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Our product is unique: A note on a delegation game with differentiated products

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Abstract

We analyze a Cournot duopoly market with differentiated goods and the separation between ownership and control. We consider a delegation game, for which the owner of a firm hires a manager who acts as if the good has a lower degree of substitutability than it really has. This is so either because managers are biased and perceive the good in this way, or because firms design an incentive scheme accordingly, which leads the manager to act in this way. Both firms rely on delegation. We discuss conditions, which lead one firm to increase its profit implying that the usual result of a prisoners´ dilemma is avoided.

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1 Introduction

Firms often claim uniqueness for their product. While this is usually seen as a sales pitch to an audience of potential consumers, in this paper, we focus on a different underlying economic rationale for this strategy, where claims of uniqueness should be studied within the competitive setting of a market. In this framework, where two firms sell substitute products, we argue that firms telegraph their intentions to competitors to adopt specific strategies. In this context, we study a delegation game and show that if the (representative) consumer weighs – in a vertical sense as in Häckner (2000) – the quality of goods differently, profit can increase for one firm. Hence, our results indicate that in a delegation game where firms sell substitute goods, the usual prisoner dilemma might be avoided. The finding applies to a relatively large degree of substitutability.

In our framework, firms delegate production to a manager to send a credible signal. We identify two possible channels, which firms might adopt, and both alternatives lead to the identical aforementioned result.

Firms hire a manager who is biased in believing that the product is more unique. This is a short way of saying that the good in question is less substitutable than it really is. That is, managers differ in the extent of their bias (i.e., their biased-type), which is known to the actors in the economy. This interpretation is in line with recent literature, reviewed below, which identifies in personality traits of managers the firms' commitment to a given strategy.

Alternatively, we argue that owners of firms write contracts, which incentivize managers to act *as if* the good is more unique than it really is and publicly announce their incentive scheme. Suppose a manager is incentivized to care about the firm's relative profit compared to the competitor. We show that if the two goals are appropriately weighted, the manager's objective is effectively adjusted such that it skews the degree of substitutability downwards.

The basic idea is rooted in the literature on delegation as strategic commitment starting with Vickers (1985) and Fershtman and Judd (1987), where the question is raised, whether owners of firms would want managers to maximize profits or something in addition. On this basis one can study the effect that this kind of delegation has on profits of firms and on the type of competition observed in markets of homogeneous goods. The basic idea is later reinterpreted by Miller and Pazgal (2002) and Englmaier (2010) in terms of personality traits of managers, e.g., aggressiveness, which leads them to maximize an objective function different from mere firm profits and therefore allows firms to use delegation as a strategic commitment. In this context, Englmaier (2010) shows that owners prompt their managers to act more aggressively in the product market, which leads to higher quantities than under profit maximization.

The common thread of these approaches is the following. Whereas a single firm utilizing strategic delegation can increase profits and act as a de facto-Stackelberg leader (Vickers, 1985; Sklivas, 1987; Basu, 1995), this vanishes as competitors apply the same strategy. That is, firms tend to maneuver themselves into a prisoners' dilemma, in which profits decrease all around (Fershtman and Judd, 1987). In addition, there are contributions that are highlighting strategic delegation under quantity competition, which can cause lower market aggressiveness and lead to an output contraction. Recently, Habiger and Kopel (2020) and Choi et al. (2020) show this including vertical relations between downstream and upstream firms. De Giovanni and Lamantia (2017) use an isoelastic demand to show a possible collusive equilibrium configuration. More closely related to our finding are Veldman et al. (2014), who study an oligopoly market with homogeneous goods and a delegating process that leads to cost-reducing activities. They show that a firm, which is more efficient in cost-reducing activities, can obtain a higher profit in the delegation game. To sum up, relying on the strategic delegation, existing research has obtained opposite results. The first set of papers is about the prisoners' dilemma and the second set is about possible collusive behavior. In contrast, our paper highlights the role of substitutability among goods. We provide conditions under which the high-quality provider of the good acts as a de facto-Stackelberg leader.

2 Model

Consider a Cournot duopoly in which firm's $i \in \{1, 2\}$ owner in the first stage delegates running the firm to a manager. In the second stage, the two hired managers simultaneously choose a quantity in the market.

