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Individual health perspective, income protection insurance coverage and human capital growth

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Abstract

Health insurance in poor developing countries has long been a hot topic. Income protection insurance (IPI) is a specific form of health insurance. In an overlapping generation economy without long time saving, we compared the attitudes of individuals with different health perspective towards IPI and analyzed the impact of IPI on human capital growth. We have the following findings: (i) from the healthiest to the most unhealthy, we can divide all individuals into 4 types: the healthiest, the second healthiest, the third healthiest, and the most unhealthy, each with different attitude towards IPI and IPI will influence their human capital accumulation differently; (ii) the healthiest individuals will choose no IPI, and IPI will not influence their human capital growth; (iii) the second healthiest individuals will choose partial IPI, and IPI will decrease their human capital accumulation; (iv) the third healthiest individuals will choose full IPI and IPI will decrease their human capital accumulation, and people will choose full life insurance when the net return from education is less than zero; (v) the individuals with the worst health will choose full IPI and IPI will increase their human capital accumulation; (vi) the impact of IPI on the human capital growth of the whole economy depends on health distribution, if more individuals have extreme health, especially extreme bad health, IPI tend to enhance human capital growth of the whole economy.

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1. Introduction

Insurance for the poor has been a hot topic for a long time. The 2019 Economics Nobel Prize winner, Banerjee and Duflo (2011), in their famous book *Poor economics: A radical rethinking of the way to fight global poverty*, use a whole chapter to discuss the risk and insurance of the poor. As they mentioned, the poor often face enormous amount of risk, which makes insurances essential for the poor. In this paper, we intend to deal with a special health insurance: income protection insurance (IPI). We want to find that if we bring IPI into some poor developing countries, how human capital accumulation and economic growth will change.

Human capital is the key element offsetting diminishing returns of physical capital and ensuring continued economic growth (Lucas, 1988). It attracts many theorists. To accumulate human capital, however, individuals usually need to invest their current resources, such as current income, for future returns (Arawatari and Ono, 2009). As a result, individuals must tradeoff between the current consumption and the long-run return.

Also, investing in human capital will not guarantee individuals to realize them fully. To what extent and on what probability individuals can realize his human capital heavily depend on his future health, or health perspective. For example, if individuals know that they will be healthy in the future, they are sure that the human capital they accumulated will be realized in the future and bring them higher income and expected utility, which will make them more willing to receive education or training in the cost of current income.

Income protection insurance (IPI) is an insurance policy, available principally in many countries such as Australia and the United Kingdom, paying benefits to policyholders who are incapacitated and hence unable to work due to illness or accident. Broadly speaking, it is a kind of life insurance. Previous research has found that well-informed, expected utility-maximizing risk-averse individuals might choose not to buy some kinds of insurance or choose lower coverage by trading-off the costs of benefits (Pauly, 1990). Also, this trading-off will also apply for IPI. In some poor developing countries, where people cannot save due to self-control problems and social circumstances, even if they have access money and accessibilities to a bank account (Banerjee et al., 2011), we can see individuals with different health perspectives have very different attitudes towards IPI.

As IPI benefits are directly relevant to individuals' gross earnings, it will certainly play an important role in human capital accumulation. Individuals will take the premium and benefits into consideration when they make their investment in education or training. However, there exist two opposite effects at the same time: "guarantee effect" and "crowding-out effect." On the one hand, IPI can guarantee some proportion of future income in spite of illness and accidents, which will make individuals more willing to invest in education and training, that is, the "guarantee effect." On the other hand, IPI needs to collect the premium, which comes from the individuals' current income. If too much money is used to pay the IPI premium, less money will be left for human capital accumulation, that is, the "crowding-out effect." Although the choice and impact of life insurance which is directly related to death and survival have attracted so much attention (Yaari, 1965; Pliska & Ye, 2007), as far as I know, very few researchers have investigated the mixed effects of IPI on human capital, which should make it a major contribution of this paper. The paper intends to provide some basic insight and policy implications for insurance in order to reduce poverty and enhance long-run economic growth in poor developing countries such as India and Kenya.

2. The model

The model is a discrete-time overlapping one. The initial period is period 0. All individuals live and work for two periods, which are called youth period and old period and denoted by 1 and 2, respectively. Individuals who spend their youth in period t are called generation t. In the beginning, there is the initial old generation, each of whom with human capital, $h_{2,0}$. Here, the first subscript represents the period of the generation, and the second refers to the current time period.

