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The desirability of the supply function competition under demand uncertainty

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## **Abstract**

In this paper, we consider an oligopolistic industry with demand uncertainty and study the welfare comparison between the supply function competition and the stochastic Cournot competition. We prove that the expected consumer surplus is always higher under the supply function competition. By numerical computations we also show that the expected profits of the oligopolistic firms can be higher under the supply function competition only if the demand uncertainty is above a critical threshold. This threshold is increasing in the number of firms, while decreasing in the slope of the demand curve and the marginal cost of producing a unit output.

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### 1. Introduction

Economists have long criticized Cournot's (1838) oligopolistic competition model for not allowing firms the flexibility to adjust their output in the face of unanticipated demand shocks. A remedy was suggested by Grossman (1981), who considered a new type of competition where firms have the possibility to commit to supply functions specifying the quantity to be supplied as a function of the price. While the suggested remedy was very wise, its (initial) success was arguable, since the supply function competition was found to lead to a problem known as the multiplicity of equilibria. In fact, this problem is very disturbing, since in any oligopoly without any type of uncertainties, it is possible to find infinitely many output profiles, each of which can be supported by infinitely many profiles of supply functions in equilibrium. Fortunately, all these multiplicities/indeterminacies, and consequently all arguments against the supply function competition, may disappear under some additional assumptions about the demand or cost curves or under some changes in the equilibrium concept, as was shown by Klemperer and Meyer (1989), Delgado and Moreno (2004), Delgado (2006), and Król (2017).

The modification devised by Klemperer and Meyer (1989) was to introduce —to the oligopolistic industry— an exogenous demand uncertainty with an unbounded support. In their model, firms simultaneously choose supply functions without knowing the realization of the demand shock/uncertainty. Right after they learn the realization of the uncertainty (hence, the actual position of the demand curve) begins the production stage, where firms can calculate —using the supply functions they chose in the previous stage—a market clearing price and their actual supplies implied by this price. Since each possible realization of the demand curve implies a distinct profit maximizing quantity of supply, the ability of firms to commit, through supply functions, to all possible realizations of the profit maximizing quantities (and all possible market clearing prices) protects them against the uncertainties they might face. Besides, the number (or the measure) of the supply function equilibria would be significantly reduced in the presence of demand uncertainty, because the supply function of each firm would need to pass through not a single but a multiplicity of profit maximizing quantities, each of which corresponding to a distinct realization of the demand curve. In fact, the uncertainty in demand can even ensure the uniqueness of equilibrium if the demand curve and the marginal cost curve of each firm become linear when the price and the supply become sufficiently large.

More recently, Delgado and Moreno (2004) and Delgado (2006), who followed a different path from that of Klemperer and Meyer (1989), realized that if the Nash equilibrium concept is changed with the coalition-proof equilibrium and if some additional conditions hold, not only the uniqueness of the supply function equilibria can be ensured in a deterministic oligopoly but also this unique equilibrium and the Cournot competition can lead to the same outcome. On the other hand, Król (2017) formalized an insight of Klemperer and Meyer (1989) to show that the enormous multiplicity of the supply function equilibria can be reduced in the absence of any uncertainties as well, if different supply functions have different costs of implementation. Specifically, Król (2017) showed that when excess capacity is always costly, there is even a one-to-one correspondence between the sets of Nash equilibria, and also (under some additional conditions about the industry) between the sets of strategies surviving the iterated elimination of weakly dominated strategies.

An important result of Klemperer and Meyer (1989), distinguishing it from the subsequent works discussed above, is that the set of supply function equilibria under the demand uncertainty can never boil down to the Cournot equilibria of a non-stochastic industry. As a matter of fact, Klemperer and Meyer (1989) showed that for any realization of the demand uncertainty, the equilibrium quantities of supply become always higher, and consequently the equilibrium profits become always lower, under the supply function competition with demand uncertainty than under the Cournot competition without uncertainties. However, they also predicted that when an uncertain demand curve is linear in price and resultingly the supply function competition yields a unique equilibrium, the expected profits at this equilibrium may be higher than the expected profits obtained under the stochastic Cournot competition where firms choose their supply quantities before observing the realization of the demand uncertainty. The reason they offer is that only under the supply function competition do firms adjust optimally to every possible realization of the demand uncertainty. While the predicted superiority of the supply function competition —in terms of the induced expected profits— over the stochastic Cournot competition may

