Economics Bulletin

Volume 37, Issue 2

Multi-objective optimization via visualization

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Abstract

This paper presents a very simple and intuitive multi-objective optimization method that makes use of interactive visualization techniques. Multiple views of the potential solutions on scatterplots allow the user to directly search acceptable solutions in biobjective spaces whereas a Venn diagram displays information about the relative scarcity of potential acceptable solutions under distinct criteria. No heuristic search algorithm is used but as an option novelty and diversity in the search space can be created at the user demand. An application to a multi-criteria strategic asset allocation optimization problem is presented.

Citation: Nicolas Wesner, (2017) "Multi-objective optimization via visualization", *Economics Bulletin*, Volume 37, Issue 2, pages 1226-1233
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Submitted: February 24, 2017. **Published:** June 05, 2017.

Introduction

Multi-objective formulations are realistic models for many complex real-life problems where objectives under consideration conflict with each other. For a nontrivial multi-objective optimization problem, there does not exist a single solution that simultaneously optimizes each objective, but there exists a possibly infinite number of optimal solutions. In this context the notion of optimum was generalized by Pareto in the 19th century: Pareto's solutions are the solutions for which improvement in any objective is impossible without impairment in some other(s). The values of objectives for the Pareto solutions form a Pareto frontier which can be readily observed when two or three objective functions are considered. This graphic representation was widely used in Economic theory for explanatory purpose: indifference curves for the representation of consumer preferences, efficient frontier in portfolio optimization (Markowitz 1952) but is less straightforward with more than three objectives.

Direct computation of the Pareto Front is not possible in most cases and traditional mathematical programming is generally inefficient or requires too high computational costs, thus metaheuristics are usually considered for providing an approximation of the Pareto frontier. Those algorithms are generally advocated as nature or bio-inspired processes such as genetic algorithm, swarm based methods or colony based algorithms (see Boussaïd et al. 2013 for a survey). Usual drawbacks are the high computation cost and lack of convergence proof. Moreover calibration can be tricky, which adds complexity to the problem to be solved. Finally user preferences are generally not part of the optimization process, but either predetermined (a priori methods) or solicited at the end of the process, where the user selects the single optimal solution on the Pareto frontier.

In interactive methods the search for the solution is iterative and the decision maker dynamically interacts with the optimization process. As Deb (2006) or Miettinen (2010) pointed out in favor of interactive approach, finding a preferred and smaller set of Pareto optimal solutions, instead of the entire frontier permits to considerably reduce computation cost, moreover interactive methods allow the user to learn about the problem and build a conviction about the solution reached. Here again a plethora of methods exists (see Miettinen et al. 2008 for a survey) and can generally be classified according to the way user preferences are integrated in the search process. This papers focuses on interactive multi-objective optimization methods that make use of multidimensional data visualization techniques. Many visualization techniques have been developed for direct visualization of the Pareto solutions in the objective space (see Mietinnen 2014 or Bandaru et al. 2017). Those graphic representations permit the user to gain insight into attainable solution, evaluate trade-off among objectives, adapt his preference and goals, and ultimately choose a unique solution on the Pareto frontier. Under this approach visualization serves as a support for decision making. In the context of interactive optimization, visualization is also perfectly suited for organizing the interplay between user analysis and metaheuristics or automated search methods. In this respect Brunato and Battini (2010) use a combination of clustering, dimensionality reduction and parallel coordinates for creating a visual interface that permits the user to guide a stochastic local search algorithm in the exploration process. Other examples include Mietinnen (2010) that uses visualization techniques in combination with the NIMBUS method, a popular interactive multi-objective optimization model, to allow the decision maker to select, compare and generate solutions, or Matkovic et al. (2008) and Stump et al. (2009) who make use of interactive visualization analysis to steer simulations of new prototypes in a process of automative engine design. In this framework the user is in the loop and the system reacts to his input, visualization serves as an interface for communicating his implicit knowledge and preferences and eventually triggering or steering an automated search algorithm or specific computation. This concept is known as semantic interaction (Endert et al. 2012) or visual steering (Fonseca et al. 2015).

This paper presents an interactive multi-objective optimization method where the user directly searches and selects preferred solutions with the use of data visualization techniques such as scatter plots and Venn diagrams. This approach stands mid-way between the brush and link technique, a visual method used in operational research for exploratory analysis of multidimensional data sets (Becker and Cleveland 1987), and interactive multi-criteria decision methods that use the concept of reference point (Wierzbicki 1980).

