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Platform competition and willingness to pay in a vertical differentiated two-sided market

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Abstract

In this paper we generalize the model of single-homing users in two-sided markets by Gabszewicz and Wauthy [2014] to the case of any logconcave distribution of the willingness to pay (WTP). Our extended model allows us to discuss how distributional assumptions affect equilibrium outcomes, as well as to highlight the role of the assumption that both sides of the market are described by the same distribution of the WTP: while equilibrium does exist when this common distribution is logconcave, our results cast some doubts on existence when the two distributions differ.

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1 Introduction

In two-sided markets with cross-network externalities, the products' perceived quality may depend on the number of users: accordingly, given heterogeneity in the willingness to pay (WTP) for network sizes, price competition in two-sided markets can be studied with models of vertical differentiation. Starting from the works on two-sided markets by Armstrong (2006) and Armstrong and Wright (2007), several scholars modeled platforms competition as based on differentiation or consumers' taste for variety (Hagiu, 2009; Reisinger, 2012; Lee, 2014; Zennyo, 2016; Roger, 2017). A related stream of literature focused on product differentiation and network externalities in one-sided markets (Lambertini and Orsini, 2002, 2005, 2006).[1]

In this perspective, Gabszewicz and Wauthy (2014), assuming that the WTP of both sides of the market is uniformly distributed, show that the qualities of the two products are endogenous to the price decisions of the platforms, and the size of the network endogenously determines the WTP of the agents to register with one of the (two) platforms.

In Gabszewicz and Wauthy (2014), users from both sides met by platforms are characterized by heterogenous preferences, which interacts with the size of network externalities, and single-home, namely each of them only trades on one of the two platforms. Following the product differentiation literature (Gabszewicz and Thisse, 1979), the authors use the uniform-distribution model of vertical differentiation: their framework is particularly suitable to analyse the firms' strategies in two-sided markets such as those of credit cards (Rysman, 2007), where consumers meet merchants in the credit card platform, of smartphone's apps (Gans, 2012) and videogames, where programmers meet users and players, of managed care plans, where patients meet health providers, and of classified advertising (Ambrus and Argenziano, 2009).[2] However, the uniform-distribution assumption downplays the role of the WTP distribution across agents, even though the shape of such distribution can in principle affect the firms' equilibrium choices and may in fact have a bearing on the very existence of equilibrium. Indeed, concerns for analytical generality as well as substantive questions about the role of inequality can be addressed within vertical differentiation models precisely when a non uniform distribution of the WTP is assumed (Benassi et al., 2006, 2016, 2019).[3]

In this paper we generalize the model of Gabszewicz and Wauthy (2014) to the case of any logconcave distribution of the WTP, to enquiry about equilibria in a two-sided market in a more general distributional setup: this should in principle enable one to assess how distributional assumptions affect equilibrium outcomes, as well as to highlight the role of the implicit assumption that heterogeneity on both sides of the market is described by the same distribution of the WTP. In particular, the latter turns out to be a sensitive issue, as our results cast some doubt on equilibrium existence when the two distributions differ.

The remainder of the paper is organised as follows. In Section 2, we introduce the model; in Section 3 the existence of an equilibrium under logconcavity is proved; in Section 4 we discuss the results. Section 5 concludes the paper.

2 The model

As in Gabszewicz and Wauthy (2014), there are two platforms and two groups of single-homing users, one for each side of the market. Platforms, denoted by i, sell products $i = 1, 2$ to both users, who respectively pay price p_i and price π_i to register to platform i. Each user can register

[1] In contrast to this literature where network size is the only vertical dimension, Baake and Boom (2001) assume that consumers' WTP increases in both the product's quality and the size of its network.

[2] As suggested by Jeitschko and Tremblay (2020), another case in which users of both sides of the market are single-homing was the early home-computer market, where the two platforms were DOS-based machines and Macintosh machines, and where softwares were available for either one or the other platform.

[3] In general, Yurko (2011) shows that income inequality has important implications for the degree of product differentiation.

with only one platform, following a one-to-one matching process. The users' utilities are given by

$$\begin{aligned} U_i^v &= \beta x_i - p_i \\ U_i^x &= \sigma v_i - \pi_i \end{aligned}$$

where the marginal WTP for $i = 1, 2$ is respectively β and σ, while x_i and v_i denote the numbers of users registered with platform i, as measures of cross-network externalities. Two types of users are accordingly identified by their marginal WTP β and σ. These are distributed according to the density functions $f(\beta)$ and $g(\sigma)$; we normalize the numbers of both types of users to one, so that the implied cumulative distributions are $F(\beta) : [0, 1] \to [0, 1]$ and $G(\sigma) : [0, 1] \to [0, 1]$.