lead each firm (that is able to correctly make this comparison) to act according to the predictions of the supply function competition, whether this type of competition can also be desirable for consumers is clearly another issue. As a matter of fact, this issue may become very relevant in oligopolistic regulatory problems where the regulatory authorities that are endowed with the goal of optimally balancing the net gains of consumers and producers have also the power of imposing on producers how they will compete with each other. Motivated by the relevance of such problems, we investigate in this paper the possible (ex-ante) welfare gains of the supply function competition—over the stochastic Cournot competition—for both the oligopolistic firms and consumers. Basically, we show that the expected consumer surplus is always higher under the supply function competition, whereas the expected profits of the oligopolistic firms can be higher under the supply function competition only if the demand uncertainty is sufficiently high.

The rest of the paper is organized as follows: Section 2 introduces an oligopolistic model borrowed from Klemperer and Meyer (1989) along with the descriptions of the supply function and stochastic Cournot competitions. Section 3 presents the results and Section 4 concludes.

## 2. Model

Borrowing from Klemperer and Meyer (1989), we consider an oligopolistic industry involving  $n \geq 2$  firms who produce a single homogeneous good. The firms have identical cost functions such that each firm producing a quantity of output q incurs the cost

$$C(q) = cq^2/2 \quad \text{for all} \quad q \ge 0, \tag{1}$$

where c > 0 denotes the marginal cost of a unit output. The industry demand curve is given by

$$D(p,\epsilon) = -mp + \epsilon, \tag{2}$$

where  $p \geq 0$  is the market price of the good, m > 0 and  $\epsilon \in [0, \infty)$ . The form of the cost and demand curves, C(q) and  $D(p, \epsilon)$ , as well as the cost and demand parameters c and m are assumed to be commonly known by the firms. On the other hand,  $\epsilon$  is a scalar random variable with a probability density  $f(\epsilon)$  that is strictly positive everywhere on the support  $[0, \infty)$ . It is also assumed that there is common knowledge about f(.). For the industry described above, we consider two types of competition.

## 2.1 Supply Function Competition

Under the supply function competition, a strategy for firm i is a function mapping price into a quantity of output for this firm, i.e.,  $S_i : [0, \infty) \to (-\infty, \infty)$ . In the pre-production stage, firms simultaneously choose supply functions without knowing the realization of the demand variable  $\epsilon$ . Right after they learn the realization of  $\epsilon$  begins the production stage, where firms calculate –using the supply functions they chose in the previous stage– a market clearing price  $p(\epsilon)$  that satisfies

$$\sum_{i=1}^{n} S_i(p(\epsilon)) = D(p(\epsilon), \epsilon). \tag{3}$$

If this price exists and if it is unique, then the actual outputs  $(S_i(p(\epsilon)))_{i=1}^n$  are produced. Otherwise, each firm earns zero profits. For the game played in the pre-production stage of the above setup we focus on the Nash equilibria in supply functions as in Grossman (1981) and Klemperer and Meyer (1989). We say that a profile (list) of supply functions  $(S_i^*(p))_{i=1}^n$  is a Nash equilibrium if for each firm i the function  $S_i^*(p)$  maximizes its expected profits when all of the remaining firms stick to their supply functions in the considered profile. Klemperer and Meyer (1989) shows that this implies that for each i

$$S_i^*(p) = q_i^0((p_i^0)^{-1}(p)) \tag{4}$$

where

$$p^{0}(\epsilon) = \arg\max_{p \ge 0} \ p\left(D(p, \epsilon) - \sum_{j \ne i} S_{j}^{*}(p)\right) - C\left(D(p, \epsilon) - \sum_{j \ne i} S_{j}^{*}(p)\right), \tag{5}$$

and

$$q_i^0(\epsilon) = D(p^0(\epsilon), \epsilon) - \sum_{j \neq i} S_j^*(p^0(\epsilon)). \tag{6}$$

## 2.2 Stochastic Cournot Competition

Here, a strategy for firm i is a nonnegative quantity of output,  $q_i \in [0, \infty)$ . Let Q denote the industry output; i.e.,  $Q = \sum_{i=1}^{n} q_i$ . Inverting (2), we obtain the inverse demand function

$$P(Q,\epsilon) = \frac{\epsilon}{m} - \frac{Q}{m},\tag{7}$$

for any  $Q \geq 0$ . Using this, we can write the expected profits of firm i as follows:

$$E\left[P\left(q_i + \sum_{j \neq i} q_j, \epsilon\right) q_i - C(q_i)\right] \tag{8}$$