The first part of the paper presents the mathematical formulation of the problem and the search and selection process. An application to strategic asset allocation optimization is performed in the second part. The conclusion summarizes the main advantages of this approach and provides some directions for future research.

Mathematical Formulation

In mathematical terms, a multi-objective optimization problem can be formulated as follows:

$$\text{Max } F(x) = [f_1(x), f_2(x), \ldots, f_k(x)]^T, \quad x \in S \subset \mathbb{R}^n, \tag{1}$$

where x is a decision vector, f(x) an objective function, k is the number of objectives, and n the number of parameters subject to optimization. S is the feasible set of decision vectors, the decision space, typically defined by some constraint functions, and F maps the decision space S on to the objective space $O \subset \mathbb{R}^k$.

The most common formulation that permits to integrate user preferences is a weighted sum decomposition:

$$\text{Max } F(x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) + \ldots + \lambda_k f_k(x), \quad \sum_{i=1}^{k} \lambda_i = 1 \tag{2}$$

When preferences can be specified in advance this approach permits to reduce the problem to a single objective optimization problem. This task is not easy when multiple criteria are considered, especially when the user is not aware of the impact of weight modifications on objective values.

A common approach in interactive multi-criteria decision making is to make use of the concept of reference point. The user specifies an ideal solution, the reference point, which refers to the goal or aspiration level for each objective and revises it through the process. The method proposed here is quite similar except that the user does not have to formulate numerically his preferences but visually selects sets of acceptable solutions with regard to pairs of criteria. Ultimately this is equivalent to determining the worst acceptable solution point, i.e. minimum requirement for each criterion. For a maximization problem with two objective functions f_i and f_j, the selection $A^{i,j}$ is defined as follows (see figure 1):

$$A^{i,j} = \{x \in S, f_i(x) > w_i \text{ and } f_j(x) > w_j, i, j = 1, \ldots, k\} \tag{3}$$

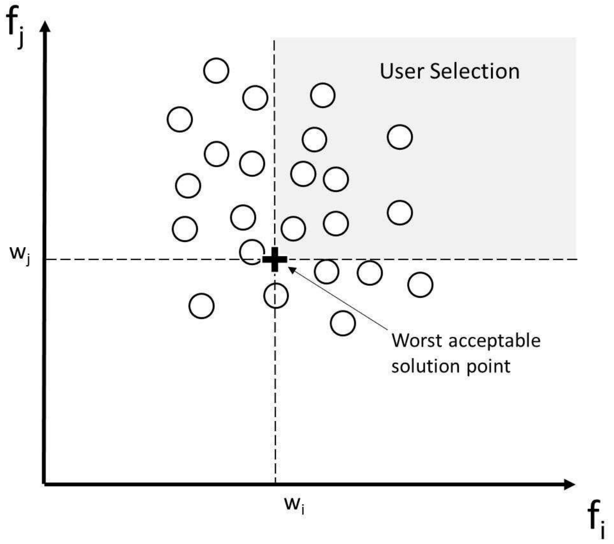

Figure 1: Decision vectors x are plotted according to their objective values for criteria i and j. The user selection is a rectangle in the upper right of the scatterplot and is characterized by the worst acceptable solution with regard to objectives i and j.

The search and selection process is iterative and makes use of interactive visual analysis. A representative precomputed set of potential solutions is displayed in multiple separate scatterplots where coordinates of points correspond to objective values. The user selects sets of acceptable solutions in those separate scatterplots and monitors his search through a Venn diagram which allows to identify the solution set S^o:

$$S^o = A^{1,2} \cap \ldots \cap A^{i,j} \cap \ldots \cap A^{k-1,k} \quad (4)$$

Discussion and related works

The method is as follows:

1. A representative set of feasible decision vectors is generated: X^i (i=1,…, N)
2. Those decision vectors are valued through objective functions: $f_j(X^i)$ (i=1,…, N; j=1,…, k)
3. Those outcomes are displayed in multiple scatterplots where the axis corresponds to pairs of different objectives
4. The decision maker selects visually/manually groups of acceptable decision vectors (rectangle zones in the scatterplots) according to his preferences and with regards to pairs of objectives.
5. The Venn diagram displays intersections of those subgroups and provides the solution set S^o. If this solution set is too large or empty, the Decision maker repeats step 4 in narrowing/enlarging selections in the scatterplots, when he is satisfied with the solution(s), the procedure stops.
6. In a sixth optional step, new decision vectors are created in the neighborhood of solution vectors S^o selected in the previous step. The procedure is then repeated from step 2.

