarize the optimum in the following proposition after solving Problems I and II and comparing the product developer's profits in these two problems (see Appendix).

Define $\delta(q,\theta,\gamma) \equiv min\left\{\theta\frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)}, (1-q)(\theta-2\gamma)-1\right\}$

$$= \begin{cases} \theta\frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)} & if\ 0 \leq \gamma \leq \frac{1-q}{2-q}\theta \\ (1-q)(\theta-2\gamma)-1 & if\ \gamma \geq \frac{1-q}{2-q}\theta \end{cases}.$$

Note that $\delta(q,\theta,\gamma) > 0$ if and only if $\theta > max\left\{\frac{1}{q(1-q)}+\gamma, \frac{1}{1-q}+2\gamma\right\}$. Since γ can be as small as zero, $\theta > \frac{1}{q(1-q)}$ ensures $\delta(q,\theta,\gamma) > 0$ for a range of γ. We further assume $\theta > \frac{2-q}{q(1-q)}$ so that when $\gamma = \frac{1-q}{2-q}\theta$, $\theta\frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)} = (1-q)(\theta-2\gamma)-1 = \frac{q(1-q)\theta-1}{(2-q)} > 0$. When $\frac{1}{q(1-q)} < \theta \leq \frac{2-q}{q(1-q)}$, the results are qualitatively same.

To simplify the analysis, we assume $2(1-q)(\theta-\gamma)>1$.[4]

Proposition 1

In the end-user piracy model with network externality,

(i) *When* $0 \le c \le max\{\delta(q,\theta,\gamma),0\}$, *the original developer accommodates piracy, the profit maximizing price is* $p^* = \frac{(1-q)(\theta-\gamma)((1-q)\theta+c)}{2(1-q)(\theta-\gamma)-1}$ *and the profit maximizing level of deterrence is* $x^* = \frac{(1-q)\theta+c}{2(1-q)(\theta-\gamma)-1}$.

(ii) *When* $c \ge \frac{q\theta}{2}$ *and* $\gamma \le \frac{\theta}{2}$, *the piracy is blockaded; the original developer chooses zero deterrence level and its profit maximizing price is the monopoly price* $p^* = \frac{\theta}{2}$.

(iii) *In all the other cases, the original developer deters piracy completely. There are four different cases.*

(a) *When* $max\left\{\theta\frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)}, q\gamma+\frac{2\gamma-\theta}{q(\theta-\gamma)}\right\} \le c \le \frac{q\theta}{2}$, *the original developer's profit maximizing price is* $p^* = \frac{\theta+qc(\theta-\gamma)}{2+q^2(\theta-\gamma)}$ *and the profit maximizing level of deterrence is* $x^* = \frac{q\theta-2c}{2+q^2(\theta-\gamma)}$.

(b) *When* $\gamma \ge \frac{1-q}{2-q}\theta$ *and* $max\{(1-q)(\theta-2\gamma)-1,0\} \le c \le q\gamma-1$, *the original developer's profit maximizing price is* $p^* = c+1+(1-q)\gamma$ *and the profit maximizing level of deterrence is* $x^* = 1$.

(c) *When* $\gamma \ge \frac{1-q}{2-q}\theta$ *and* $q\gamma-1 \le c \le min\left\{q\gamma+\frac{2\gamma-\theta}{q(\theta-\gamma)}, q\gamma\right\}$, *the original developer's profit maximizing price is* $p^* = \gamma$ *and the profit maximizing level of deterrence is* $x^* = q\gamma-c$.

(d) *When* $c \ge q\gamma$ *and* $\gamma \ge \frac{\theta}{2}$, *the original developer's profit maximizing price is* $p^* = \gamma$ *and the profit maximizing level of deterrence is* $x^* = 0$.

Thus, the only situation where we find support to allow piracy is when consumers' taste variety is sufficiently large compared to the strength of network externality ($\theta > max\left\{\frac{1}{q(1-q)}+\gamma, \frac{1}{1-q}+2\gamma\right\}$) and IPR protection is low ($0 \le c \le max\{\delta(q,\theta,\gamma),0\}$). Otherwise, piracy will be always deterred. Therefore, the existence of strong network externality itself cannot be a sufficient condition for the copyright holder to allow piracy in this general setup.

In the following comparative statics analyses, we investigate further the impact of network externality on the extent of piracy.

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[4]See Case I1 in the appendix for the role of this assumption.

