Economics Bulletin

Volume 40, Issue 4

# Efficiency in a search and matching model with endogenous labor participation and different skill groups

Makoto Masui
*Soka University*

## Abstract

This study addresses endogenous labor force participation based on a search and matching model with workers' productivity dispersion within different skill groups. I show that equilibrium vacancy creation can be both excessive and insufficient from the viewpoint of economic efficiency, depending on the relative share of workers in a high-skill group.

I thank the referee for carefully reading my manuscript. This work is supported by JSPS KAKENHI Grant Number 17K03779. All errors are mine.
**Citation:** Makoto Masui, (2020) "Efficiency in a search and matching model with endogenous labor participation and different skill groups", *Economics Bulletin*, Volume 40, Issue 4, pages 2648-2653
**Contact:** Makoto Masui - mmasui@soka.ac.jp.
**Submitted:** July 06, 2020. **Published:** October 12, 2020.

# 1. Introduction

Albrecht et al. (2010) (hereafter, ANV) construct a search and matching model in which workers are ex ante heterogeneous in their market productivity and labor force participation is endogenous. They show that even under the rule proposed by Hosios (1990),[1)] constrained efficiency in vacancy creation is no longer achieved and too many vacancies are created in equilibrium; this is because the marginal participants do not take into account the negative effect of their participation on the average match productivity.[2)]

This study examines whether ANV's result will hold in an environment where workers' productivity is dispersed within different skill groups. ANV and its related studies do not consider such heterogeneous skill groups.[3)] However, within-group heterogeneity as well as between-group heterogeneity is an important factor in explaining the recent trends of wage inequality in the U.S. (Autor et al. (2008)) and a rise in overeducation as a consequence of a skill-biased technological change (Cuadras-Morató and Mateos-Planas (2013)). I introduce such productivity dispersion to obtain new insights into the argument of efficiency in vacancy creation. A numerical exercise demonstrates that vacancy creation in equilibrium can be both excessive and inefficiently low, depending on the share of workers in a high-skill group among the total workers and that too few vacancies are created if this share of workers and the value of their non-participation are low. The latter result indicates that for a given share of workers in each skill group, the state of vacancy creation could switch from excessive to insufficient and vice versa.

# 2. Model

This study follows the framework of ANV. Therefore, a one-period search and matching model with ex ante heterogeneous potential workers is considered. The measure of these workers is one. I assume that potential workers belong to either a high-skill or low-skill group. A different educational attainment among workers is interpreted as a difference in skill. The fraction of workers in the high-skill group is denoted by $\mu \in [0, 1]$, which is exogenously given. Workers' market productivity $y$ follows a continuous distribution function $F_i(\cdot)$ for $i = H, L$ (the subscript could be high or low).[4)] Note that $F_H(\cdot)$ has a density function $f_H(\cdot)$ and support $[\underline{y}_H, 1]$ with $\underline{y}_H \in (0,\ 1)$ and that $F_L(y)$ has a density function $f_L(\cdot)$ and support $[0, \bar{y}_L]$ with $\bar{y}_L \in (0,\ 1)$. Workers know their productivity and the group to which they belong, and firms can observe these features when they meet a job

1) Job creation in a decentralized equilibrium is constrained efficient when the worker share of the net surplus is equal to the elasticity of the matching function with respect to unemployment, which is called the Hosios rule.

2) Based on ANV's framework, Julien and Mangin (2017) specify a modified version of the Hosios rule to achieve an efficient level of job vacancies.

3) Other related works such as Lehmann et al. (2011) and Baalen and Müller (2014), which deal with endogenous participation under the general equilibrium framework, also assume that worker's skill is heterogeneous, but it follows a single distribution. These works consider the heterogeneous value of non-market activities (or leisure) among workers with different skills, as well.