Platforms choose prices p_i and π_i, taking the users' expectations x_i^e and v_i^e as given, such that $x_2^e > x_1^e$ and $v_2^e > v_1^e$.[4] Therefore, the marginal users are identified by

$$\beta_1 = \frac{p_1}{x_1^e},\ \beta_2 = \frac{p_2 - p_1}{x_2^e - x_1^e} \tag{1}$$

$$\sigma_1 = \frac{\pi_1}{v_1^e},\ \sigma_2 = \frac{\pi_2 - \pi_1}{v_2^e - v_1^e} \tag{2}$$

and the demand functions are defined as

$$\begin{aligned} D_1^v &= F(\beta_2) - F(\beta_1),\ D_2^v = 1 - F(\beta_2) \\ D_1^x &= G(\sigma_2) - G(\sigma_1),\ D_2^x = 1 - G(\sigma_2) \end{aligned}$$

Finally, by definition, the platform payoffs are

$$\begin{aligned} \Pi_1 &= p_1 D_1^v + \pi_1 D_1^x = p_1 [F(\beta_2) - F(\beta_1)] + \pi_1 [G(\sigma_2) - G(\sigma_1)] \\ \Pi_2 &= p_2 D_2^v + \pi_2 D_2^x = p_2 [1 - F(\beta_2)] + \pi_2 [1 - G(\sigma_2)] \end{aligned}$$

Given this general framework, our key assumption about the distributions of the users' WTP is logconcavity:[5]

Assumption 1 *The distributions $F(\beta)$ and $G(\sigma)$ are logconcave.*

Letting primes denote derivatives, this can be cast in a more convenient form by means of the following elasticities:

$$\theta^f(\beta) = 1 + \frac{\beta f'(\beta)}{f(\beta)},\ \theta^g(\sigma) = 1 + \frac{\sigma g'(\sigma)}{g(\sigma)}$$

and

$$\eta^F(\beta) = \frac{\beta f(\beta)}{1 - F(\beta)},\ \eta^G(\sigma) = \frac{\sigma g(\sigma)}{1 - G(\sigma)}$$

The former are the Esteban elasticities of the relevant densities (Esteban, 1986), while the latter are the (positive) elasticities of $1 - F(\beta)$ and $1 - G(\sigma)$. It is then easy to prove that logconcavity of any continuous distribution $J = F, G$ $(j = f, g)$ amounts to the constraint

$$\eta^J(\cdot) + \theta^j(\cdot) > 1$$

[4]As in Gabszewicz and Wauthy (2014), we do not explicitly take into consideration the case where $v_1^e = v_2^e$ since this would imply zero profit in equilibrium. Moreover, since $v_1^e > 0$, it is impossible that a firm is excluded from the market, that is, the two firms both enjoy a positive demand.

[5]As is well known, this is a usual assumption in product differentiation models, following Caplin and Nalebuff (1991).

which in turn implies that $\eta^J(\cdot)$ in an increasing function (An, 1998).[6]

3 Price equilibrium

In this section, we find a Nash equilibrium in the two-sided market duopoly as the solution of the price game. We adopt the definition of equilibrium given by Gabszewicz and Wauthy (2014):

Definition 1 *A (pure-strategy) Nash Equilibrium is defined by two quadruples* $(\mathbf{p}^*, \mathbf{x}^*)$*, with* $\mathbf{p}^* = \{p_i^*, \pi_i^*\}$ *and* $\mathbf{x}^* = (v_i^*, x_i^*)$*,* $i = 1, 2$*, such that: (i) given expectations* $\mathbf{x}^*$*,* (p_i^*, π_i^*) *is a best reply against* (p_j^*, π_j^*)*,* $i \neq j$*, and vice-versa; (ii)* $D_i^v(p_1^*, p_2^*) = x_i^*$*;* $D_i^x(\pi_1^*, \pi_2^*) = v_i^*$*,* $i = 1, 2$*.*