Proportional human capital inheritance is an assumption that has been applied by many researchers (Yakita, 2003; Lu & Yanagihara, 2013; Viaene and Zilcha's 2002). Here, we borrow the idea of proportional human capital inheritance from those researches. That is, children's generations' human capital in their youth is proportional to their parents' generations' human capital in old periods. Also, for simplicity, we push Viaene and Zilcha's (2002) assumption of average parents' human capital further by assuming that children's generations' human capital in the first period, $h_{1,t}$, is proportional to their parents' generations' average human capital in the old period, $\bar{h}_{2,t}$.

$$h_{1,t} = \chi \bar{h}_{2,t}, \chi > 0 \tag{1}$$

We assume individuals in the economy will consume in the current period and invest in education for human capital in the future. For example, individuals can use their current income to buy some books or take some skill training classes. We assume there are no other ways of storage. This may seem unrealistic at first sight, but as Banerjee et al. (2011) pointed out, "Unable to save" is the real situation in many poor developing economies. For example, the fruit vendors from Chennai borrow about 1,000 rupees ($45.75) each morning at the rate of 4.69 percent per day, and although they have good opportunity to save enough money to get rid of that debt, they never try to do so (Banerjee et al., 2011) [1].

We assume they get their income equal to their human capital. So, we can get the following budget constraint[2]:

$$c_{1,t} + e_{1,t} = h_{1,t} \tag{2}$$

Following Krebs (2003) and Grossmann (2008), the human capital in generation t in their old period, $h_{2,t+1}$, is proportionally influenced by the amount of education investment:

$$h_{2,t+1} = \theta e_{1,t} \tag{3}$$

$where\ \ \theta > 1$, measures the efficiency of education.

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[1] According to Banerjee et al.'s theory and research (2011), there are three main reasons: first, individuals who have been living in poverty for a long time have suffered too much stress and have to restraint continuously, which make it very difficult for them to control themselves when they have some excess money; second, when they really have some excess money, the circumstance around them will drive the money away from them quickly: for example, when their friends get married, or guests come, they will be forced to spend that excess money, and those things happen very often; third, as banks don't want to deal with small amount of money and the deposit fees are usually very high for the poor, individuals prefer to put the money at home, which make money even more vulnerable against the above self-control problem and social circumstance problem. We can see that the "unable to save" problem in those poor economies involves many things and cannot be solved in a short time. For those reasons, we believe it is appropriate to assume people cannot save in those economies.

[2] If individuals can save, the individuals who have negative net return will save their money in the bank and the conclusions will need some modification. But as we have mentioned, "no saving" is just the reality in those poor countries.

We assume individuals are risk-averse, and the utility function is has a constant relative risk aversion (CRRA), that is, isoelastic function for utility, here we assume the relative risk aversion is 1/2 for the simplicity of calculation, and any risk aversion that is between 0 and 1 will not change the main results[3].

$$\mathrm{u}\left(c_{1,t}, c_{2,t+1}\right) = 2\sqrt{c_{1,t}} + 2\frac{\sqrt{c_{2,t+1}}}{1+\beta} \tag{4}$$

where $\beta > 0$ is the discount rate.

In the old period, individuals may fall ill or not, and the illness can be light and heavy, depending on individuals. For example, if individuals have a probability of π to fall ill, and the illness can cause a fraction of ϕ of income loss, then his consumption in the old period will be:

$$c_{2,t+1} = \begin{cases} h_{2,t+1}, & if\ healthy \\ (1-\phi)h_{2,t+1}, & if\ unhealthy \end{cases} \tag{5}$$

And his expected utility function should be

$$E\left[u\left(c_{1t}, c_{2,t+1}\right)\right] = 2\sqrt{c_{1,t}} + \frac{2}{1+\beta}\left[(1-\pi)\sqrt{h_{2,t+1}}\right] + \frac{2}{1+\beta}\left[\pi\sqrt{(1-\phi)h_{2,t+1}}\right] \tag{7}$$