4) Although endogenous participation is not considered, Dolado et al. (2007) assume that firms face a different distribution of idiosyncratic productivity shock according to worker's skill, which takes either high or low, in a search and matching model with endogenous job destruction.

seeker. The value of non-participation (or leisure) differs among skill groups and is denoted by $z_i > 0$.[5)]

Workers choose to either search for a job or engage in home production (or enjoy leisure). If participation is chosen, they find a job with the probability $p(\theta)$ (the corresponding probability for recruiting firms is $p(\theta)/\theta$) where $p(\cdot)$ is twice differentiable, increasing, and concave. Market tightness $\theta$ is defined as the ratio of vacant firms to participating workers. I assume that workers in each skill group search for a job in the same labor market (i.e., random matching is considered).

Following standard approach, wages are determined by generalized Nash bargaining with workers' bargaining power $\beta$, which is common to all workers in a skill group. An individual worker and firm split the surplus of a match based on their bargaining power. Employed workers with productivity $y$ receive $\beta\, y$ as compensation. The firm receives $(1-\beta)\, y$ as profits. The value of searching for a job but failing is $b$, which is assumed to be zero for simplicity.

Potential workers enter the labor market if participation provides them with higher payoffs than non-participation: $\beta\, p(\theta)\, y \geq z_i$. The cutoff productivity for participation in each skill group, denoted by $y_i^*$, is

$$y_i^* = \frac{z_i}{\beta\, p(\theta)} \equiv \kappa_i(\theta). \tag{1}$$

Note that $\kappa_i(\theta)$ is decreasing in $\theta$ because $p(\theta)$ is increasing in $\theta$. Since workers with $y \geq y_i^*$ participate, $1-F_i(y_i^*)$ is the participation rate of workers in skill group $i$. The total number of workers who seek employment is then represented by $\mu\,(1-F_H(\tilde{y}_H^*))+(1-\mu)(1-F_L(\tilde{y}_L^*))$, where $\tilde{y}_H^* = \max\{\underline{y}_H,\, y_H^*\}$ and $\tilde{y}_L^* = \min\{\bar{y}_L,\, y_L^*\}$. Labor market tightness is represented by

$$\theta = \frac{v}{\mu\,(1-F_H(\tilde{y}_H^*)) + (1-\mu)(1-F_L(\tilde{y}_L^*))}, \tag{2}$$

where $v$ is the measure of vacant jobs. By differentiating implicitly, it is shown that $\theta$ increases with $v$.

The value of a vacant job should be zero in equilibrium. This free entry condition yields a condition for the value of $\theta$ given $\tilde{y}_H^*$ and $\tilde{y}_L^*$:

$$c = \frac{(1-\beta)\, p(\theta)}{\theta}\left[\mu \int_{\tilde{y}_H^*}^{1} \frac{y\, f_H(y)}{1-F_H(\tilde{y}_H^*)}\, dy + (1-\mu)\int_{\tilde{y}_L^*}^{\bar{y}_L} \frac{y\, f_L(y)}{1-F_L(\tilde{y}_L^*)}\, dy\right], \tag{3}$$

where $c$ is the cost of recruiting and the term in the square bracket is the expected productivity obtained from forming a match. The equilibrium in this model is characterized by a condition for the workers' optimal choice of participation and the free entry condition. Thus, (1) and (3) jointly determine the equilibrium value of $y_i^*$ and $\theta$.

## 3. Efficiency

As in ANV, this study examines the economic efficiency of vacancy creation. I focus on a case where the number of participants and nonparticipants in each skill group are both

5) Even if $z_H$ is equal to $z_L$, the main result of this study will not change.

strictly positive: $0 < F_i(y_i^*) < 1$ for $i = H, L$. Then $\tilde{y}_H^* = y_H^*$ and $\tilde{y}_L^* = y_L^*$ are obtained. For convenience, let us introduce the following notations:

$$\begin{aligned}
\Gamma(\theta) &\equiv \mu \int_{y_H^*}^{1} y\, f_H(y) dy + (1-\mu) \int_{y_L^*}^{\bar{y}_L} y\, f_L(y) dy, \\
R(\theta) &\equiv \mu\,(1 - F_H(y_H^*)) + (1-\mu)(1 - F_L(y_L^*)).
\end{aligned}$$

