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Collusion stability in a differentiated Cournot duopoly with payoff interdependence

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Abstract

This study considers an infinitely repeated Cournot (quantity-setting) duopoly with product differentiation in which each firm is concerned about not only its own profit but also that of the opponent. Assuming that both firms use the grim trigger strategy, we prove that tacit collusion between the firms (cartel) may become more sustainable as product substitutability increases when the objective of each firm is to maximize its own profit compared to that of the opponent. On the other hand, it is shown that when each firm seeks to maximize its own "absolute" profit without taking the profit of the opponent into account, increased product substitutability necessarily destabilizes collusion between the firms. These results imply that the impact of increasing product substitutability on collusion stability crucially hinges on whether duopoly firms seek to maximize absolute or relative profits.

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1. Introduction

There has been extensive research interest in tacit collusion among oligopolistic firms (i.e., cartels). In particular, much attention has been paid to the relationship between the stability of tacit collusion and product substitutability. For example, Deneckere (1983) considers a differentiated Cournot (quantity-setting) duopoly with constant marginal costs and finds that collusion becomes more difficult to sustain when the products become more substitutable in an infinitely repeated interaction with grim trigger strategies.[1] However, succeeding studies (e.g., Wernerfelt [1989], Tyagi [1999], and Song and Wang [2017]) reveal that Deneckere's (1983) result is not always true, as increased product substitutability facilitates collusion in some cases.

This study provides novel confirmation of the view that collusion can become more sustainable as product substitutability increases by analyzing an infinitely repeated Cournot duopoly with product differentiation. In this analysis, we employ the "payoff interdependence approach" and assume that the objective of each firm is to maximize its payoff, which depends not only on its own profit but also on that of the opponent.[2] The payoff interdependence approach is increasingly being applied in studies of the theory of industrial organization. For example, Matsumura and Matsushima (2012) take this approach to analyze the relationship between market competitiveness and collusion stability in an infinitely repeated Cournot duopoly, assuming perfect product substitutability.[3] We extend the analysis of Matsumura and Matsushima (2012) to include product differentiation for the purpose of (re)examining the impact of increasing product substitutability on collusion stability.

This study contributes to the related literature by proving that increased product substitutability does not necessarily destabilize collusion when the objective of each firm is to maximize its own profit compared to that of the rival firm, whereas collusion always becomes less sustainable under increased product substitutability when firms maximize "absolute" profits. In other words, we show that the impact of increasing product substitutability on collusion stability crucially depends on whether duopoly firms seek to maximize absolute or relative profits. This result is important for setting competition policy because it implies that competition policy that ignores payoff interdependence may fail severely.

[1] Collie (2006) revisits Deneckere (1983), assuming increasing linear marginal costs (i.e., quadratic total costs).

[2] By contrast, previous studies usually assume that each firm seeks to maximize its "absolute" profit without taking the profit of the opponent(s) into account.

[3] See also Lundgren (1996), Matsumura *et al.* (2013), Tanaka (2013), and Matsumura and Okamura (2015), among others.

The remainder of this paper is organized as follows. We set up our model in Section 2. Section 3 provides preliminary results for the analysis in Section 4, in which we present the primary results. Concluding remarks are provided in Section 5, and the technical details of each result are explained in the Appendix.

2. Model

Suppose that there exist only two firms (A and B) producing differentiated goods. Let x_i and p_i denote the quantity supplied by firm $i \in \{A, B\}$ and the price of good i, respectively. In this model, p_i is determined according to the inverse demand function for good i,

$$p_i = a - x_i - \gamma x_j,\ j \in \{A, B\},\ j \neq i,$$

where $a > 0$ and $\gamma \in (0, 1)$ are constants. Note that γ is regarded as a parameter that captures the degree of product differentiation.[4] That is, a larger (smaller) γ implies that the two goods are more (less) substitutable.

For analytical simplicity, we assume that the total cost of firm i is given by cx_i, where $c \in (0, a)$ denotes the marginal cost, which is assumed to be constant. Therefore, the *profit* of firm i is given by $\pi_i := (a - c - x_i - \gamma x_j)x_i$.