2.3 Comparative Statics

2.3.1 The relationship between the network externality (γ) and $\delta(q,\theta,\gamma)$

When $0 \le c \le max\{\delta(q,\theta,\gamma),0\}$ and $> max\left\{\frac{1}{q(1-q)}+\gamma, \frac{1}{1-q}+2\gamma\right\}$, the pirate is accommodated. Clearly, as γ increases, the condition $\theta > max\left\{\frac{1}{q(1-q)}+\gamma, \frac{1}{1-q}+2\gamma\right\}$ is less likely to be satisfied. We are also interested in how $\delta(q,\theta,\gamma)$ changes as γ increases.

Recall $\delta(q,\theta,\gamma) \equiv min\left\{\theta\frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)}, (1-q)(\theta-2\gamma)-1\right\}$.

Since $\frac{\partial}{\partial\gamma}\left(\theta\frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)}\right) = -\frac{\theta}{(2-q)(\theta-\gamma)^2} < 0$, and $\frac{\partial}{\partial\gamma}\left((1-q)(\theta-2\gamma)-1\right) = -2(1-q) < 0$, as γ increases, the condition $0 \le c \le max\{\delta(q,\theta,\gamma),0\}$ is less likely to be satisfied.

We have shown that both $0 \le c \le max\{\delta(q,\theta,\gamma),0\}$ and $\theta > max\left\{\frac{1}{q(1-q)}+\gamma, \frac{1}{1-q}+2\gamma\right\}$ are less likely to be satisfied as γ increases, thus the following proposition holds.

Proposition 2

Given the regime of IPR protection and the degree of consumers' taste variety, the possibility to accommodate end-user pirates by the copyright holder reduces as the network externality becomes stronger.

Main Intuition:

The main intuition behind this and subsequent results (Propositions 3 and 4) comes from the structure of the utility function considered in the analysis. Let us define the gross utility of consuming the original product as U_o and the gross utility of consuming the pirated product as U_p. Observe that $U_o - U_p = (1-q)(X+\gamma D_o)$. The difference in gross utilities is increasing in D_o and independent of D_p. This implies that an additional consumer of the original product is more valuable for a consumer of the original product than for a consumer of the pirated product, while an additional consumer of the pirated product is as valuable for both. Hence, more piracy (i.e. higher D_p) does not confer any additional advantage to the original product through network externalities while more users of the original product (i.e. higher D_o) does.

It also explains why stronger network externalities do not make piracy more profitable to the original producer. This feature is also reflected when the original producer adjusts it deterrence level as network effect gets stronger as we show below.

2.3.2 The relationship between the network externality (γ) and the profit maximizing level of deterrence (x)

When $0 \le c \le max\{\delta(q,\theta,\gamma),0\}$, i.e., when the original firm accommodates the pirate, $\frac{\partial x^*}{\partial\gamma} = \frac{2(1-q)[c+(1-q)\theta]}{[2(1-q)(\theta-\gamma)-1]^2} > 0$.

When the original firm deters the pirate completely, i.e., in the four cases (a)-(d) in Proposition 1(iii), $\frac{\partial}{\partial\gamma}\left(\frac{q\theta-2c}{q^2(\theta-\gamma)+2}\right) = \frac{q^2(q\theta-2c)}{[q^2(\theta-\gamma)+2]^2} > 0$, $\frac{\partial}{\partial\gamma}(1) = 0$, $\frac{\partial}{\partial\gamma}(q\gamma-c) = \mathrm{q} > 0$, $\frac{\partial}{\partial\gamma}(0) = 0$.

We can also show that as γ increases, the original developer's profit maximizing deterrence level is continuous and non-decreasing when the original developer moves from accommodation to complete deterrence, or moves from blockade to complete deterrence, or moves between different cases of complete deterrence.

Thus, we have the result summarized in the following proposition:

Proposition 3

The copyright holder's profit maximizing deterrence level to the end-user pirates is continuous and non-decreasing in the effect of network externality.

2.3.3The relationship between the network externality (γ) and the rate of piracy

We define the ratio of $D_p / (D_o + D_p)$ to measure the rate of piracy. Thus, the higher the ratio, the higher will be the rate of piracy. When $0 \leq c \leq max\{\delta(q,\theta,\gamma),0\}$, i.e. when the original firm accommodates the pirate, we get $\frac{D_p}{D_o+D_p} = \frac{(\theta-\gamma)[(1-q)q\theta-(2-q)c]-\theta}{2(\theta-\gamma)(1-q)(q\theta-c)-\theta}$ (when the market is partially covered, i.e., $D_o + D_p < 1$) or $\frac{D_p}{D_o+D_p} = 1 - \frac{c+(1-q)\theta}{2(1-q)(\theta-\gamma)-1}$ (when the market is fully covered, i.e., $D_o + D_p = 1$). In all the other cases, entry is either completely deterred or blockaded; thus, the rate of piracy is zero.