A social planner who is constrained by workers' participation decision chooses $v$ to maximize the following social surplus: [6)]

$$\begin{aligned}
\max_{v}\ \Omega &= [\mu\, F_H(y_H^*)\, z_H + (1-\mu) F_L(y_L^*)\, z_L] + p(\theta) \Bigg[ \mu\,(1 - F_H(y_H^*)) \int_{y_H^*}^{1} \frac{y\, f_H(y)}{1 - F_H(y_H^*)} dy \\
&\quad + (1-\mu)(1 - F_L(y_L^*)) \int_{y_L^*}^{\bar{y}_L} \frac{y\, f_L(y)}{1 - F_L(y_L^*)} dy \Bigg] - c\, v, \\
&= [\mu\, F_H(y_H^*)\, z_H + (1-\mu) F_L(y_L^*)\, z_L] + p(\theta)\, \Gamma(\theta) - c\, v. \qquad (4)
\end{aligned}$$

The first term on the RHS of (4) is the value for nonparticipation accruing to workers, the second term is the expected value of market output, and the third is the cost of creating vacancies. The derivative of $\Omega$ with respect to $v$ is

$$\begin{aligned}
\frac{d\,\Omega}{d\,v} &= \Big[ \mu\,(z_H - p(\theta)\, y_H^*)\, f_H(y_H^*)\, \kappa_H'(\theta) + (1-\mu)(z_L - p(\theta)\, y_L^*)\, f_L(y_L^*)\, \kappa_L'(\theta) \Big] \frac{d\,\theta}{d\,v} \\
&\quad + p'(\theta) \left[ \frac{\partial\,\theta}{\partial\,v} + \left( \frac{\partial\,\theta}{\partial\, y_H^*}\, \kappa_H'(\theta) + \frac{\partial\,\theta}{\partial\, y_L^*}\, \kappa_L'(\theta) \right) \frac{d\,\theta}{d\,v} \right] \Gamma(\theta) - c. \qquad (5)
\end{aligned}$$

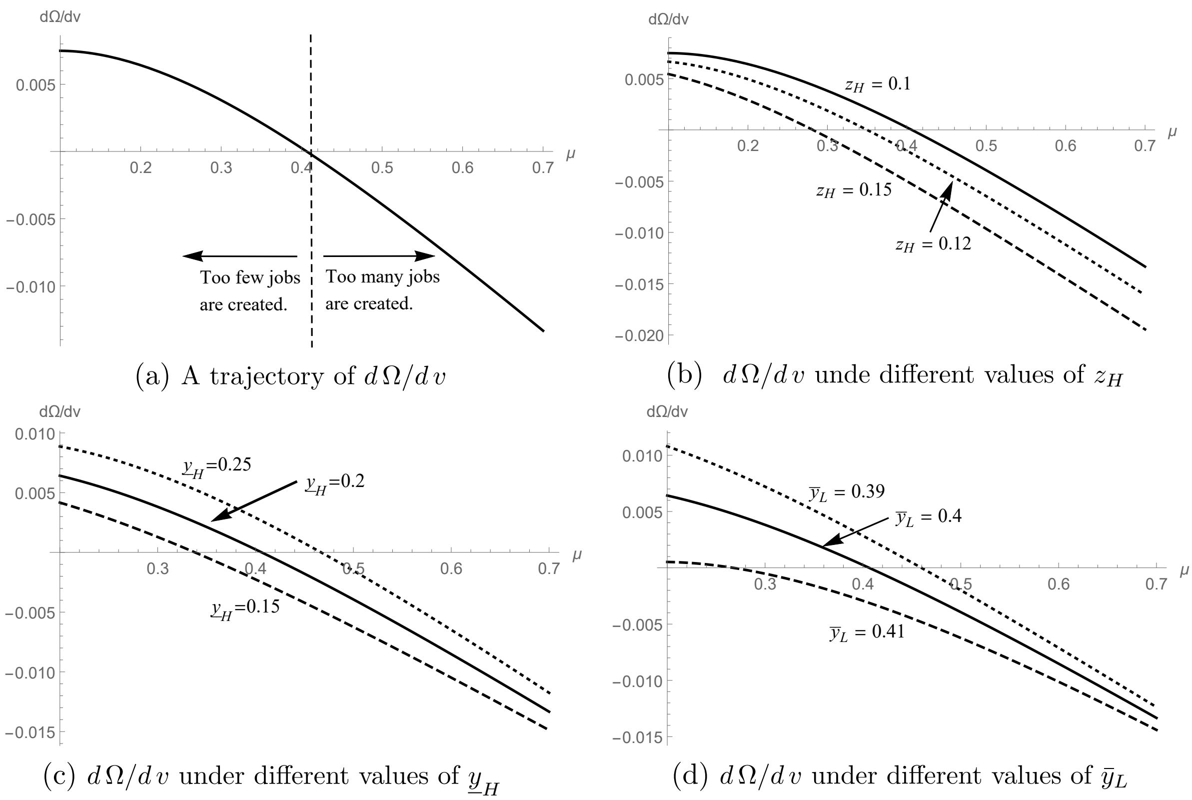
Using the results of $\partial\,\theta / \partial\, v$ and $\partial\,\theta/\partial\, y_i^*$ from (2), the Hosios rule $1 - \beta = \theta\, p'(\theta) \,/\, p(\theta)$ and condition (3), (5) becomes