According to (3) and (4), we get the simplified budget constraint:

$$c_{1,t} + \frac{h_{2,t+1}}{\theta} = h_{1,t} \tag{8}$$

Maximizing (7) subject to (8), we get those individuals optimal education investment and human capital in the old period without IPI:

$$e_{1,t} = \frac{h_{1,t}}{\frac{(1+\beta)^2}{\left[(1-\pi)+\pi\sqrt{1-\phi}\right]^2}+\theta}; h_{2,t+1} = \frac{h_{1,t}}{\frac{(1+\beta)^2}{\left[(1-\pi)+\pi\sqrt{1-\phi}\right]^2\theta^2}+\frac{1}{\theta}} \tag{9}$$

Their human capital growth from his previous generation will be:

$$G^O = \frac{h_{2,t+1}}{\bar{h}_{2,t}} = \frac{\chi}{\frac{(1+\beta)^2}{\left[(1-\pi)+\pi\sqrt{1-\phi}\right]^2\theta^2}+\frac{1}{\theta}} \tag{10}$$

3. The impact of IPI on human capital growth

Nowadays, income protection insurance (IPI) is typically implemented as an industry-specific employment benefit. In Australia, for example, such insurance packages are often bundled together with other benefits in the Superannuation contributions, which is a compulsory system of placing a minimum percentage of ones' income into a fund to support their financial needs in retirement. When we want to bring IPI into the developing world, however, we may have to make some modifications. First, it is often the microfinance institutions (MFIs) that provide insurance (Banerjee et al., 2011), so it cannot be compulsory. Also, IPI in developed industrial countries is part of the social welfare system, which may be in favor of those with worse health at the cost of those healthy ones. But MFIs don't have that power and have to charge the premium actuarially fairly. So, if we bring IPI into those poor developing countries, although the name is the same, it will become a very different thing to the counterpart in the developed world. For those reasons, we assume individuals in our model can choose whether to enroll and the specific level of coverage.

One may ask that if the poor are unable to save money, why would they be able to invest in IPI, which

[3] The isoelastic function for utility has the form: $u(c) = \begin{cases} \frac{c^{1-\eta}-1}{1-\eta}, \eta \geq 0, \eta \neq 1 \\ ln(c), \eta = 1 \end{cases}$, in our case, we take $\eta = 1/2$.

also needs to sacrifice current consumption. Banerjee et al. (2011) have given the example of "building the house brick by brick," that is, the poor crystalize their money in house building whenever they have a little excess money, but the houses will remain unfinished for many years. At least, the example showed that if there is some way to crystalize excess money, the poor are willing to try. If they invest their money in IPI little by little, it will be a much safer and smarter way of financing excess money.

Now, if there is an IPI which allows individuals to choose coverage freely, then individuals will choose the optimal coverage δ to maximize their expected utility.

We assume individuals need to pay $x_{1,t}$ to the insurance company to cover some risks. Here, $\pi h_{2,t+1}\phi$ is his expected loss in the future, but they choose to cover a δ fraction of the full risk. So, we have

$$\delta\pi h_{2,t+1}\phi = x_{1,t} \tag{11}$$

Here, to guarantee the equation in (11), we assume the IPI companies are perfectly competitive, and we ignore their functioning costs to simplify our analysis because our main focus of this paper is the effects of IPI on human capital. For simplicity, here we assume both the IPI companies and individuals have perfect information and rational expectation, so the reverse selection and moral hazard are ruled out.

If they actually fall ill in the future, they can get compensation $X_{2,t+1}$ based on the chosen coverage:

$$X_{2,t+1} = \delta h_{2,t+1}\phi \tag{12}$$

His budget constraint is:

$$c_{1,t} + e_{1,t} + x_{1,t} = h_{1,t} \tag{13}$$

So, his second-period consumption when they are unhealthy will be:

$$c_{2,t+1}^{u} = (1-\phi)h_{2,t+1} + X_{2,t+1} = (1-\phi+\delta\phi)h_{2,t+1} \tag{14}$$

His expected utility function will be

$$E\left[u\left(c_{1,t}, h_{2,t+1}, \delta\right), \pi\right] = 2\sqrt{c_{1,t}} + \frac{2}{1+\beta}\left[(1-\pi)\sqrt{h_{2,t+1}}\right] + \frac{2}{1+\beta}\left[\pi\sqrt{(1-\phi+\delta\phi)h_{2,t+1}}\right] \tag{15}$$

According to (4) (11) and (13), we can get the simplified budget constraint:

$$c_{1,t} + \frac{h_{2,t+1}}{\theta} + \delta\pi\phi h_{2,t+1} = h_{1,t} \tag{16}$$

Besides current consumption, Individuals also need to tradeoff between human capital accumulation and IPI. For example, individuals can choose to use their current income to buy more books and taking more training classes, or invest more money in IPI.