Under accommodation, simple computation yields

$$\frac{\partial}{\partial\gamma}\left(\frac{D_p}{D_o+D_p}\right) = -\frac{q\theta[c+\theta(1-q)]}{[2(\theta-\gamma)(1-q)(q\theta-c)-\theta]^2} < 0 \text{ (when the market is partially covered),}$$

$$\frac{\partial}{\partial\gamma}\left(\frac{D_p}{D_o+D_p}\right) = -\frac{2(1-q)[c+(1-q)\theta]}{[2(1-q)(\theta-\gamma)-1]^2} < 0 \text{ (when the market is fully covered),}$$

so the rate of piracy is decreasing in γ.

The above also verifies our main intuition described before.

Proposition 4

The rate of piracy and the degree of network externality is monotonically decreasing in the degree of network externality, i.e. the higher network effect, the lower is the piracy rate.

3. Conclusion

In this paper, we revisit the issue of whether a strong presence of network externality in the digital products market could always be a reason for the copyright holders to allow piracy in their market. A sizeable amount of previous literature answers this question in an affirmative way, saying that allowing piracy under the presence of strong network externality is profitable to the original producer. In this paper, we demonstrate that this result is generally not true in a framework of piracy which involves IPR protection and copyright holder's costly effort to prevent piracy. There we show it is not only the *strength* of network externality that matters, but the *relative difference* between the consumers' taste variety and the degree of network externality matters in the decision whether the copyright holder will allow piracy or not. Moreover, in the comparative statics analysis, we further show that as the degree of network externality increases, the strategic piracy deterrence level of the copyright holder actually increases, and the rate of piracy decreases. So the higher the strength of network effect, the lower will be the actual incidence of piracy.

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Appendix: Choice of Profit Maximizing Price and Level of Deterrence by the Product Developer

Problem I

Given the constraints $qp \geq c + x$ and $x \geq 0$, $p \geq 0$ is automatically satisfied. Define Lagrangian

$$L_1(p,x,\lambda,\eta,\mu) = p\left(\frac{(1-q)\theta - p + (c+x)}{(1-q)(\theta-\gamma)}\right) - \frac{1}{2}x^2 + \lambda(qp-c-x) + \eta\big(p-c-x-(1-q)\gamma\big) + \mu x.$$

The sufficient and necessary conditions for the optimum are the following:

$$\frac{\partial L_1(p,x,\lambda,\eta,\mu)}{\partial p} = \frac{(1-q)\theta - 2p + (c+x)}{(1-q)(\theta-\gamma)} + \lambda q + \eta = 0, \tag{A1}$$

$$\frac{\partial L_1(p,x,\lambda,\eta,\mu)}{\partial x} = \frac{p}{(1-q)(\theta-\gamma)} - x - \lambda - \eta + \mu = 0, \tag{A2}$$

$$\lambda(qp-c-x) = 0,\ \lambda \geq 0,\ qp \geq c+x, \tag{A3}$$

$$\eta\big(p-c-x-(1-q)\gamma\big) = 0,\ \eta \geq 0,\ p-c-x-(1-q)\gamma \geq 0, \tag{A4}$$

$$\mu x = 0,\ \mu \geq 0,\ x \geq 0. \tag{A5}$$

When solving this problem, we assume $\mu = 0$. If a negative value of x is obtained, then we will address the constraint $x \geq 0$.

Case I1: $\lambda = 0, \eta = 0$

Solving for p and x from (A1) and (A2) after plugging $\lambda = 0$ and $\eta = 0$ into these equations yields $p = \dfrac{(1-q)(\theta-\gamma)\big((1-q)\theta+c\big)}{2(1-q)(\theta-\gamma)-1}$ and $x = \dfrac{(1-q)\theta+c}{2(1-q)(\theta-\gamma)-1}$. Note that when $2(1-q)(\theta-\gamma) > 1$, $x > 0$. This is why we assume $2(1-q)(\theta-\gamma) > 1$ in Section 2.2 (before Proposition 1). To simplify analysis, we will not explore the original producer’s profit maximizing price and level of deterrence when this assumption does not hold true.