In order to establish the existence of such an equilibrium, we start by considering the price setting problem of platform $i = 2$. Given the price pair (p_1, π_1) set by the firm 1, as well as expectations (v_i^e, x_i^e), $v_1^e < v_2^e$, $x_1^e < x_2^e$, p_2 and π_2 are charged by platform 2 to maximize its profit:

$$
\begin{aligned}
\frac{\partial \Pi_2}{\partial p_2} &= 1 - F(\beta_2) - p_2 f(\beta_2) \frac{\partial \beta_2}{\partial p_2} = 0 \\
\frac{\partial \Pi_2}{\partial \pi_2} &= 1 - G(\sigma_2) - \pi_2 g(\sigma_2) \frac{\partial \sigma_2}{\partial \pi_2} = 0
\end{aligned}
$$

These first order conditions can be cast in elasticity terms as[7]

$$
\begin{aligned}
\eta^F(\beta_2)\varepsilon_{2,2}^{\beta} &= 1 \qquad (3) \\
\eta^G(\sigma_2)\varepsilon_{2,2}^{\sigma} &= 1 \qquad (4)
\end{aligned}
$$

where $\varepsilon_{i,j}^k$, $(k = \beta, \sigma)$, denotes the (relevant) price elasticity of the (relevant) marginal user.[8] It is easily seen that

$$
\begin{aligned}
\varepsilon_{1,2}^{\beta} &= -\frac{p_1}{p_2 - p_1} < 0,\ \varepsilon_{2,2}^{\beta} = \frac{p_2}{p_2 - p_1} > 1 \\
\varepsilon_{1,2}^{\sigma} &= -\frac{\pi_1}{\pi_2 - \pi_1} < 0,\ \varepsilon_{2,2}^{\sigma} = \frac{\pi_2}{\pi_2 - \pi_1} > 1
\end{aligned}
$$

such that the sum of the elasticities of the marginal users are (standardly) constant and equal to one: $\varepsilon_{1,2}^k + \varepsilon_{2,2}^k = 1$, while $\varepsilon_{1,1}^k = 1$, $k = \beta, \sigma$.

We now turn to platform 1, which maximizes its profit with respect to its prices p_1 and π_1, for any given price pair (p_2, π_2):

$$
\begin{aligned}
\frac{\partial \Pi_1}{\partial p_1} &= F(\beta_2) - F(\beta_1) + p_1 \left[f(\beta_2) \frac{\partial \beta_2}{\partial p_1} - f(\beta_1) \frac{\partial \beta_1}{\partial p_1} \right] = 0 \\
\frac{\partial \Pi_1}{\partial \pi_1} &= G(\sigma_2) - G(\sigma_1) + \pi_1 \left[g(\sigma_2) \frac{\partial \sigma_2}{\partial \pi_1} - g(\sigma_1) \frac{\partial \sigma_1}{\partial \pi_1} \right] = 0
\end{aligned}
$$

[6]For a similar approach to modeling distributions of the WTP in the framework of vertical differentiation with uncovered markets, see Benassi et al. (2016, 2019).

[7]Second order conditions check is reported in the Appendix.

[8]For example, $\varepsilon_{i,j}^{\beta} = (\partial \beta_j / \partial p_i)(p_i / \beta_j)$, $\varepsilon_{i,j}^{\sigma} = (\partial \sigma_j / \partial \pi_i)(\pi_i / \sigma_j)$.

Again, these can be cast in elasticity terms

$$\frac{1-F(\beta_2)}{1-F(\beta_1)} = \frac{1-\eta^F(\beta_1)}{1-\eta^F(\beta_2)\,\varepsilon^{\beta}_{1,2}} \tag{5}$$

$$\frac{1-G(\sigma_2)}{1-G(\sigma_1)} = \frac{1-\eta^G(\sigma_1)}{1-\eta^G(\sigma_2)\,\varepsilon^{\sigma}_{1,2}} \tag{6}$$

Accordingly, in a price equilibrium it has to be the case that (3), (4), (5), and (6) hold. This, combined with the requirement that expectations are fulfilled, i.e.

$$x^e_1 = F(\beta_2) - F(\beta_1)\,,\; x^e_2 = 1 - F(\beta_2) \tag{7}$$

$$v^e_1 = G(\sigma_2) - G(\sigma_1)\,,\; v^e_2 = 1 - G(\sigma_2) \tag{8}$$

allows us to prove the following

Proposition 1 *Suppose the distribution of both types β and σ is the same, i.e. $F(\cdot) = G(\cdot)$. Then under Assumption 1, there exists a price equilibrium $(\mathbf{p}^*, \mathbf{x}^*)$.*