We suppose that in the absence of collusion, the objective of firm i is to maximize the *payoff* given by

$$U_i := \pi_i - \alpha\pi_j,$$

where $\alpha \in (-1, 1)$ denotes the degree to which each firm is concerned about its rival's profits.[5] When $\alpha > 0$ ($\alpha < 0$), firm i becomes worse off (better off) as the profit of firm j, π_j, increases. When $\alpha = 0$, this analysis is the same as that of a standard differentiated Cournot duopoly.

3. Preliminaries

Before deriving our main results on collusion stability, we derive the payoff of each firm in the case of collusion and the payoffs at the Cournot-Nash equilibrium in the case of no collusion, both of which will be useful for stating our main results.

[4]If $\gamma = 1$ (i.e., if there is perfect product substitutability), the present analysis is the same as that of Matsumura and Matsushima (2012).

[5]The present specification of U_i follows that of Matsumura and Matsushima (2012), among others. Matsumura and Matsushima (2012) provide a convincing argument for considering a non-zero $\alpha \in (-1, 1)$. See also the studies cited therein. In the present study, for analytical simplicity, we assume that the value of α is exogenously given. Miller and Pazgal (2002), for example, consider a two-stage oligopoly game to endogenize α (denoted as ϕ in their analysis).

3.1. Collusion (Joint Payoff Maximization)

Suppose that the two firms collude. Then, each firm maximizes the joint payoff

$$\begin{aligned} U_A + U_B &= (1-\alpha)(\pi_A + \pi_B) \\ &= (1-\alpha)\{(a - c - x_A - \gamma x_B)x_A + (a - c - \gamma x_A - x_B)x_B\} \end{aligned}$$

by producing

$$x_{\mathrm{C}} = \frac{a-c}{2(1+\gamma)},$$

where the suffix C stands for collusion. The payoff of each firm under collusion is derived as:

$$U_{\mathrm{C}} = \frac{(1-\alpha)(a-c)^2}{4(1+\gamma)}. \tag{1}$$

3.2. Cournot-Nash Equilibrium

Suppose that the two firms do *not* collude. Then, taking x_j as given, firm i chooses x_i in order to maximize its own payoff,

$$U_i = (a - c - x_i - \gamma x_j)x_i - \alpha\{(a - c - \gamma x_i - x_j)x_j\}.$$

The reaction function of firm i is derived as

$$x_i = \frac{a-c}{2} - \frac{(1-\alpha)\gamma}{2}x_j.$$

Therefore, at a Cournot-Nash equilibrium, each firm produces

$$x_{\mathrm{N}} = \frac{a-c}{2+(1-\alpha)\gamma},$$

where the suffix N stands for non-collusion. As expected, $x_{\mathrm{N}} > x_{\mathrm{C}}$ for any $\alpha \in (-1, 1)$. The payoff of each firm at the Cournot-Nash equilibrium is

$$U_{\mathrm{N}} = \frac{(1-\alpha)(1-\alpha\gamma)(a-c)^2}{\{2+(1-\alpha)\gamma\}^2}. \tag{2}$$

4. Results on Collusion Stability

Suppose that the interaction between the two firms is repeated infinitely. In addition, following previous studies, we assume that each firm uses the grim trigger strategy, in which one firm's unilateral deviation from collusion triggers the "forever Nash reversion" of the other firm.

If firm i decides to deviate from collusion unilaterally at some point in time, then firm i maximizes

$$U_i = (a - c - x_i - \gamma x_{\mathrm{C}})x_i - \alpha\{(a - c - \gamma x_i - x_{\mathrm{C}})x_{\mathrm{C}}\}$$

by producing

$$x_{\mathrm{D}} = \frac{\{2 + (1 + \alpha)\gamma\}(a - c)}{4(1 + \gamma)},$$

where the suffix D stands for deviation. The temporary payoff of the deviating firm is

$$U_{\mathrm{D}} = \frac{\{4(1 - \alpha)(1 + \gamma) + (1 + \alpha)^2\gamma^2\}(a - c)^2}{16(1 + \gamma)^2}. \tag{3}$$

Note that once deviation is detected, the Cournot-Nash equilibrium arises, and, thus, each firm obtains U_{N} ($< U_{\mathrm{C}}$) forever. As long as no deviation takes place, each firm sticks to collusion and earns U_{C}.