We also need to check whether $qp \geq c+x$ and $p-c-x-(1-q)\gamma \geq 0$ are satisfied and we find that the former condition is satisfied when $c \leq \dfrac{\theta\big(q(1-q)(\theta-\gamma)-1\big)}{(2-q)(\theta-\gamma)}$ and the latter condition is satisfied when $c \le (1-q)(\theta-2\gamma)-1$. So this case is the optimum if $c \le min\left\{\theta \frac{q(1-q)(\theta-\gamma)-1}{(2-q)(\theta-\gamma)}, (1-q)(\theta-2\gamma)-1\right\}$. In this case, the developer's profit is $\pi_o^A = \frac{\left((1-q)\theta+c\right)^2}{2\left(2(1-q)(\theta-\gamma)-1\right)}$, where the superscript A indicates this is an accommodation case.

Similarly, we can solve for *p* and *x* in all the other cases and also derive the conditions on *c* under which the respective case is the optimum. The details are omitted here and we list the results only.

Case I2: $\lambda = 0, \eta \ge 0$

$$p = c+(1-q)\gamma+1, x=1, \pi_o = c+(1-q)\gamma+\frac{1}{2}.$$

Conditions: $c \le q\gamma-1$, $c \ge (1-q)(\theta-2\gamma)-1$.

Case I3: $\lambda \ge 0, \eta = 0$

Subcase I3-1: $p = \frac{\theta+qc(\theta-\gamma)}{2+q^2(\theta-\gamma)}$, $x = \frac{q\theta-2c}{2+q^2(\theta-\gamma)}$, $\pi_o = \frac{\theta^2+2(\theta-\gamma)c(q\theta-c)}{2(\theta-\gamma)\left(2+q^2(\theta-\gamma)\right)}$.

Conditions: $c \ge \frac{\theta\left(q(1-q)(\theta-\gamma)-1\right)}{(2-q)(\theta-\gamma)}$, $c \ge q\gamma+\frac{2\gamma-\theta}{q(\theta-\gamma)}$, $c \le \frac{q\theta}{2}$.

Subcase I3-2: $p = \frac{c}{q}$, $x=0$, $\pi_o = \frac{c(q\theta-c)}{q^2(\theta-\gamma)}$.

Conditions: $c \ge q\gamma$, $c \ge \frac{q\theta}{2}$.

Case I4: $\lambda \ge 0, \eta \ge 0$

$$p = \gamma, x = q\gamma-c, \pi_o = \gamma-\frac{1}{2}(q\gamma-c)^2.$$

Conditions: $c \ge q\gamma-1$, $c \le q\gamma+\frac{2\gamma-\theta}{q(\theta-\gamma)}$, $c \le q\gamma$.

Problem II

Given the constraint $p \ge \gamma$, $p \ge 0$ is automatically satisfied. Define Lagrangian $L_2(p,x,\kappa,\phi,\nu) = p\left(\frac{\theta-p}{\theta-\gamma}\right)-\frac{1}{2}x^2-\kappa(qp-c-x)-\phi(\gamma-p)+\nu x$. The sufficient and necessary conditions for the optimum are the following:

$$\frac{\partial(p,x,\kappa,\phi,\nu)}{\partial p} = \frac{\theta-2p}{\theta-\gamma}-\kappa q+\phi = 0, \quad \text{(A6)}$$

$$\frac{\partial L_2(p,x,\kappa,\phi,\nu)}{\partial x} = -x+\kappa+\nu = 0, \quad \text{(A7)}$$

$$\kappa(qp-c-x) = 0,\ \kappa \ge 0,\ qp \le c+x, \quad \text{(A8)}$$

$$\phi(\gamma - p) = 0,\ \phi \geq 0,\ p \geq \gamma\ , \tag{A9}$$

$$\nu x = 0,\ \nu \geq 0,\ x \geq 0\ . \tag{A10}$$

Again, there are four cases to consider, we omit the details here and list the results only.

Case II1: $\kappa = 0, \phi = 0$

$p = \frac{\theta}{2}, x = 0, \pi_o^B = \frac{\theta^2}{4(\theta - \gamma)}$ (where the superscript B indicates this is a blockade case).

Conditions: $c \geq \frac{q\theta}{2}\ ,\ \gamma \leq \frac{\theta}{2}$.

Case II2: $\kappa = 0, \phi \geq 0$

$p = \gamma\ , x = 0\ , \pi_o = \gamma$.