Proposition 1: Individuals choose not to buy IPI when $\mathrm{T} = (1-\pi)\sqrt{1-\phi} + \pi(1-\phi) - \frac{1}{\theta} > 0$.

The proof is shown in Appendix A.

We can also regard $\mathrm{T} = 0$ as the zero-insurance boundary (ZIB). Any combination of (ϕ, π) below ZIB means the individual will not buy IPI.

ZIB also has another form:

$$\pi = \frac{\frac{1}{\theta} - \sqrt{1-\phi}}{1-\phi-\sqrt{1-\phi}} \tag{17}$$

Next, we investigate whether we can get the corner solution at $\delta^{EN} = 1$. According to Appendix A, when there are interior solutions:

$$\delta - 1 = \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} - 1 = \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2}{(1-\pi)^2\phi} \tag{18}$$

To find whether $\delta - 1 < 0$, we just need to find the value of the following equation:

$$\frac{1}{\theta} - \pi(1-\phi) - (1-\pi) = \frac{1}{\theta} - 1 + \pi\phi = \frac{1-\theta(1-\pi\phi)}{\theta} \tag{19}$$

Notice that $\theta(1-\pi\phi) - 1$ is the net expected return from investing in education; for some people, it is positive. But we can't deny that for some people else, it can be negative.

Proposition 2: If individuals, can get a positive expected net return from investing in education, that is, $\theta(1-\pi\phi) - 1 > 0$, then they will not cover all his risks when buying IPI. If they can't get a positive expected net return from education, that is $\theta(1-\pi\phi) - 1 \leq 0$, they will choose to cover all his risks. The full covered IPI boundary (FIB) is $\theta(1-\pi\phi) - 1 = 0$, or

$$\pi = \frac{1-\frac{1}{\theta}}{\phi} \tag{20}$$

The proof is shown in the above analysis.

Figure 1 shows how the ZIB and FIB divide the area $\pi \in (0,1), \phi \in (0,1)$ when $\theta = 1.5, 2, 3$ *and* 4. The curve on the left is ZIB, and the curve on the right is FIB.

If the equation holds when $\delta^{EN} = \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} \in (0,1)$, then there is the interior solution of δ. Now we want to find out whether IPI will increase his investment in education and human capital or decrease them.

Proposition 3: If individuals can choose their IPI coverage freely when they choose to buy IPI to cover part of the risks, their individual human capital growth rates in the second period will be lower than the case without IPI.

Proof: See Appendix B.

When individuals choose full coverage, we have the expected utility function and simplified budget constraint:

$$E(c_{1,t}, h_{2,t+1}) = 2\sqrt{c_{1,t}} + \frac{2}{1+\beta}\sqrt{h_{2,t+1}} \tag{21}$$

$$c_{1,t} + \frac{h_{2,t+1}}{\theta} + \pi\phi h_{2,t+1} = h_{1,t} \tag{22}$$

The Lagrangian equation will be:

$$L = 2\sqrt{c_{1,t}} + \frac{2}{1+\beta}\sqrt{h_{2,t+1}} + \lambda[c_{1,t} + \frac{h_{2,t+1}}{\theta} + \pi\phi h_{2,t+1} - h_{1,t}]$$

Finally, we get the human capital in the second period and the personal growth rate

$$h_{2,t+1} = \frac{h_{1,t}}{(1+\beta)^2\left[\frac{1}{\theta}+\pi\phi\right]^2 + \frac{1}{\theta} + \pi\phi}; G^F = \frac{h_{2,t+1}}{h_{1,t}} = \frac{\chi}{(1+\beta)^2\left[\frac{1}{\theta}+\pi\phi\right]^2 + \frac{1}{\theta} + \pi\phi} \tag{23}$$

To compare G^F with G^O, we define:

$$S = \frac{\chi}{G^F} - \frac{\chi}{G^O} = (1+\beta)^2\left(\frac{1}{\theta} + \pi\phi\right)^2 - \frac{(1+\beta)^2}{\left((1-\pi)+\pi\sqrt{1-\phi}\right)^2\theta^2} + \pi\phi \tag{24}$$

The sign of S is ambiguous. For example, when $\pi = \phi = 0.8$, $\theta = 2$, $\beta = 0.2$, $\frac{1-\frac{1}{\theta}}{\phi} = \frac{1-\frac{1}{2}}{0.8} = 0.625 <$

$\pi = 0.8$, so this individual is in the area above FIB and will choose full coverage. We get $S = 1.3524 > 0$, $G^F < G^O$. On the contrary, when $\pi \to 1^-, \phi \to 1^-$, $-\frac{(1+\beta)^2}{[(1-\pi)+\pi\sqrt{1-\phi}]^2\theta^2} \to -\infty, S < 0$, $G^F > G^O$.