Firms are assumed to simultaneously choose and implement their supplies without knowing the realization of the uncertain demand variable  $\epsilon$ . In this game we focus on the (Cournot) Nash equilibria in quantities. We say that a profile of quantities  $(\hat{q}_i)_{i=1}^n$  is a Nash equilibrium if for each firm i the quantity  $\hat{q}_i$  maximizes its expected profits when all of the remaining firms stick to their quantities in the considered profile. That is, for each i the quantity  $\hat{q}_i$  solves

$$\max_{q_i \ge 0} E\Big[P\Big(q_i + \sum_{j \ne i} \hat{q}_j, \epsilon\Big)q_i - C(q_i)\Big]. \tag{9}$$

#### 3. Results

Below, in Propositions 1 and 2, we will present the characterizations of the equilibria arising under the supply function competition and the stochastic Cournot competition, respectively.

**Proposition 1 (Klemperer and Meyer 1989).** The supply function competition with demand uncertainty has a unique Nash equilibrium characterized by

$$S_i(p) = \alpha p \quad for \ all \ i,$$
 (10)

where

$$\alpha = \frac{1}{2(n-1)} \left( -m + \frac{n-2}{c} + \sqrt{\left( -m + \frac{n-2}{c} \right)^2 + \frac{4m(n-1)}{c}} \right). \tag{11}$$

**Proof.** See the proof of Proposition 8a in Klemperer and Meyer (1989).

**Proposition 2.** The stochastic Cournot competition with demand uncertainty has a unique Nash equilibrium characterized by

$$q_i^C = \frac{E[\epsilon]}{n+1+mc} \quad \text{for all } i. \tag{12}$$

**Proof.** Inserting (1) and (7) into (8), we can rewrite the expected profits of firm i as

$$E[\pi_i(\epsilon)] = \frac{E[\epsilon]}{m} q_i - \frac{1}{m} \left( q_i + \sum_{i \neq i} q_i \right) q_i - \frac{c}{2} \left( q_i \right)^2.$$

$$(13)$$

Differentiating (13) with respect to  $q_i$  we obtain the first-order necessary condition

$$\frac{E[\epsilon]}{m} - \frac{1}{m} \left( 2 \, q_i + \sum_{j \neq i} q_j \right) - c \, q_i = 0, \tag{14}$$

which implies that the best-response (reaction) function for firm i is given by

$$q_i = \frac{1}{2 + mc} \left( E[\epsilon] - \sum_{j \neq i} q_j \right). \tag{15}$$

Since the reaction functions of the firms are symmetric, we must have  $q_j = q_i \equiv q_i^C$  for all  $j \neq i$  in equilibrium. Inserting this into (15) we obtain

$$q_i^C = \frac{1}{2 + mc} \Big( E[\epsilon] - (n - 1) \, q_i^C \Big). \tag{16}$$

Solving for  $q_i^C$  yields equation (12). Finally, we calculate the second-order differential of (13) with respect to  $q_i$  to obtain

$$\frac{\partial^2 E[\pi_i(\epsilon)]}{\partial (q_i)^2} = -\frac{2}{m} - c,\tag{17}$$

which is always negative. Therefore, the second-order sufficiency condition also holds, implying that the profile  $(q_i^C)_{i=1}^n$  satisfying (12) solves the maximization problem of each firm, constituting a Nash equilibrium.

Below, we will calculate the expected profits of each firm and the expected consumer surplus obtained at the equilibrium of each type of competition we are studying. Let us first consider the supply function competition. We can use the equilibrium supply functions given by (10) and (11), the demand function in (2), and the market clearing condition in (3) to calculate for any realization of  $\epsilon$  the market clearing price

$$p^{SF}(\epsilon) = \frac{\epsilon}{n\alpha + m} \tag{18}$$

and the equilibrium quantities

$$q^{SF}(\epsilon) = S_i(p^{SF}(\epsilon)) = \frac{\alpha \epsilon}{n\alpha + m}$$
 for all  $i$ . (19)