In the first step there is a natural tradeoff between ensuring the representativeness of the set of feasible solutions, which is necessary in order to find a global optimum, and reduce computational cost especially when objective values are simulation based. Monte Carlo simulations can be used but as can be seen in the next application a grid search approach can also be an efficient strategy.

Scatterplots allows the user to directly observe true objective values for each potential solution and translate his preferences and aspirations into explicit goals. Deb (2001) among other authors advocated the use of scatterplot matrices (matrix that displays each unique pair of criteria in scatterplots) for exploring the Pareto frontier. The use of complete scatterplot matrix is not necessary here since each objective has to be shown only once to the user. Moreover, unlike parallel coordinates, a popular graphical technique used for the visualization of solutions in the objective space (Deb 2016, Bruanto and Berrati 2010, Matvokic et al. 2009), no specific order is needed for the display of objectives in Scatterplots and it is not necessary that objectives should be conflicting.

The use of a Venn diagram is central to the interactive process, it displays intersections of acceptable solution sets and permits to identify solutions that meet all criteria. The Venn diagram also displays information about the relative scarcity of acceptable solutions under distinct criteria and permits the user to guide and revise his selections. The combination with scatterplots provides a useful interactive tool that allows the user to learn about the optimization problem.

The last optional step allows to create diversity and novelty in the search space. As in the first step, many algorithms can be used to populate the decision space in the neighborhood of the selected solutions. As an interesting example, Stump et al. (2007) use a differential evolution algorithm where the fitness function is defined by the Euclidean distance between the potential solution and the preselected solutions, in both decision and objective spaces. In the following example a simple grid search approach is used.

Multi-objective Optimal Strategic Asset Allocation

In this section the proposed approach is applied to a strategic asset allocation optimization problem with 7 asset classes (Equity, Nominal and inflation indexed government bonds, high quality AA and BBB corporate bonds, real estate and cash) and 6 objectives related to risk and expected returns. One supposes that the decision maker is interested portfolio allocation that allows to avoid large downward deviation in the short term. Thus the criteria used for measuring the performance of portfolios are the mean, standard deviation and first quartile of historical monthly returns. In order to take into account the changing market and economic conditions and focus on recent history, those statistics are measured over a 10-year and a 3-year period. More precisely monthly returns for various portfolios were computed on the basis of historical data over the period 2000-2015 for yields on Nominal and real Government Bonds, Iboxx £ AA 15+ and Iboxx £ BBB 15+ indexes, UK house prices and FTSE index returns.

In a first step the representative set of acceptable solutions is generated through a grid search approach. The decision maker specifies his preference according to an acceptable range for each asset class (i.e. between 15% and 60% for equity, between 0% and 25% for real estate,…,etc.) , those preferences are then translated into a matrix where possible choices are displayed by bin of 5% (see table 1). The representative feasible set is defined by the arrangements of those possible choices whose sum is 100%, which results in 8 050 feasible portfolios.

The 6 criteria are computed for each portfolio and objective values are displayed in scatterplots. The optimization process is described in figure 2: the decision maker selects subsets of acceptable solutions in those multiple views and analyzes intersections in a Venn diagram. On scatterplots a) and b) he selects portfolios in the upper-left, those subsets regroup allocations that maximize average and minimize standard deviation of historical monthly returns, respectively measured over 3 or 10 years. On scatterplots c) the user selects portfolios that maximize the first quartile of monthly returns, which are located in the upper-right.