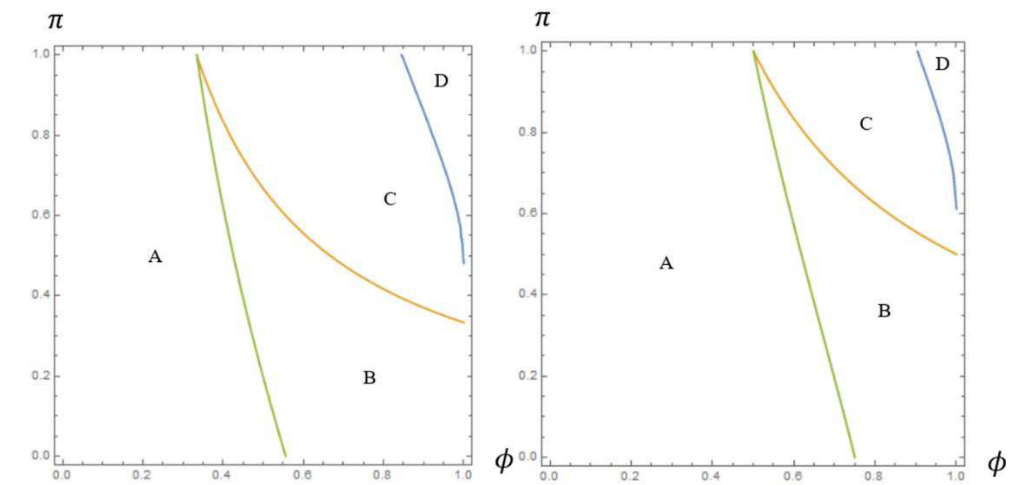
By the latter extreme example, we know that when individuals have an extremely bad health perspective, full IPI will surely increase his human capital accumulation.

Proposition 4: For those individuals with extreme bad health perspectives, in other words, when $\pi \to 1^-, \phi \to 1^-$, they will always choose full IPI that covers all the risks, and IPI can increase their human capital accumulation in their second life periods.

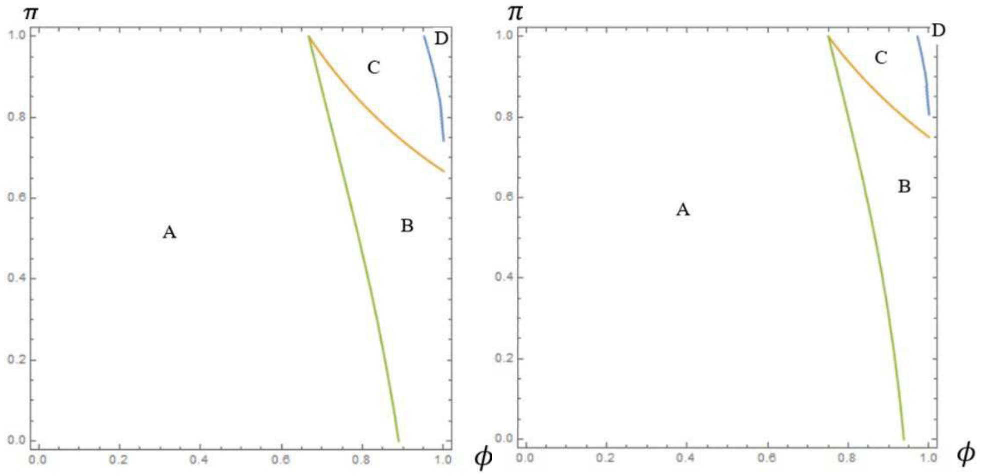
The proof is shown in the above analysis.