Proof. See Appendix. ■

It can be seen that Proposition 1 leads to equilibrium prices

$$(p^*_1, p^*_2) = \left(1, \frac{1}{1-\eta^F(\beta^*_2)}\right)[F(\beta^*_2) - F(\beta^*_1)]\,\beta^*_1 \tag{9}$$

$$(\pi^*_1, \pi^*_2) = \left(1, \frac{1}{1-\eta^G(\sigma^*_2)}\right)[G(\sigma^*_2) - G(\sigma^*_1)]\,\sigma^*_1 \tag{10}$$

where $(\beta^*_1, \beta^*_2) = \beta^*$ and $(\sigma^*_1, \sigma^*_2) = \sigma^*$, are such that (7) and (8) hold. One obvious corollary is that when the two distributions of the WTP are uniform, one recovers the results by Gabszewicz and Wauthy (2014), that is $\beta^* = \sigma^* = \left(\frac{1}{7}, \frac{3}{7}\right)$.

4 Discussion

The fairly broad distributional setup underpinning Proposition 1 raises two main questions. The first is, how different distributional shocks affect equilibrium – i.e., what happens to prices, market shares and profits if the distribution of WTP is altered by some exogenous shock; the second concerns our assumption that the WTP is identically distributed on the two sides of the market.

As to distributional shocks, the framework provided by equation (9) and (10), though not easily amenable to comparative statics propositions, does allow to perform numerical simulations which may shed some light on this issue. Here we limit ourselves to considering two simple cases *vis à vis* the standard uniform distribution: (a) a symmetric constant-mean change in dispersion; and (b) a first order (stochastic dominance) shifts, which obviously implies higher average WTP.[9]

Case (a) can be captured by a simple generalization of the uniform distribution such that $F(\beta) = (2\beta - 1 + 2\rho)/4\rho$: $\rho \in (0, 1/2]$ is a dispersion parameter, which in fact ranks distributions by second order stochastic dominance.[10] It is easily seen that in this example prices, quantities and profits all decrease with ρ;[11] more generally, lower concentration of the WTP

[9] It can be checked that equilibrium is unique in all examples discussed in this section.

[10] Clearly, $\rho = 1/2$ yields the standard uniform distribution over $[0, 1]$. The mean is obviously $1/2$ and the variance of this distribution is $\rho^2/3$.

[11] Let $c = c(\rho) = (2\rho + 1)/\rho$, a decreasing function such that $c > 4$ for $\rho \in (0, 1/2]$. Then $x_1 = c/14$ and $x_2 = c/7$; $p_1 = \pi_1 = c^2/[196(2-c)]$ and $p_2 = \pi_2 = c^2/[49(c-2)]$; $\Pi_1 = c^3/[1372(c-2)]$, $\Pi_2 = 2c^3/[343(c-2)]$.

entails a shrinking market, as the effects of the density falling in the middle of the support is not compensated by higher density near the outer boundaries – a mean-preserving higher spread of a symmetric distribution has asymmetric effects on served demand.[12] In this particular instance, it is also the case that higher dispersion narrows the difference between the market shares of the two platforms, so that in this respect the uniform distribution case examined by Gabszewicz and Wauthy (2014) identifies (within this framework) minimum heterogeneity.

We take up case (b) by turning to a numerical simulation: we compare the uniform distribution with two simple triangular distributions: the former is dominated by, and the latter dominates, the uniform distribution.[13] Table I gives a summary picture.

Table I: First Order Stochastic Dominance

	triangular F_1	uniform F	triangular F_2
average WTP	1/3	1/2	2/3
$x_1 = v_1$	0.302	0.286	0.236
$x_2 = v_2$	0.541	0.571	0.709
$p_1 = \pi_1$	0.025	0.041	0.055
$p_2 = \pi_2$	0.088	0.163	0.310
Π_1	0.015	0.023	0.026
Π_2	0.095	0.187	0.440

While clearly limited in scope, this exercise seems to identify a general pattern: unsurprisingly, higher average WTP is unambiguously associated with higher values of all relevant variables but one: the exception is the decreasing value of x_1 – the first order dominance shift associates higher mean with a lower density at low values of the WTP. A noteworthy implication is that this appears to drive a sort of polarization, as the market shares of the two platform diverge in size.