It follows that for any realization of  $\epsilon$ , the 'realized' equilibrium profits of each firm will be equal to

$$\pi^{SF}(\epsilon) = p^{SF}(\epsilon) q^{SF}(\epsilon) - \frac{c}{2} (q^{SF}(\epsilon))^2 = \frac{\alpha \epsilon^2}{(n\alpha + m)^2} \left( 1 - \frac{c\alpha}{2} \right). \tag{20}$$

Then, the equilibrium profits that each firm can expect before it learns the realization of the demand uncertainty must be equal to

$$E[\pi^{SF}(\epsilon)] = \frac{\alpha}{(n\alpha + m)^2} \left(1 - \frac{c\alpha}{2}\right) E[\epsilon^2]. \tag{21}$$

On the other hand, for any realization of  $\epsilon$ , the consumer surplus under the supply function competition is given by

$$CS^{SF}(\epsilon) = \int_0^{n q^{SF}(\epsilon)} P(x, \epsilon) dx - n p^{SF}(\epsilon) q^{SF}(\epsilon) = \frac{n^2 (q^{SF}(\epsilon))^2}{2 m} = \frac{n^2 \alpha^2 \epsilon^2}{2 m (n \alpha + m)^2}.$$
 (22)

Thus, the expected consumer surplus under the supply function becomes

$$E[CS^{SF}(\epsilon)] = \frac{n^2 \alpha^2}{2 m (n \alpha + m)^2} E[\epsilon^2].$$
 (23)

Now, we will consider the stochastic Cournot competition. Let  $q^C = E[\epsilon]/(n+1+mc)$ . From (12), we know that  $q_i^C = q^C$  for all i. So, the equilibrium output of the industry must be equal to

$$Q^{C} = n q^{C} = \frac{n E[\epsilon]}{n + 1 + mc}.$$
(24)

Using (7) and (24), we can calculate for any realization of  $\epsilon$  the corresponding market clearing price:

$$p^{C}(\epsilon) = \frac{1}{m} \left( \epsilon - \frac{nE[\epsilon]}{n+1+mc} \right) \tag{25}$$

It follows that for any realization of  $\epsilon$  the equilibrium profits of each firm are equal to

$$\pi^{C}(\epsilon) = p^{C}(\epsilon) q^{C} - \frac{c}{2} (q^{C})^{2} = \left(\frac{\epsilon}{m}\right) \left(\frac{E[\epsilon]}{n+1+mc}\right) - \left(\frac{(n/m) + (c/2)}{(n+1+mc)^{2}}\right) (E[\epsilon])^{2}. \tag{26}$$

It is easy to check that the expected profits of each firm under the stochastic Cournot competition will then be equal to

$$E[\pi^C(\epsilon)] = \left(\frac{2+mc}{2m}\right) \frac{(E[\epsilon])^2}{(n+1+mc)^2}.$$
(27)

On the other hand, for any realization of  $\epsilon$ , the consumer surplus under the stochastic Cournot competition can be calculated as

$$CS^{C}(\epsilon) = \int_{0}^{n \, q^{C}} P(x, \epsilon) dx - n \, p^{C}(\epsilon) \, q^{C} = \frac{n^{2} \, (q^{C})^{2}}{2 \, m} = \frac{n^{2}}{2 \, m \, (n+1+mc)^{2}} \, (E[\epsilon])^{2}. \tag{28}$$

Since  $CS^C(\epsilon)$  is independent of the realization of  $\epsilon$ , the expected consumer surplus under the stochastic Cournot competition,  $E[CS^C(\epsilon)]$ , is also given by (28). Below, we will first show that the expected consumer surplus is always higher under the supply function competition.

**Proposition 3.** The expected consumer surplus under the supply function competition is always higher than under the stochastic Cournot competition.

**Proof.** Recall from (28) that  $E[CS^C(\epsilon)] = CS^C(\epsilon)$  since  $CS^C(\epsilon)$  is independent of  $\epsilon$ . Then, comparing (23) and (28), we observe that  $E[CS^{SF}(\epsilon)] > E[CS^C(\epsilon)]$  if and only if

$$\frac{n^2 \alpha^2}{2 m (n \alpha + m)^2} E[\epsilon^2] > \frac{n^2}{2 m (n + 1 + mc)^2} (E[\epsilon])^2$$
 (29)

or

$$\frac{1}{(n+\frac{m}{\alpha})^2} E[\epsilon^2] > \frac{1}{(n+1+mc)^2} (E[\epsilon])^2.$$
 (30)