In the following analysis, collusion is said to be *stable* (or *sustainable*) if and only if each firm cannot become better off by deviating from collusion unilaterally. Let $\delta \in (0, 1)$ denote the discount factor common to both firms. Collusion is stable when the firms apply grim trigger strategies if and only if

$$\frac{U_{\mathrm{C}}}{1 - \delta} \geq U_{\mathrm{D}} + \frac{\delta}{1 - \delta}U_{\mathrm{N}} \text{ or } \delta \geq \frac{U_{\mathrm{D}} - U_{\mathrm{C}}}{U_{\mathrm{D}} - U_{\mathrm{N}}} =: \delta^*(\alpha, \gamma), \tag{4}$$

where U_{C}, U_{N}, and U_{D} are given by (1), (2), and (3), respectively.

Proposition 1: $\delta^*(\alpha, \gamma)$ is given by

$$\delta^*(\alpha, \gamma) = \frac{\{2 + (1 - \alpha)\gamma\}^2}{4(2 - \alpha) + 8(1 - \alpha)\gamma + (1 - \alpha)^2\gamma^2}. \tag{5}$$

Proof. See Appendix.

We note that our equation (5) replicates equation (8) in Matsumura and Matsushima (2012) if $\gamma = 1$ (i.e., if there is perfect product substitutability).

Following previous studies, we use $\delta^*(\alpha, \gamma)$ as a measure of collusion stability. A larger (smaller) δ^* implies that collusion is less (more) stable.

Proposition 2: For each degree of product substitutability, $\gamma \in (0, 1)$, $\delta^*(\alpha, \gamma)$ is strictly increasing in α, which implies that collusion becomes more difficult to sustain when both firms are more strongly motivated to compete (i.e., when α is larger).

Proof. See Appendix.

Proposition 2 states that the result of Matsumura and Matsushima (2012) continues to hold even when their basic model is extended to include product differentiation. The intuitive explanation for this result is the same as that provided by Matsumura and Matsushima (2012) regarding Proposition 1 of their analysis.

Proposition 3 (Main Result): (a) If $\alpha \in (-1, 0]$, then $\delta^*(\alpha, \gamma)$ is strictly increasing in $\gamma \in (0, 1)$. (b) If $\alpha \in (0, 1/3)$, then there exists $\bar{\gamma} := 2\alpha/(1 - \alpha) \in (0, 1)$ such that $\delta^*(\alpha, \gamma)$ is strictly decreasing in γ over the interval $(0, \bar{\gamma})$ and is strictly increasing in γ over the interval $(\bar{\gamma}, 1)$. Therefore, $\delta^*(\alpha, \gamma)$ is minimized at $\gamma = \bar{\gamma}$ for each $\alpha \in (0, 1/3)$. (c) If $\alpha \in [1/3, 1)$, then $\delta^*(\alpha, \gamma)$ is strictly decreasing in $\gamma \in (0, 1)$.

Proof. See Appendix.

According to Proposition 3, collusion becomes less stable as products become closer substitutes (i.e., as γ increases) for $\alpha \in (-1, 0]$. For $\alpha \in (0, 1)$, however, this result does not necessarily hold. In particular, if $\alpha \in [1/3, 1)$, then increased product substitutability necessarily *stabilizes* collusion.