Obviously, the boundary where the IPI begins to enhance human capital accumulation, or human capital enhancing boundary (HEB) is influenced not only by π, ϕ, θ but also β. And the HEB is surely located beyond FIB. Figure 1 shows the relative location of ZIB, FIB, and HEB when $\beta = 0.2$ and $\theta = 1.5, 2, 3, 4$. The three curves from left to right are ZIB, FIB, and HEB. The three curves divide the whole area into four small areas: A, B, C, and D.

We notice that as θ increases, the ZIB, FIB and, HEB all move rightward, when θ is extremely high, for example, when $\theta = 4$, all the three lines will crowd in a small area in the right, and most people will not want to invest in IPI. That means that as people can get higher returns from education, they will not want to sacrifice current resources in insurance, rather they would like to invest those resources in human capital accumulation.



(a) $\theta = 1.5$ (b) $\theta = 2$



(c) $\theta = 3$ (d) $\theta = 4$

Figure 1 The relative positions of ZIB, FIB, and HEB

If we know the distribution density function of individual health perspectives $f(\phi, \pi)$, we can get the total growth rate with and without IPI:

$$\mathrm{G}^{IT} = \iint_A G^O f(\phi,\pi) d\phi d\pi + \iint_B G^{EN} f(\phi,\pi) d\phi d\pi + \iint_C G^F f(\phi,\pi) d\phi d\pi + \iint_D G^F f(\phi,\pi) d\phi d\pi$$

$$G^{OT} = \iint_{\substack{\pi\in(0,1)\\ \phi\in(0,1)}} G^O f(\phi,\pi) d\phi d\pi \qquad (25)$$

The total impact of IPI can be seen by comparing G^{IT} and G^{OT}.

For the whole economy, whether IPI is good or bad for economic growth depends on the distribution of the whole population on life perspectives. For example, if most individuals in an economy located in area A, B, C or D, the impact of IPI will be very clear, which is shown in Figure 2 and Table 1

Table 1 The preferred coverages and the impact of insurance on human capital in different areas

	A	B	C	D
Coverage	None	Partial	Full	Full
Impact	None	Negative	Negative	Positive

If the individuals are scattered on all four areas, things will become a bit complex. For example, if more individuals' perspectives are polarized: either very good or very bad, then individuals in area A or D, especially D will have a higher weight in the whole economy, and the IPI will be more likely to enhance human capital growth, or at least not deter it, $G^{IT} > G^{OT}$. On the contrary, if more individuals have normal or moderate health perspectives: neither very good nor very bad, then individuals in area B and C will have a higher weight, and the IPI will be more likely to decrease human capital growth, $G^{IT} < G^{OT}$.

4. Conclusion

Through our analysis, we got the following 6 conclusions:

(i) From the healthiest to the most unhealthy, we can divide all individuals into 4 types: the healthiest, the second healthiest, the third healthiest, and the most unhealthy, each with different attitude towards

IPI and IPI will influence their human capital accumulation differently; (ii) the healthiest individuals will choose no IPI, and IPI will not influence their human capital growth; (iii) the second healthiest individuals will choose partial IPI, and IPI will decrease their human capital accumulation; (iv) the third healthiest individuals will choose full IPI and IPI will decrease their human capital accumulation, and people will choose full life insurance when the net return from education is less than zero; (v) the individuals with the worst health will choose full IPI and IPI will increase their human capital accumulation; (vi) the impact of IPI on the human capital growth of the whole economy depends on health distribution, if more individuals have extreme health, especially extreme bad health, IPI tend to enhance human capital growth of the whole economy.

As for the policy implication, this paper shows that if we bring IPI into those poor developing countries to reduce poverty, we need to consider the health distribution in that country. For those countries with relatively bad health, such as in Western Africa (where the life expectancy is very low, only 56 for men and 58 for women, according to statista), IPI will tend to improve economic growth and reduce poverty. For those countries with moderate health distribution, such as some country in Northern Africa (where the life expectancy is moderate, 71 for men and 74 for women, according to statista), although IPI will surely improve current generations' welfare, it may deter the long-run economic growth and increase the long-run poverty due to the "crowding-out" effect.