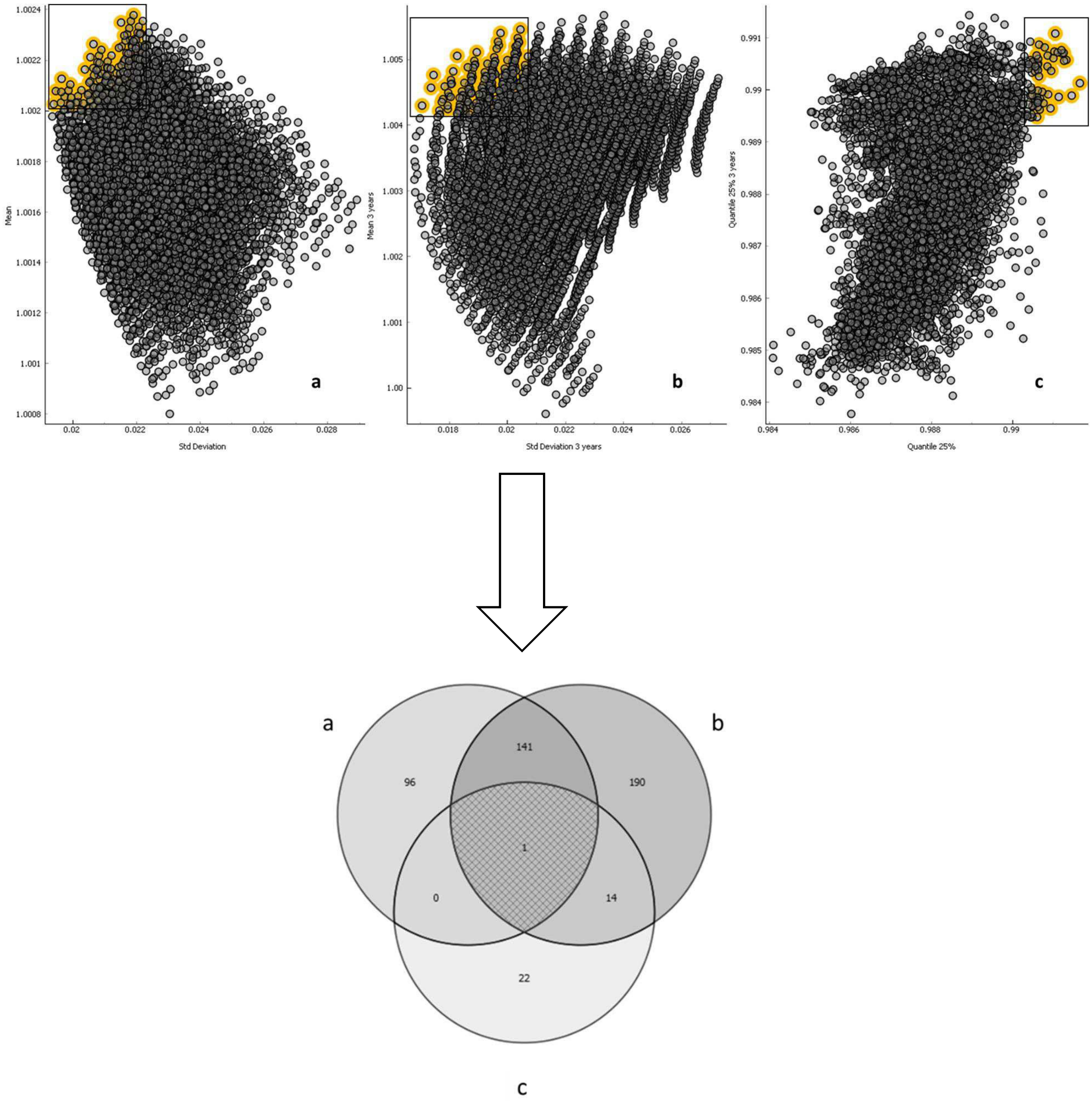

Figure 2: Feasible portfolios are displayed in the mean-variance space for a period of 3 and 10 years (respectively on scatterplots *a* and *b*) and in a scatterplot that presents quartiles measured over a 3 and 10-year period. Intersections of those 3 subsets are presented in a Venn diagram.

After few iterative refinements guided by information gained from the Venn diagram, the solution set S°, here a single solution, is determined:

	Equity	Gov. Bonds	Inflaiton linked Gov. Bonds	AA Corp. Bonds	BBB Corp. Bonds	Real Estate	Monetary
Acceptable Range	15%-60%	15%-60%	0%-25%	15%-60%	0%-25%	0%-25%	0%-10%
Optimal Solution S°	15%	25%	25%	15%	5%	10%	5%

Table 1: Acceptable allocation ranges and optimal solution reached.

In order to perform a sensitivity analysis and explore a possible threshold effect, new feasible solutions are generated in the neighborhood of this single solution with the aid of a permutation approach. For each pair of asset classes, the allocation in the former asset class is augmented by an arbitrary small margin of 2%, whereas the latter allocation is reduced by the same margin. This procedure is repeated for a margin of 3% and leads to 84 new allocations that are displayed in the scatterplots with other potential solutions (see figure 3). As can be seen in figure 3 the new portfolios created do not allow to improve performance relative to mean and variance of returns but some yield higher quartile returns. Those results can serve as a new basis for the optimization process and possibly lead the decision maker to revise his preferences in term of allocation or confirm his former choice.