Both firms face the linear inverse demand functions:[1]

$$p_i = a_i - q_i - \gamma q_{-i} \tag{1}$$

and goods are substitute; i.e., $0 \leqslant \gamma < 1$. In case of $\gamma = 0$, the two firms are in separate industries. We normalize $a_2 = 1$ and define $a_1 \equiv \xi \geqslant 1$ to differentiate firms according to the quality of the product. In fact, the parameter a_i measures quality in the vertical sense. Other things equal, the marginal utility is increasing in a_i. In the interpretation of Häckner (2000), firm 1 can be seen as the high-quality provider of the good.[2] W.l.o.g. there are no costs of production and therefore firms' profits are $\pi_i = p_i q_i$.

In the first stage, the owner of firm i hires a risk neutral manager to whom they delegate the production. Firms select a manager with a certain bias θ_i. In particular, biased managers believe that the degree of substitutability between products is $(\gamma - \theta_i)$ with $\theta_i \in [0, \gamma]$. The higher θ_i is the smaller the perceived degree of substitutability. Each owner of the firm observes the bias, and both managers agree to disagree (Morris, 1995) on the perceived level of substitutability. As in Englmaier (2010), we refrain from agency problems and assume that a θ_i-manager maximizes the following:

$$D_i = (a_i - q_i - (\gamma - \theta_i) q_{-i}) q_i. \tag{2}$$

Clearly, the case of $\theta_i = 0$ for both firms refers back to the standard competition without delegation. A value $\theta_i > 0$ constitutes a delegation game.

There is an alternative to assuming that the owners hire biased managers. The machinery of the model would be the same if firms incentivize an unbiased manager to act as if the good i is more special in the eyes of consumers than it really is. One way to put this incentive scheme into practice would involve weighing profits similar to Vickers (1985), where the owner appropriately rescales the weight with the degree of substitutability. To be precise, the owner incentivizes manager i not only to care about profits of firm i but also about the relative position of the firm to the profits of the other firm, that is, $(1 - x_i)\pi_i + x_i(\pi_i - \pi_{-i})$, where $x_i \in [0, 1]$

[1] The demand for each firm, as in Dixit (1979), can be traced back to a representative consumer whose preferences are represented by the following utility function:

$$U(q_0, q_1, q_2) = q_0 + a_1 q_1 + a_2 q_2 - \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2)$$

All parameters are positive. $\gamma^2 < 1$ must be satisfied to ensure strict concavity of the utility. The consumer maximizes U subject to the budget constraint $p_1 q_1 + p_2 q_2 + q_0 \leqslant m$, where q_0 is the numéraire and m the income.

[2] Assuming $\xi \geqslant 1$ is innocuous. With $\xi < 1$, firm 2 would be the high-quality provider.

represents the weight put on the difference between profits. This incentive scheme would be made public. For $x_i = \theta_i/\gamma$, this becomes $D_i - x_i(a_{-i} - q_{-i})q_{-i}$, where the last term does not affect the quantity chosen by manager i. In this scenario, in order to be able to interpret x_i as a weight, $0 \leqslant \theta_i \leqslant \gamma$ must hold.[3] Hence, in this way, managers take distorted output decisions by considering that the linear demand slope is different compared to real one.

In what follows, we pursue interpreting θ_i as the extent to which a manager is biased. Importantly, observe that as shown by Basu (1995), the delegation represents a credible commitment because it gives firm i a *Stackelberg advantage*.

The second stage has both managers simultaneously choose a quantity. The game is solved by backward induction. The Nash equilibrium of the second-stage game is characterized by the two quantities q_i^n, with the subscript n denoting the Nash equilibrium outcome: each manager will choose $q_i = \arg\max_{\tilde{q}_i} D_i = (a_i - \tilde{q}_i - (\gamma - \theta_i)q_{-i})\tilde{q}_i$. This is summarized in the following:

Lemma 1. *In equilibrium, the second-stage quantities are:*

$$q_1^n = \frac{2\xi + \theta_1 - \gamma}{4 - \gamma^2 + \gamma(\theta_1 + \theta_2) - \theta_1\theta_2}, \tag{3}$$

$$q_2^n = \frac{2 - \gamma\xi + \theta_2\xi}{4 - \gamma^2 + \gamma(\theta_1 + \theta_2) - \theta_1\theta_2}. \tag{4}$$

This proof and all subsequent ones can be found in the appendix.