Appendix A:

Maximizing (15) subject to (16), we can get the following Lagrangian function:

$$\mathcal{L} = 2\sqrt{c_{1,t}} + \frac{2}{1+\beta}\left[(1-\pi)\sqrt{h_{2,t+1}}\right] + \frac{2}{1+\beta}\left[\pi\sqrt{\delta\phi h_{2,t+1} + (1-\phi)h_{2,t+1}}\right]$$

$$+\lambda\left[c_{1,t} + \frac{h_{2,t+1}}{\theta} + \delta\pi\phi h_{2,t+1} - h_{1,t}\right] \quad \text{(A1)}$$

When there are interior solutions, the first-order conditions should be:

$$\begin{cases} \frac{\partial\mathcal{L}}{\partial c_{1,t}} = \frac{1}{\sqrt{c_{1,t}}} + \lambda = 0 \\ \frac{\partial\mathcal{L}}{\partial h_{2,t+1}} = \frac{1-\pi}{1+\beta}\frac{1}{\sqrt{h_{2,t+1}}} + \frac{1}{1+\beta}\left[\pi\frac{\sqrt{\delta\phi+(1-\phi)}}{\sqrt{h_{2,t+1}}}\right] + \lambda\left[\frac{1}{\theta} + \delta\pi\phi\right] = 0 \\ \frac{\partial\mathcal{L}}{\partial\delta} = \frac{1}{1+\beta}\left[\pi\sqrt{h_{2,t+1}}\right]\frac{\phi}{\sqrt{\delta\phi+(1-\phi)}} + \lambda\pi\phi h_{2,t+1} = 0 \\ c_{1,t} + \frac{h_{2,t+1}}{\theta} + \delta\pi\phi h_{2,t+1} - h_{1,t} = 0 \end{cases} \quad \text{(A2)}$$

From the second and third equation of (A2), we further have

$$\frac{1}{\left[\frac{1}{\theta}+\delta\pi\phi\right]}\left[(1-\pi) + \pi\sqrt{\delta\phi + (1-\phi)}\right] = \frac{1}{\sqrt{\delta\phi+(1-\phi)}} \quad \text{(A3)}$$

After some transformation, we get

$$(1-\pi)\sqrt{\delta\phi + (1-\phi)} = \frac{1}{\theta} - \pi(1-\phi) \quad \text{(A4)}$$

However, if $\frac{1}{\theta} - \pi(1-\phi) < 0$, $(1-\pi)\sqrt{\delta\phi + (1-\phi)} > \frac{1}{\theta} - \pi(1-\phi)$, there is only a corner solution $\delta^{EN} = 0$.

When $\frac{1}{\theta} - \pi(1-\phi) \geq 0$, we can get the δ value that can satisfy the equation (21):

$$\delta = \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} \tag{A5}$$

However, we must notice that $\delta^{EN} \in (0,1)$, so when

$$\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi) \leq 0 \tag{A6}$$

Or we can say, when

$$0 \leq \frac{1}{\theta} - \pi(1-\phi) \leq (1-\pi)\sqrt{1-\phi} \tag{A7}$$

$\frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} < 0$, we can only have the corner solution $\delta^{EN} = 0$.

We can sum up the above two cases and get Proposition 1.

Appendix B

The proof of proposition 3:

We have got the endogenous coverage in the above section:

$$\delta^{EN} = \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} \in (0,1) \tag{B1}$$

Then the human capital accumulation is:

$$h_{2,t+1} = \frac{h_{1,t}}{\frac{(1+\beta)^2\left[\frac{1}{\theta} + \delta\pi\phi\right]^2}{\left[(1-\pi) + \pi\sqrt{\delta\phi + (1-\phi)}\right]^2} + \frac{1}{\theta} + \delta\pi\phi} \tag{B2}$$

Their personal human capital growth rate is:

$$G^{ENP} = \frac{h_{2,t+1}}{h_{1,t}} = \frac{1}{\frac{(1+\beta)^2\left[\frac{1}{\theta} + \delta\pi\phi\right]^2}{\left[(1-\pi) + \pi\sqrt{\delta\phi + (1-\phi)}\right]^2} + \frac{1}{\theta} + \delta\pi\phi} \tag{B3}$$

where $\delta = \delta^{EN} = \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} \in (0,1)$

The human capital growth between generations is

$$G^{EN} = \frac{h_{2,t+1}}{\bar{h}_{2,t}} = \frac{\chi}{\frac{(1+\beta)^2\left[\frac{1}{\theta} + \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2}\pi\right]^2}{\left[(1-\pi) + \pi\frac{\frac{1}{\theta} - \pi(1-\phi)}{(1-\pi)}\right]^2} + \frac{1}{\theta} + \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2}\pi} \tag{B4}$$