First note that

$$E[\epsilon^2] = (E[\epsilon])^2 \left[ 1 + \left( \frac{\sigma(\epsilon)}{E[\epsilon]} \right)^2 \right], \tag{31}$$

where  $\sigma(\epsilon) \geq 0$  denotes the standard deviation of  $\epsilon$ . Thus, we have  $E[\epsilon^2] \geq (E[\epsilon])^2$ , implying that the inequality in (30) holds if

$$\frac{1}{(n+\frac{m}{\alpha})^2} > \frac{1}{(n+1+mc)^2}. (32)$$

On the other hand, the above inequality holds if

$$\alpha > \frac{m}{1 + mc}.\tag{33}$$

Using equation (11), the last inequality can be reduced to

$$\sqrt{\left(-m + \frac{n-2}{c}\right)^2 + \frac{4m(n-1)}{c}} > \frac{2m(n-1)}{(1+mc)} + m - \frac{(n-2)}{c},\tag{34}$$

implying that

$$\left(-m + \frac{n-2}{c}\right)^{2} + \frac{4m(n-1)}{c} - \left(\frac{2m(n-1)}{1+mc} + m - \frac{(n-2)}{c}\right)^{2} > 0 \tag{35}$$

or

$$(1+mc)^{2} (2mnc + (n-2)^{2}) - 2mc(1+mc)(mnc + 2-n) - (mnc + 2-n)^{2} > 0.$$
 (36)

After some simple algebra, one can show that the left hand side of the above inequality reduces to

$$4 m n c (n-2) + 4 m c.$$
 (37)

This expression is always positive, since  $n \geq 2$  and m, c > 0 by assumption. Thus, (36) holds, implying that  $E[CS^{SF}(\epsilon)] > E[CS^{C}(\epsilon)]$ .

On the side of producers, we will see that neither the supply competition nor the stochastic Cournot competition can always become ex-ante the superior mode of competition under demand uncertainty. To show this we will compare  $E[\pi^{SF}(\epsilon)]$  and  $E[\pi^{C}(\epsilon)]$ , respectively given by (21) and (27), using equation (31). Note that the ratio  $\sigma(\epsilon)/E[\epsilon]$  in equation (31) is known as the coefficient of variation, which is a unitless measure of relative variability. Moreover, it is independent of the realization of  $\epsilon$ . Let us denote this ratio by  $\mathcal{CV}$ . Then, equation (31) can be rewritten as

$$E[\epsilon^2] = (E[\epsilon])^2 \left[ 1 + \mathcal{CV}^2 \right]. \tag{38}$$

Let us denote by  $CV^*$  the value of the coefficient of variation at which the expected equilibrium profits obtained in the stochastic Cournot competition and in the supply function competition become equal. By equating equations (21) and (27), this value can be calculated as:

$$CV^* = \sqrt{\frac{(2+mc)(n\alpha+m)^2}{\alpha m(2-c\alpha)(n+1+mc)^2} - 1},$$
(39)

where  $\alpha$  satisfies (11). When the coefficient of variation in demand,  $\mathcal{CV}$ , is above (below) the threshold value  $\mathcal{CV}^*$ , the supply function competition leads to higher (lower) expected profits for each firm than the stochastic Cournot competition. Also, as it should be apparent from (39) along with (11), the threshold  $\mathcal{CV}^*$  depends on various attributes of the industry structure, involving the number of firms (n), the slope of the demand curve (m), and the marginal cost of producing a unit output (c). Below, we will explore how these attributes affect  $\mathcal{CV}^*$ . However, due to the complex analytical form of  $\mathcal{CV}^*(c,m,n)$ , characterized by equation (39) along with (11), we conduct our comparative statics analysis with the help of a computer. Specifically, we change the demand slope parameter m in the set  $\{1/81, 1/27, 1/9, 1/3, 1, 3\}$ , and for each value of m we plot the graph of  $\mathcal{CV}^*$  as a function of n and c, when n takes 15 integer values between 2 and 30 and c takes 15 real values between 0.01 and 30.00. These graphs are drawn in Figure 1, showing that the threshold value of the coefficient of variation,  $\mathcal{CV}^*$ , is always positive for all considered values of m, n, and c. This is not surprising since an observation with  $\mathcal{CV}^* = 0$  would be in contradiction with Klemperer and Meyer (1989), who showed that in the absence of any (demand) uncertainty (i.e., when  $\mathcal{CV} = 0$ ), the profits from the supply function competition must be always below the profits from the Cournot competition.