An intuitive explanation of Proposition 3 is as follows. If product substitutability, γ, increases for a given α, then there arise two conflicting effects on δ^*.[6] The first effect is called the *deviation effect*, and it makes collusion more difficult to sustain because it increases the gain from unilateral deviation from collusion (i.e., $U_{\mathrm{D}} - U_{\mathrm{C}}$):

$$\frac{\gamma}{(U_{\mathrm{D}} - U_{\mathrm{C}})} \cdot \frac{\partial(U_{\mathrm{D}} - U_{\mathrm{C}})}{\partial \gamma} = \frac{2}{1 + \gamma} > 0. \tag{6}$$

The other effect is called the *penalty effect*, and it makes collusion easier to sustain because it increases the loss triggered by unilateral deviation from collusion (i.e., $U_{\mathrm{C}} - U_{\mathrm{N}}$):

$$\frac{\gamma}{(U_{\mathrm{C}} - U_{\mathrm{N}})} \cdot \frac{\partial(U_{\mathrm{C}} - U_{\mathrm{N}})}{\partial \gamma} = \frac{4 + \{2 - (1 - \alpha)\gamma\}\gamma}{(1 + \gamma)\{2 + (1 - \alpha)\gamma\}} > 0. \tag{7}$$

Therefore, the impact of increasing product substitutability on collusion stability is generally ambiguous. However, we should note that the penalty effect (7) is stronger for

[6]Equations (6) and (7) are derived from (8) and (9) in the Appendix, respectively.

a greater degree of rivalry, α. Intuitively, as the product substitutability, γ, increases, the price of each good at the Cournot-Nash equilibrium decreases, and each firm becomes worse off (i.e., U_{N} is strictly decreasing). For a larger α, this fall in equilibrium prices is larger, so it serves as a more effective penalty for deviating from collusion. Hence, for a larger α, the penalty effect is more likely to dominate the deviation effect, and δ^* becomes smaller (i.e., collusion becomes more sustainable) for closer substitutes.

5. Concluding Remarks

In order to (re)examine how collusion stability is affected by increased product substitutability, we considered an infinitely repeated Cournot (quantity-setting) duopoly in which each firm is concerned about not only its own profit but also that of the opponent.

By extending the analysis of Matsumura and Matsushima (2012) to include product differentiation, we found the following:

1. For each level of product substitutability, $\gamma \in (0, 1)$, collusion always becomes less stable as both firms are more motivated to compete (i.e., α is larger).

2. Suppose that both firms seek to maximize either the "symbiotic" payoff (i.e., $\alpha \in (-1, 0)$) or the absolute profit (i.e., $\alpha = 0$) in the absence of collusion. Then, increased product substitutability (i.e., a larger γ) destabilizes collusion with certainty.

3. Suppose that each firm seeks to maximize its own profit compared to that of the opponent (i.e., $\alpha \in (0, 1)$) in the absence of collusion. Increased product substitutability can either stabilize or destabilize collusion when $\alpha \in (0, 1/3)$, whereas if $\alpha \in [1/3, 1)$, collusion is necessarily easier to sustain as the products become closer substitutes.

These results imply that the impact of increasing product substitutability on collusion stability crucially depends on what duopoly firms seek to maximize (i.e., the value of α) and that a competition policy that ignores payoff interdependence (i.e., assuming $\alpha = 0$) may severely fail if $\alpha \in [1/3, 1)$.

Finally, we note that a similar analysis to this one could be conducted for the case of a differentiated Bertrand (price-setting) duopoly with payoff interdependence.[7] This analysis is left for future work.

[7] In the supplemental material to Matsumura and Matsushima (2012), which is available at: `http://norick.sakura.ne.jp/research/boer-supplement.pdf`, Matsumura and Matsushima analyze a differentiated Bertrand (price-setting) duopoly with payoff interdependence. However, they do not consider the relationship between collusion stability and product substitutability.