It is convenient to compare the inverses:

$$\frac{\chi}{G^{EN}} - \frac{\chi}{G^{O}} = \frac{(1+\beta)^2\left[\frac{1}{\theta} + \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2}\pi\right]^2}{\left[(1-\pi) + \pi\frac{\frac{1}{\theta} - \pi(1-\phi)}{(1-\pi)}\right]^2} + \frac{\left[\frac{1}{\theta} - \pi(1-\phi)\right]^2 - (1-\pi)^2(1-\phi)}{(1-\pi)^2}\pi - \frac{(1+\beta)^2}{\left[(1-\pi) + \pi\sqrt{1-\phi}\right]^2\theta^2}$$

(B5)

We define

$$\frac{\frac{1}{\theta} - \pi(1-\phi)}{(1-\pi)} = x; \sqrt{1-\phi} = y \tag{B6}$$

Then

$$\frac{\chi}{G^{EN}} - \frac{\chi}{G^{O}} = \frac{(1+\beta)^2}{\theta^2}\left(\frac{1+(x^2-y^2)\pi\theta}{(1-\pi)+\pi x} - \frac{1}{(1-\pi)+\pi y}\right) + \frac{\left[\frac{1}{\theta}-\pi(1-\phi)\right]^2-(1-\pi)^2(1-\phi)}{(1-\pi)^2}\pi \qquad \text{(B7)}$$

We define

$$T = \frac{1+(x^2-y^2)\pi\theta}{(1-\pi)+\pi x} - \frac{1}{(1-\pi)+\pi y} \qquad \text{(B8)}$$

Through transformation, we get

$$T = \frac{(x-y)\pi((x+y)\theta((1-\pi)+\pi y)-1)}{((1-\pi)+\pi x)((1-\pi)+\pi y)} \qquad \text{(B9)}$$

When $\delta = \delta^{EN} = \frac{\left[\frac{1}{\theta}-\pi(1-\phi)\right]^2-(1-\pi)^2(1-\phi)}{(1-\pi)^2\phi} > 0$, $\left[\frac{1}{\theta}-\pi(1-\phi)\right]^2-(1-\pi)^2(1-\phi) > 0$, then

$$x - y = \frac{\frac{1}{\theta}-\pi(1-\phi)}{(1-\pi)} - \sqrt{1-\phi} > 0$$

Also, $\big((1-\pi)+\pi x\big)\big((1-\pi)+\pi y\big) > 0$. So, the sign of T is the same with

$Q = (x+y)\theta((1-\pi)+\pi y) - 1$

or

$$Q = \left(\frac{\frac{1}{\theta}-\pi(1-\phi)}{(1-\pi)} + \sqrt{1-\phi}\right)\theta\left[(1-\pi)+\pi\sqrt{1-\phi}\right] - 1$$

We define $t = \sqrt{1-\phi}$

$$Q = \left(\frac{1}{(1-\pi)} - \frac{\pi t^2}{1-\pi}\theta + \theta t\right)\big((1-\pi)+\pi t\big) - 1$$

Through transformation, we know when $Q = 0$

$$Q = t\frac{\pi+\theta-2\pi\theta+\pi^2(1-t^2)\theta}{1-\pi} = 0$$

We know that $\phi = 1-t^2$, and we know the minimum ϕ (when $\pi = 1$) that individual choose to buy IPI is: $\underline{\phi} = 1-\frac{1}{\theta}$, so we know

$$\pi+\theta-2\pi\theta+\pi^2\theta\phi \geq \pi+\theta-2\pi\theta+\pi^2\theta\left(1-\frac{1}{\theta}\right) = \pi-\pi^2+\theta(1-\pi)^2$$

When $\pi \in (0,1)$, $\pi-\pi^2+\theta(1-\pi)^2 > 0$, $\pi+\theta-2\pi\theta+\pi^2\theta\phi > 0$

When $t = \sqrt{1-\phi} \in (0,1)$

$Q = t\frac{\pi+\theta-2\pi\theta+\pi^2(1-t^2)\theta}{1-\pi} > 0,\ T > 0$

Also, we know $\frac{\left[\frac{1}{\theta}-\pi(1-\phi)\right]^2-(1-\pi)^2(1-\phi)}{(1-\pi)^2}\pi > 0$

As a result, $\frac{1}{G^{EN}} - \frac{1}{G^{O}} > 0,\ G^{EN} < G^{O}$ ■

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