Figure 1 also illustrates that an increase in the slope of the demand curve, m, reduces the threshold value of the coefficient of variation,  $\mathcal{CV}^*$ , at all values of n and c in their domains. Similarly, the cost parameter c has a negative impact on  $\mathcal{CV}^*$  at all values of m and n in their domains. On the other hand, the number of firms n is found to have a positive effect on  $\mathcal{CV}^*$  for all values of m and c. As a matter of fact, this effect becomes larger when the cost parameter c is not very high.

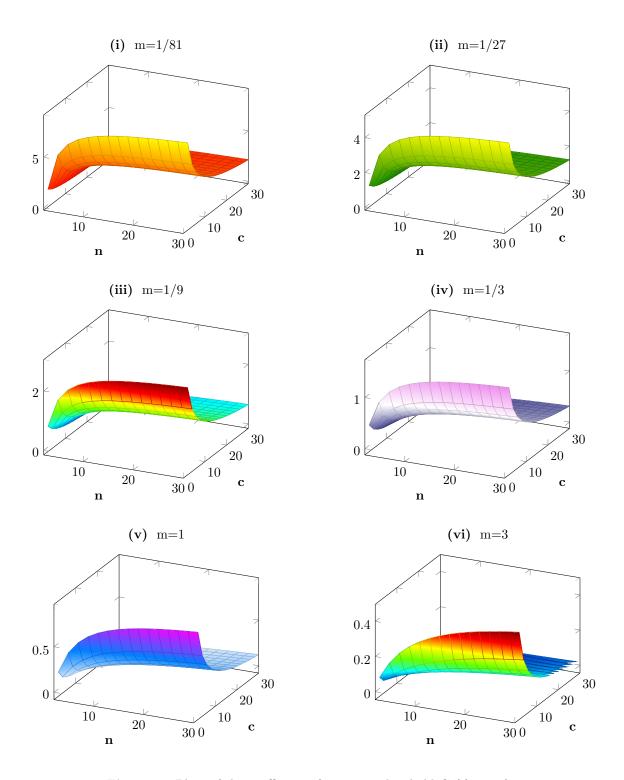


Figure 1. Plots of the coefficient of variation threshold  $\mathcal{CV}^*(c,m,n).$ 

### 4. Conclusion

In this paper, we have compared the welfares obtained under the supply function competition and the stochastic Cournot competition in the presence of demand uncertainty. We have showed that the expected consumer surplus is always higher under the supply function competition, whereas the expected profits of the oligopolists can be higher under the supply function competition only if the demand uncertainty is sufficiently high. In particular, we have found that the higher the slope of the industry demand curve or the higher the marginal cost of producing a unit output or the smaller the number of firms in the industry, the more likely that at any given level of demand uncertainty the supply function competition yields higher expected profits than the stochastic Cournot competition in equilibrium. We should note that both an increase in the slope of the demand curve and an increase in the marginal cost of a unit output result in a decrease in the potential (maximal) social surplus that could be attained in a perfectly competitive industry. Given this, our findings suggest that when the potential social surplus in the industry -some part of which the oligopolistic firms can expect to extract when they compete in quantities or in supply functions- is sufficiently small, the supply function competition with demand uncertainty becomes -from the viewpoints of firms-inferior to the stochastic Cournot competition only when the size of the demand uncertainty is also sufficiently low. On the other hand, when the number of firms in the industry is not sufficiently small, the supply function competition with demand uncertainty can become a superior mode of competition for the oligopolistic firms only at very high levels of uncertainties.

Our findings have real life implications especially in power (electricity) markets, where the supply function competition is believed to model the strategic interaction between power generators much more realistically than price (Bertrand) competition and quantity (Cournot) competition (see, for example, Green and Newbery 1992, and Rudkevich and Duckworth 1998). Our results imply that when power markets are faced with a sufficiently high uncertainty in demand, the supply function competition becomes a very desirable type of competition, being ex-ante Pareto superior to the quantity competition. Our results also imply that regulators of power markets should not intervene to impose the quantity competition except in situations they can estimate the demand uncertainty to be sufficiently low. Even, in such situations, regulators may prefer not to intervene if the social welfare function they use for regulatory purposes attaches a sufficiently high weight to the welfare of consumers, who are –unlike producers–'always' better off under the supply function competition independent of the size of demand uncertainty.

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