We finally turn to a problem which seems to us quite relevant in the analysis of two-sides markets: indeed, at a theoretical level the natural question presents itself as to what extent the existence result of Proposition 1 carries over to the case where G and F are different distributions. The following provides some perspective on this issue:

Remark 1 *Let $F : [0, 1] \to [0, 1]$ be a (continuously differentiable) logconcave distribution, and let $t : [0, 1] \to [0, 1]$ be an increasing concave transformation, such that $G : [0, 1] \to [0, 1]$, $G(\sigma) = F(t(\sigma))$, is also logconcave. If $\beta^* = (\beta_1^*, \beta_2^*)$ and $\sigma^* = (\sigma_1^*, \sigma_2^*)$ are equilibrium values such that $\beta_i^* = t(\sigma_i^*)$, $i = 1, 2$, then $t(\sigma) = \sigma$.*

Proof. See Appendix. ■

Though admittedly confined to the arguably specific case of concave transformations, this can be looked at as a sort of baseline scenario, as such transformations preserve logconcavity (Bagnoli and Bergstrom, 2005). Overall, the perspective one gains from Remark 1 is essentially negative: the existence of an equilibrium in this framework is inconsistent with t being strictly concave – indeed, it is easily seen that the logic of the proof rules out existence also in the case where t is (strictly) convex: even disregarding logconcavity, a necessary condition for existence is that the second derivative of t changes sign over $[0, 1]$. All this points to the fact that, while assuming that the WTP is identically distributed on both sides of the market may

[12]Indeed, this is generally the case in vertical differentiated markets, as high-WTP customers are served anyway, while the extent of market access for low-WTP depends on how the distributive shock operates in specific cases (Benassi et al., 2019).

[13]These two triangular distribution, $F_1(\beta) = 2\beta - \beta^2$ and $F_2(\beta) = \beta^2$, are one the 'mirror image' of the other. Their means are respectively $\mu_1 = 1/3 < \mu^U = 1/2 < \mu_2 = 2/3$, with obvious notation. The same applies to $G = F$.

seem unduly restrictive, the existence of equilibrium is likely to constrain severely the degree of heterogeneity between the distributions of users. In other words, some degree of consistency between these distributions seems to be required for equilibrium to exist. At equilibrium each firm is setting its prices in such a way that the marginal revenue is the same on the sellers' and the buyers' side, marginal revenue itself depending crucially on both the set prices and the shape of the two distributions: i.e., profit maximization (which with zero variable costs delivers unit demand elasticity at equilibrium) constrains the relationship between ε (the way the position of the marginal consumer is affected by a marginal change in prices) and η (the way that very change in turn affects the willingness to pay). If the two distributions are the same, marginal changes in (p_1, π_1) have the same effect on profits as marginal changes in (p_2, π_2): equilibrium existence ensures that along this dimension the two firms' choices are consistent, and indeed prices, profits and demand are the same for both firms. However, if the two distributions differ, the combination of ε and η consistent with one firm making positive profits does not necessarily entail positive profits for the other firm.

5 Conclusion

Two-sided markets and platforms are playing an ever increasing role in the economy; they are characterized by network externalities between the two sides of the market and, potentially, by vertical differentiation of the products provided by the platforms. Analysing the effect of these externalities on platform competition in vertical differentiated two-sided markets needs taking into account the heterogeneity of the users on both sides, which up to now has been mainly addressed by assuming a uniform distribution of the WTP of agents.

In our paper we generalize the model by Gabszewicz and Wauthy (2014), which studies the price competition between two platforms in a two-sided market with uniform distribution of WTP of single-homing agents on both sides, to consider the case of any logconcave distribution of WTP.[14] Logconcavity of the income (or WTP) distribution may in fact affect the equilibrium conditions with respect to the uniform case, or influence its very existence (Benassi et al., 2006, 2016, 2019).

We prove the existence of an equilibrium when both sides of the market have the same logconcave distribution of WTP, and we find that when the distribution is not the same on both sides this equilibrium may not exist. Moreover, by way of a suitable example we study how a symmetric constant-mean change in dispersion and a first order stochastic dominance shift of the distribution affect equilibrium prices and profits: lower concentration of the WTP shrinks the market and narrows the difference between the market shares of the two platforms, while a higher mean WTP is associated with a lower density at low values of the WTP.

Though admittedly limited in scope, these examples show that the standard assumption of a uniform distribution of the WTP is not neutral; on the other hand, our general results casts some doubt on the robustness of the (often) implicit assumption that the distribution of WTP on both sides of the market is the same, which should be relevant to the assessment of the results in platform markets analyses.

[14] As a further extension of the model by Gabszewicz and Wauthy (2014) where users are assumed to join only one platform (single-homing), the results could be checked when users join more than one platform (multi-homing). We thank an anonymous reviewer for this suggestion.

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