In the first stage, each owner anticipates the equilibrium productions and therefore takes the quantities (3) and (4) into consideration in the *delegation game*. Hence, the strategy of firms consist of selecting an (appropriate) type of manager to whom delegate production. Therefore, each firm's owner chooses a θ_i to maximize the respective profit:

$$\pi_1(\cdot;\theta_1,\theta_2) = [\xi - q_1^n(\cdot) - \gamma q_2^n(\cdot)]q_1^n(\cdot), \tag{5}$$

$$\pi_2(\cdot;\theta_1,\theta_2) = [1 - q_2^n(\cdot) - \gamma q_1^n(\cdot)]q_2^n(\cdot). \tag{6}$$

This leads to the following optimal types.[4]

Lemma 2. *Each owner chooses a θ_i according to:*

$$\theta_1^* = \frac{\gamma^2(\xi - \gamma)}{2 - \gamma^2 - \gamma\xi} \tag{7}$$

$$\theta_2^* = \frac{\gamma^2(1 - \gamma\xi)}{2\xi - \gamma - \gamma^2\xi}. \tag{8}$$

In order to state the main results we first analyze the effect of the degree of substitutability, γ, on the choice of both owners, represented by the variable (bias-type) θ_i.

[3] Instead of the proposed incentive scheme, the result would be the same if we assumed that firms make a take-it-or-leave-it offer for the observable output. In our set-up the manager is the residual claimant who provides the desired quantity.

[4] In order to avoid corner solutions, we restrict $\xi \leqslant 1/\gamma$. In the appendix, we show that for θ_i not to exceed γ, θ_{-i} must be non negative.

Proposition 1. *The following holds:*

i) $\frac{\partial \theta_1^*}{\partial \gamma} > 0$*;*

ii) there exists a $\overline{\xi}$ *such that if* $\xi \leqslant \overline{\xi}$ *then* $\frac{\partial \theta_2^*}{\partial \gamma} \geqslant 0$ *(with equality if* $\xi = \overline{\xi}$*); otherwise for* $\xi > \overline{\xi}$*,* θ_2^* *is an inverted U-shaped function of* γ*.*

The proposition summarizes the response that firms using the delegation can give if they are faced with increased substitutability of goods. Each firm's owner aims at balancing the negative impact that a higher γ would have on the price using the delegation to compensate for the decrease in quantity. Evidently, an increase in γ leads both managers to decrease the quantity to compensate the direct negative effect on the price. If the degree of substitutability is relatively low, a higher θ_i is the commitment to produce more than in the case of no delegation. However, if the absolute advantage of firm 1 is relatively high, an increase of the degree of substitutability can lead firm 2 to opt for a manager with lower θ_2 and hence a lower quantity. In this situation, firm 2 effectively becomes a Stackelberg follower to firm 1, because the detrimental impact of a high quantity by firm 1 and a high γ on the price of firm 2 can only be countered by a decrease of q_2.

There is a corollary to this reasoning, which summarizes the paper's contribution. Previous results have shown that the delegation game always implies a prisoners' dilemma. In our case, this is not always true anymore. In the case of both ξ and γ being sufficiently large, (only) firm 1 as de-facto Stackelberg leader is better off.

Corollary 1. *There exists a* $\xi > \overline{\xi}$ *such that for a relatively large* γ *firm 1's profit is higher under the delegation game than the standard competition.*

The result holds when the goods are relatively similar. For the high-quality provider hiring a biased manager increases profits. Intuitively, as can be seen from Proposition 1, being a high-quality provider and adopting such a strategy leads the firm to increase its own quantity, which, in turn, leads the competitor to opt for a manager with a low θ_2 and therefore a quantity similar to the one, which would be produced in absence of delegation.

3 Conclusion

In this note, we study a delegation game in which the owner of a firm hires a biased manager who perceives the good as more unique. We show that in a market with substitute goods, the common pitfall of a prisoners' dilemma resulting from strategic delegation can be avoided. Whenever goods are relatively strong substitutes, a high-quality provider can increase profits. Furthermore, we show that the same result occurs in the absence of biased managers when the owner of the high-quality provider implements a managerial incentive scheme which skews the degree of substitutability downwards.