Appendix

A.1. Proof of $U_{\mathbf{D}} > U_{\mathbf{C}} > U_{\mathbf{N}}$

Subtracting (2) from (1) yields

$$U_{\mathrm{C}} - U_{\mathrm{N}} = \frac{(1-\alpha)(1+\alpha)^2\gamma^2(a-c)^2}{4(1+\gamma)\{2+(1-\alpha)\gamma\}^2}. \tag{8}$$

Because $\alpha \in (-1, 1)$, $\gamma \in (0, 1)$, and $a > c$, it is clear that $U_{\mathrm{C}} > U_{\mathrm{N}}$.

Subtracting (1) from (3) yields

$$U_{\mathrm{D}} - U_{\mathrm{C}} = \frac{(1+\alpha)^2\gamma^2(a-c)^2}{16(1+\gamma)^2}, \tag{9}$$

from which $U_{\mathrm{D}} > U_{\mathrm{C}}$ clearly follows.

A.2. Proof of Proposition 1

By the definition of δ^* (given in (4)), we have

$$\delta^* = \frac{U_{\mathrm{D}} - U_{\mathrm{C}}}{U_{\mathrm{D}} - U_{\mathrm{N}}} \Leftrightarrow \frac{1}{\delta^*} - 1 = \frac{U_{\mathrm{C}} - U_{\mathrm{N}}}{U_{\mathrm{D}} - U_{\mathrm{C}}} =: F(\alpha, \gamma). \tag{10}$$

Using (8) and (9), we obtain

$$F(\alpha, \gamma) = \frac{4(1-\alpha)(1+\gamma)}{\{2+(1-\alpha)\gamma\}^2} > 0.$$

Therefore, it is straightforward to derive

$$\delta^* = \frac{1}{1+F(\alpha,\gamma)} = \frac{\{2+(1-\alpha)\gamma\}^2}{4(2-\alpha)+8(1-\alpha)\gamma+(1-\alpha)^2\gamma^2} \tag{11}$$

from (10). This result is (5) in the main text. Note that $\delta^* \in (0, 1)$ because $F > 0$.

A.3. Proof of Proposition 2

Differentiating $F(\alpha, \gamma)$ with respect to α yields

$$F_\alpha := \frac{\partial F(\alpha,\gamma)}{\partial \alpha} = \frac{-4(1+\gamma)\{2-(1-\alpha)\gamma\}}{\{2+(1-\alpha)\gamma\}^3} < 0$$

because

$$2-(1-\alpha)\gamma > 2-(1-\alpha) = 1+\alpha > 0.$$

Therefore, it follows from (11) that as α increases, δ^* increases (because F decreases).

A.4. Proof of Proposition 3

Differentiating $F(\alpha, \gamma)$ with respect to γ yields

$$F_\gamma := \frac{\partial F(\alpha, \gamma)}{\partial \gamma} = \frac{4(1-\alpha)\{2\alpha - (1-\alpha)\gamma\}}{\{2 + (1-\alpha)\gamma\}^3}. \tag{12}$$

(a) If $\alpha \in (-1, 0]$, $F_\gamma < 0$ for any $\gamma \in (0, 1)$. Therefore, it follows from (11) that as γ increases, δ^* increases (because F decreases), which is stated in part (a) of Proposition 3.

(b) If $\alpha \in (0, 1/3)$, it follows from (12) that there exists $\bar{\gamma} := 2\alpha/(1-\alpha) \in (0, 1)$ such that

$$F_\gamma \gtreqless 0 \Leftrightarrow \gamma \lesseqgtr \bar{\gamma}.$$

Because δ^* is strictly decreasing in F, we have

$$\frac{\partial \delta^*(\alpha, \gamma)}{\partial \gamma} \lesseqgtr 0 \Leftrightarrow \gamma \lesseqgtr \bar{\gamma},$$

which leads to part (b) of Proposition 3.

(c) If $\alpha \in [1/3, 1)$, $F_\gamma > 0$ for any $\gamma \in (0, 1)$, because

$$2\alpha - (1-\alpha)\gamma > 2\alpha - (1-\alpha) = 3\alpha - 1 \geq 0.$$

Therefore, it follows from (11) that as γ increases, δ^* decreases (because F increases), which is stated in part (c) of Proposition 3.

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