$$\begin{aligned}
&\mu\, f_H(y_H^*)\, \kappa_H'(\theta) \left[ z_H - p(\theta)\, y_H^* + \frac{\theta\, p'(\theta)\, \Gamma(\theta)}{R(\theta)} \right] \frac{d\,\theta}{d\,v} \\
&+ (1-\mu)\, f_L(y_L^*)\, \kappa_L'(\theta) \left[ z_L - p(\theta)\, y_L^* + \frac{\theta\, p'(\theta)\, \Gamma(\theta)}{R(\theta)} \right] \frac{d\,\theta}{d\,v} \\
&+ \frac{\mu\,(1-\mu)(F_L(y_L^*) - F_H(y_H^*))\, p'(\theta)}{R(\theta)} \left[ \int_{y_H^*}^{1} \frac{y\, f_H(y)}{1 - F_H(y_H^*)}\, dy - \int_{y_L^*}^{\bar{y}_L} \frac{y\, f_L(y)}{1 - F_L(y_L^*)}\, dy \right]. \qquad (6)
\end{aligned}$$

Note that this is the expression of $d\,\Omega / d\,v$ evaluated at the equilibrium level of vacancies when the Hosios rule is satisfied.

An increase in market tightness resulting from creation of one more vacancy influences the value of social surplus through two channels. First, an increase in $\theta$ decreases the cutoff productivity in each skill group, leading to more labor force participation. Second, a higher $\theta$ increases the probability of finding a job, contributing to an increase in the

6) The planner takes into account that workers choose to participate in line with (1). Alternatively, Masters (2015) considers an environment in which the planner can choose both the cutoff productivity and level of vacancies.




(a) A trajectory of $d\,\Omega/d\,v$

(b) $d\,\Omega/d\,v$ unde different values of $z_H$

(c) $d\,\Omega/d\,v$ under different values of $\underline{y}_H$

(d) $d\,\Omega/d\,v$ under different values of $\bar{y}_L$

Figure 1. Efficiency of vacancy creation

expected value of market productivity. If there exists only one skill group as in ANV (i.e., either $\mu = 0$ or $\mu = 1$), the third line in (6) disappears and $d\,\Omega/d\,v$ becomes negative. However, in the present model accompanied with productivity dispersion within the skill groups, $d\,\Omega/d\,v$ can be positive for a lower value of $\mu$ because a smaller share of high-skill workers makes employers reluctant to create vacancies. In this case, a socially insufficient number of vacancies will be created in equilibrium.