This very simple example has primarily an illustrative purpose and can find a lot of improvements. First many other performance criteria for risk and expected returns, even simulation based with the use of economic scenario generator, could be considered. Finally the solution reached can serve as initial conditions for a population based optimization evolutionary algorithm whose solutions can be injected back in the visual interactive optimization process.

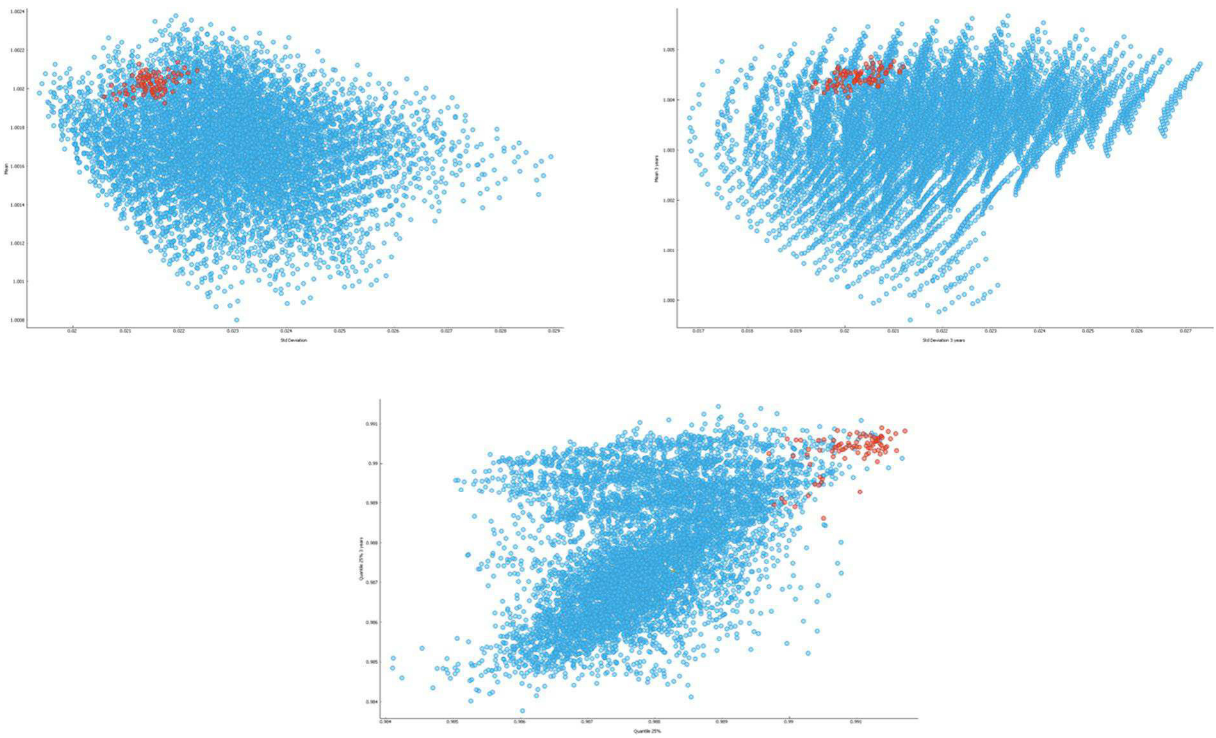

Figure 3: New potential solutions are presented in red.

Conclusion

The use of multiple scatterplots views as a support for decision making or computation steering in the context of multi-objective optimization is not new, but here computation is steered on the basis of a Venn diagram and scatterplots serves for exploration and the selection process. The combination of those graphic representations serve as a tool for interactive learning and allows the user to gain insight into attainable solutions. Moreover those very intuitive data visualization techniques allow for comprehensive interpretation and permit to communicate the results efficiently. The method presented here can serve as a solid starting point for the implementation of a more sophisticated evolution algorithm.

More generally, this approach follows the recommendations of Shneiderman (2002) who suggested to combine information visualization with data mining: novel methods should allow the user to specify what he is looking for, results should be easily reportable and human responsibility should be respected. In the multi-objective optimization method presented here the user selects the solutions he is interested in and possibly revises his preference and goal to reach a unique solution through the use of an efficient combination of interactive data visualization techniques. The development of such visual interactive methods should permit to make full use of the combination of the processing power of computers with superior learning and pattern recognition capabilities of the human user.

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