A Appendix

Proof of Lemma 1. Each manager maximizes D_i in choosing quantity. Hence, it is straightforward to see i) that the Nash equilibrium is as in (3) and (4), ii) that the conditions are necessary and sufficient. ■

Proof of Lemma 2. Profits that owners maximize are (5) and (6). Using the expressions as in (3) and (4), and differentiating, we have:

$$\frac{\partial \pi_1}{\partial \theta_1} = -\frac{(\gamma\xi - \theta_2\xi - 2)\left(\gamma^3 - \gamma^2(\theta_1 + \theta_2 + 2\xi) + \gamma\theta_1(\theta_2 - 2\xi) + 2\gamma\theta_2\xi + 2\theta_1(\theta_2\xi + 2)\right)}{\left(\gamma^2 - \gamma(\theta_1 + \theta_2) + \theta_1\theta_2 - 4\right)^3}$$

$$\frac{\partial \pi_2}{\partial \theta_2} = -\frac{(\gamma - \theta_1 - 2\xi)\left(\gamma^3\xi - \gamma^2(\theta_1\xi + \theta_2\xi + 2) + \gamma\theta_1(\theta_2\xi + 2) - 2\gamma\theta_2 + 2\theta_2(\theta_1 + 2\xi)\right)}{\left(\gamma^2 - \gamma(\theta_1 + \theta_2) + \theta_1\theta_2 - 4\right)^3}$$

The system:

$$\begin{cases} \frac{\partial \pi_1}{\partial \theta_1} = 0, \\ \frac{\partial \pi_2}{\partial \theta_2} = 0, \end{cases}$$

is only satisfied by (7) and (8). It remains to prove that conditions are also sufficient in the relevant range (i.e., $\theta_i \in [0, \gamma]$). We prove it only for π_1 since for π_2 the proof follows the exact same steps. Observe first that to have $\theta_i \geqslant 0$ ($\theta_{-i} \leqslant \gamma$), it must be that $\theta_{-i} \leqslant \gamma$ ($\theta_i \geqslant 0$) and therefore for having $\theta_2 \geqslant 0$ ($\theta_1 \leqslant \gamma$), the relation $\xi \leqslant \frac{1}{\gamma}$ has to be satisfied. Hence, we prove that the first derivative $\frac{\partial \pi_i(\cdot;\theta_i)}{\partial \theta_i}$ changes sign from positive to negative when we evaluate it at the lower bound ($\theta_1 = 0$) and upper bound ($\theta_1 = \gamma$). In fact,

$$\lim_{\theta_1 \to 0} \frac{\partial \pi_1(\cdot;\theta_1)}{\partial \theta_1} = \frac{\gamma(\gamma - \theta_2)(2\xi - \gamma)(2 + \theta_2\xi - \gamma\xi)}{\left(4 + \gamma\theta_2 - \gamma^2\right)^3} > 0$$

$$\lim_{\theta_1 \to \gamma} \frac{\partial \pi_1(\cdot;\theta_1)}{\partial \theta_1} = -\frac{1}{16}\gamma\left(\gamma^2\xi^2 - \gamma\xi(2\theta_2\xi + 3) + \theta_2^2\xi^2 + 3\theta_2\xi + 2\right) < 0$$

The sign of the first limit is immediate. To see the second, define:

$$\Omega(\xi) \triangleq \gamma^2\xi^2 - \gamma\xi(2\theta_2\xi + 3) + \theta_2^2\xi^2 + 3\theta_2\xi + 2 \tag{A.1}$$

We have to show that $\Omega(\xi) > 0$. Observe that:

$$\frac{\partial \Omega(\xi)}{\partial \xi} = (\gamma - \theta_2)(2\gamma\xi - 2\theta_2\xi - 3) < 0 \tag{A.2}$$

and

$$\Omega(\xi = \frac{1}{\gamma}) = \frac{\theta_2(\gamma + \theta_2)}{\gamma^2} > 0. \tag{A.3}$$

■

Proof of Proposition 1. To prove i), consider the cross partial derivative:

$$\frac{\partial^2 \theta_1^*}{\partial \xi \partial \gamma} = \frac{4\gamma[2 - 3\gamma^2 + \gamma^3\xi]}{\left(2 - \gamma^2 - \gamma\xi\right)^3} > 0. \tag{A.4}$$

It is clearly positive because $\gamma \in [0,1)$ and $\xi \leqslant \frac{1}{\gamma}$. To conclude, observe that $\frac{\partial \theta_1^*}{\partial \gamma}(\xi = 1) = \frac{\gamma(\gamma+4)}{(\gamma+2)^2} > 0$.