## 4. A numerical example

To confirm the existence of $\mu \in (0,\,1)$ such that $d\,\Omega/d\,v$ can be positive, I consider the following example. Suppose the distribution $F_i(\cdot)$ is uniform ($F_H(y) = (y - \underline{y}_H)/(1 - \underline{y}_H)$ and $F_L(y) = y/\bar{y}_L$), $p(\theta) = \theta^{1/2}$, $\beta = 0.5$, $z_H = 0.1$, $z_L = 0.12$, $\underline{y}_H = 0.2$, $\bar{y}_L = 0.4$ and $c = 0.35$. Figure 1 illustrates how the value of $d\,\Omega/d\,v$ varies as $\mu$ and $z_H$ change. In panel $(a)$, $d\,\Omega/d\,v$ has a downward-sloping curve and takes a positive value for a lower value of $\mu$; the curve intersects with the horizontal line at $\mu = 0.406$ in the baseline case. In panel $(b)$, a decrease in the value of non-participation of high-skill workers shifts the curve upward and $d\,\Omega/d\,v$ takes a positive sign in a broader range of $\mu$. As a lower $z_H$ decreases the cutoff productivity of workers in the high-skill group, firms are likely to hire less productive workers than before, resulting in too few vacancy creations. In panels $(c)$ and $(d)$, how a change in $\underline{y}_H$ and $\bar{y}_L$ affects the value of $d\,\Omega/d\,v$ is also examined. Panel $(c)$ represents the impact of changing the minimum productivity of workers in the high-skill group. As the least productive workers' ability increases, the trajectory of $d\,\Omega/d\,\mu$ shifts upward and widens the range of $\mu$, which yields a positive $d\,\Omega/d\,\mu$. Panel ($d$) demonstrates that lowering the upper bound of the distribution for workers in the low-skill group has a similar effect. The last two exercises demonstrate that if workers' productivity becomes less dispersed within a group by expanding a productivity difference between groups, vacancy creation in equilibrium is likely to be insufficient.

## 5. Concluding remarks

This study extends Albrecht et al. (2010), which construct a search and matching model with ex ante heterogeneous workers in their market productivity and endogenous labor force participation, by incorporating productivity dispersion within different skill groups. I show that vacancy creation can be insufficient from the viewpoint of economic efficiency if the share of workers in a high-skill group and the value of their non-participation are low. Policymakers need to understand the occurrence of such a change in the state of vacancy creation that is between excessive and insufficient because that would determine the direction of labor market policies.

For further research, the present study will be useful as a reference for studying the participation decision of workers with different educational backgrounds and for examining the impact of labor market policies on economic efficiency. According to the OECD database, the inactivity rate among women in 2018 differed significantly across educational attainments. Japan is one of the countries that is seeing a high inactivity rate among women with tertiary education. Increasing the participation and employment of these women will be an important policy objective for securing the labor force and improving labor productivity. The results of this study will contribute towards policy redesign to induce inactive women to participate and get better employment opportunities. Technological progress is another factor that could be incorporated into the present model. Specifically, skill-biased technological change, which increases the productivity of workers with higher skills more rapidly than that of workers with lower skills, will change both within-group and between-group dispersion of workers' productivity. Introducing technological progress allows us to deal with a more realistic change in productivity dispersion and conduct a more detailed analysis of economic efficiency in a model with endogenous labor force participation and heterogeneous skills.

## References

Albrecht et al. (2010) Efficiency in a search and matching model with endogenous participation. *Economics Letters*, 106, 48-50.

Autor, H. D., and Katz, L. F., and Kearney, M. S. (2008) Trends in U.S. wage inequality: Revising the revisionists. *Review of Economics and Statistics*, 90, 300-323.

Baalen, B., and Müller, T. (2014) Social welfare effects of tax-benefit reform under endogenous participation and unemployment: an ordinal approach. *International Tax and Public Finance,* 21, 198-227.

Cuadras-Morató, X., and Mateos-Planas, X. (2013) Overeducation and skill-biased technical change. *The B.E. Journal of Macroeconomics*, 13, 1-18.

Dolado, J., Jansen, M., and Serrano, J. J. (2007) A positive analysis of targeted employment protection legislation. *The B.E. Journal of Macroeconomics*, 7, 1-28.

Hosios, A. (1990) On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, 57, 279-298.

Julien, B., and Mangin, S. (2017) Efficiency of job creation in a search and matching model with labor force participation. *Economics Letters*, 150, 149-151.

Lehmann, E., Parmentier, A., and Van Der Linden, B. (2011) Optimal income taxation with endogenous participation and search unemployment. *Journal of Public Economics*, 95, 1523-1537.

Masters, A.(2015) Efficiency in a search and matching model with participation policy. *Economics Letters*, 134, 111-113.