Conditions: $c \geq q\gamma\ ,\ \gamma \geq \frac{\theta}{2}$.

Case II3: $\kappa \geq 0, \phi = 0$

$$p = \frac{\theta + qc(\theta - \gamma)}{2 + q^2(\theta - \gamma)},\ x = \frac{q\theta - 2c}{2 + q^2(\theta - \gamma)},\ \pi_o = \frac{\theta^2 + 2(\theta - \gamma)c(q\theta - c)}{2(\theta - \gamma)(2 + q^2(\theta - \gamma))}.$$

Conditions: $c \geq q\gamma + \frac{2\gamma - \theta}{q(\theta - \gamma)},\ c \leq \frac{q\theta}{2}$.

Case II4: $\kappa \geq 0, \phi \geq 0$

$p = \gamma\ , x = q\gamma - c\ , \pi_o = \gamma - \frac{1}{2}(q\gamma - c)^2$.

Conditions: $c \leq q\gamma + \frac{2\gamma - \theta}{q(\theta - \gamma)},\ c \leq q\gamma$.

Profit Comparison

To obtain the product developer's profit maximizing price and deterrence level, we have to compare its profits in Problem I and Problem II. For any given q and θ, we draw a figure in (c, γ) space to show the areas specified by the conditions given above for each case and then find out which expressions of the product developer's profit are relevant in profit comparison. Profit comparison yields the results summarized by the following figure and Proposition 1.

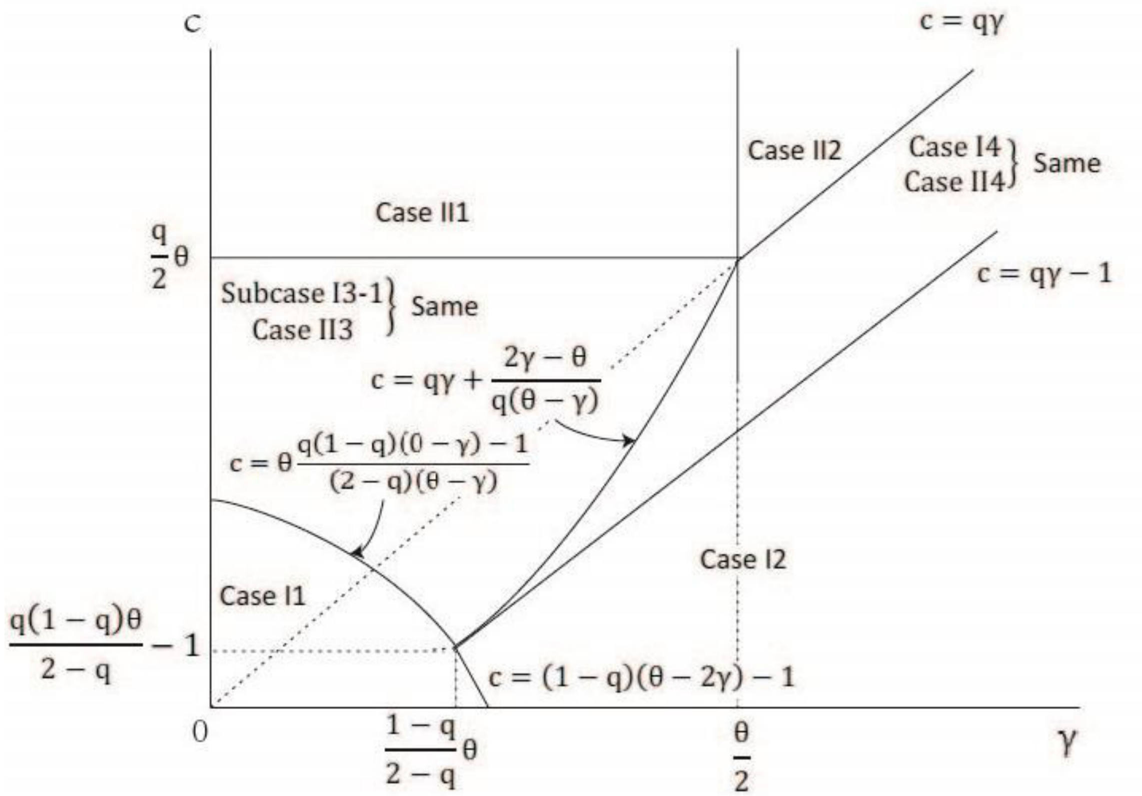

Figure A1 The distribution of the product developer's profit maximization case