For ii), to simplify tedious algebra, denote with H the denominator of (8); it is straightforward to see that $H > 0$. In what follows, given that $y(x) = \frac{D(x)}{N(x)} \Leftrightarrow D(x) = y(x)N(x)$, in computing $\frac{\partial y(x)}{\partial x}$, we will write:

$$\frac{\partial y(x)}{\partial x} = \frac{D'(x)N(x) - D(x)N'(x)}{[N(x)]^2} = \frac{D'(x)N(x) - y(x)N(x)N'(x)}{[N(x)]^2}$$

Hence,

$$\frac{\partial \theta_2^*}{\partial \gamma} = \frac{\gamma\left(\gamma^3 + 2\gamma^2\xi + 4\xi - \gamma\left(6 + \xi^2\right)\right)}{\left(\gamma^2 + \gamma\xi - 2\right)^2} = \tag{A.5}$$

$$\frac{(2\gamma - 3\gamma\xi)H + \theta_2^* H(1 - 2\gamma\xi)}{H^2} = \frac{(2\gamma - 3\gamma\xi) + \theta_2^*(1 - 2\gamma\xi)}{H} \tag{A.6}$$

and

$$\begin{aligned}\frac{\partial^2 \theta_2^*}{\partial \gamma^2} &= \frac{(2 - 3\xi - 2\theta_2^*\xi)H + \frac{\partial \theta_2^*}{\partial \gamma} H(1 - 2\gamma\xi)}{H^2} = \\ &\frac{(2 - 3\xi - 2\theta_2^*\xi) + \frac{\partial \theta_2^*}{\partial \gamma}(1 - 2\gamma\xi)}{H}.\end{aligned} \tag{A.7}$$

To prove the first part of ii), observe that the numerator of (A.5) is strictly concave in ξ and positive for $\xi = 1$; hence there exists at the most one $\xi > 1$ for which $\frac{\partial \theta_2^*}{\partial \gamma} = 0$. For the second part of ii), observe from (A.7) that in $\frac{\partial \theta_2^*}{\partial \gamma} = 0$ the function is locally concave and it holds $\theta_2^*(\gamma = 0) = \theta_2^*(\gamma = \frac{1}{\xi}) = 0$. ■

Proof of Corollary 1. Denote with $\Pi_i \equiv \pi_i(\theta_i = 0, \theta_{-i} = 0)$ firms' profit in absence of delegation and with $\pi_i^* = \pi_i(\theta_i = \theta_i^*, \theta_{-i} = \theta_{-i}^*)$ if they both engage in the delegation game. We have:

$$\begin{aligned}\Pi_1 &= \frac{(2\xi - \gamma)^2}{\left(4 - \gamma^2\right)^2} \\ \Pi_2 &= \frac{(2 - \gamma\xi)^2}{\left(4 - \gamma^2\right)^2} \\ \pi_1^* &= \frac{(2\xi - \gamma)\left(2\xi - \gamma^2\xi - \gamma\right)}{16\left(1 - \gamma^2\right)} \\ \pi_2^* &= \frac{(2 - \gamma\xi)\left(2 - \gamma^2 - \gamma\xi\right)}{16\left(1 - \gamma^2\right)}\end{aligned}$$

The differences are:

$$\pi_1^* - \Pi_1 = \frac{\gamma^3(2\xi - \gamma)\left(10\gamma\xi - 8 - \gamma^3\xi - \gamma^2\right)}{16\left(4 - \gamma^2\right)^2\left(1 - \gamma^2\right)} \tag{A.8}$$

$$\pi_2^* - \Pi_2 = \frac{\gamma^3(2 - \gamma\xi)\left(10\gamma - 8\xi - \gamma^3 - \gamma^2\xi\right)}{16\left(4 - \gamma^2\right)^2\left(1 - \gamma^2\right)} \tag{A.9}$$

Hence, in the relevant range, (A.9) can never be positive, but (A.8) is positive for ξ and γ sufficiently large.

It is straightforward to see that all the fundamentals of the market can be positive despite (A.8) being positive. In fact,

$$
\begin{aligned}
q_1^* &= \frac{2\xi - \gamma^2\xi - \gamma}{4\,(1-\gamma^2)} \\
q_2^* &= \frac{2 - \gamma^2 - \gamma\xi}{4\,(1-\gamma^2)} \\
p_1^* &= \frac{1}{4}(2\xi - \gamma) \\
p_2^* &= \frac{1}{4}(2 - \gamma\xi).
\end{aligned}
$$

Which are clearly positive given that $\xi \leqslant \frac{1}{\gamma}$. ■

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