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Progressive tax and responsiveness to changes in investment projects: no loss offset

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Abstract

This study shows the comprehensive impact of tax progressiveness on investment. First, tax progressiveness makes an entrepreneur less responsive to changes in investment projects. When a tax becomes less (more) progressive, an entrepreneur becomes more (less) responsive to changes in the profitability and riskiness of investment projects. Second, tax asymmetries of progressive taxes and proportional taxes have opposite effects on the responsiveness of an entrepreneur to changes in investment projects. Confronting a progressive tax burden with no loss offset available, an entrepreneur less sharply adjusts the amount of investment in response to changes in the profitability and riskiness of investment projects while confronting a proportional tax burden with no loss offset available, an entrepreneur more sharply adjusts. The results suggest empirical and policy implications. 1) There exist interaction effects to measure between tax progressiveness and the profitability and riskiness of investment projects. 2) By adjusting tax asymmetries and progressiveness, tax jurisdictions are able to induce investors to be more (less) responsive to changes in investment projects which is to take a more (less) aggressive strategy.

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1. Introduction

Income taxes are more often than not progressive. On the other hand, studies on progressive taxes are rare and most of the literature focuses on proportional taxes. This study shows that tax progressiveness has comprehensive effects on investment by affecting the responsiveness of an entrepreneur to changes in the profitability and riskiness of investment projects.

Although tax consequences of losses have a profound impact on investment decisions, studies on progressive taxes do not consider losses for the analysis of investment decisions. This study considers both the loss and income sides and analyzes the asymmetric tax treatment of losses and incomes of an entrepreneur under the burden of progressive taxation. An entrepreneur is subject to progressive taxation when an investment generates incomes, but the entrepreneur has no loss offset when an investment incurs losses. This study shows that a progressive tax burden coupled with no loss offset and a proportional tax burden coupled with no loss offset have opposite effects on the responsiveness of an entrepreneur to changes in the profitability and riskiness of investment projects.

The literature largely focuses on proportional taxes although progressive taxes are widely practiced. Domar and Musgrave (1944), Tobin (1958), Mossin (1968), Feldstein (1969), Stiglitz (1969), Singer (1979), Langenmayr and Lester (2018), and Armstrong et al. (2019) use only proportional taxes in analyzing the effects of taxes on investment and risk-taking. Ahsan (1974) and Cowell (1975) introduce tax progressiveness by allowing marginal tax rates to discretely change according to the pre-tax income levels and keeping the total tax revenue fixed. Fellingham and Wolfson (1978, 1984), Waterson (1985), and Taylor (1986) adopt fully progressive tax rates that continuously increase in the pre-tax income. This study assumes progressive taxation under which the marginal tax rate continuously increases in the pre-tax income.

Most of the literature exogenously assumes that an investor is averse to risk, using concave utility functions (Tobin 1958, Mossin 1968, Feldstein 1969, Stiglitz 1969, Ahsan 1974, Cowell 1975, Singer 1979, Waterson 1985, Langenmayr and Lester 2018, Armstrong et al. 2019). On the other hand, Fellingham and Wolfson (1978, 1984) and Taylor (1986) assume risk neutrality of an investor and show that aversion to risk is induced by a progressive tax. This study shows that a risk neutral entrepreneur effectively turns into aversion to risk at the burden of progressive taxation.

The literature often measures an investor's risk-taking by the investment in a risky asset relative to the investment in a riskless asset in the context of investment portfolio (Tobin 1958, Mossin 1968, Feldstein 1969, Stiglitz 1969, Ahsan 1974, Cowell 1975, Singer 1979, Fellingham and Wolfson 1984, Waterson 1985). Some papers measure managers' risk-taking by their choice of riskier project (Langenmayr and Lester 2018, Armstrong et al. 2019). This study measures a risk neutral entrepreneur's limited risk-taking by the amount of investment in a project. In addition, this study is distinguished from the literature in that it analyzes the responsiveness of an entrepreneur to changes in investment projects, that is the change in investment amounts in response to updated information on the profitability and riskiness of investment projects.

A strand of literature analyzes the impact of asymmetries and progressiveness of taxation on irreversible investment under uncertainty. Alvarez and Koskela (2008) use a tax exemption threshold below which no tax is imposed such that the average tax rate increases in the pre-tax income (i.e., tax progressiveness) while the marginal tax rate is invariant. This study is differentiated from Alvarez and Koskela (2008) in that whereas the impact of taxes in Alvarez and Koskela (2008) depends on the relative magnitude of a tax exemption threshold and sunk cost of investment as well as volatility and other parameters, the impact of tax progressiveness on investment in this study keeps the same nature regardless of parameter values. Panteghini (2005) analyzes the effect of tax asymmetry on irreversible and sequential investment under uncertainty. Whereas the asymmetric tax treatment of losses and incomes in Panteghini (2005) is to show that taxation has no impact (i.e., tax neutrality) on irreversible investment under uncertainty which results from Bernanke's (1983) Bad News Principle, this study shows that the asymmetric progressive tax treatment and the asymmetric proportional tax treatment of losses and incomes have opposite impacts on investment.

The main results are as follows. First, a progressive tax induces a risk neutral entrepreneur to optimally invest a finite amount in a project. The optimal investment amount increases in the profitability of a project, but decreases in the riskiness of a project and the tax burden in terms of tax progressiveness and a base tax rate. The responsiveness of an entrepreneur to changes in the profitability and riskiness of investment projects decreases in tax progressiveness. As a result, when a tax becomes less (more) progressive, not only the optimal amount of investment increases (decreases) but also an entrepreneur more (less) sharply adjusts the amount of investment in response to changes in the profitability and riskiness of investment projects.

Second, the asymmetric progressive tax treatment of losses and incomes makes an entrepreneur less responsive to changes in the profitability and riskiness of investment projects while the asymmetric proportional tax treatment of losses and incomes makes an entrepreneur more responsive. That is, confronting a progressive tax burden coupled with no loss offset, an entrepreneur less sharply adjusts the amount of investment in response to changes in the profitability and riskiness of investment projects while confronting a proportional tax burden coupled with no loss offset, an entrepreneur more sharply adjusts.

This study contributes by showing the comprehensive impact of tax progressiveness on investment. Tax progressiveness lowers the responsiveness of an investor to changes in investment projects, and tax asymmetries of progressive taxes and proportional taxes have opposite effects on the responsiveness of an investor to changes in investment projects. Whereas prior studies use either proportional taxes in considering losses or progressive taxes while excluding losses from the analysis, this study employs progressive taxes *and* includes losses for the analysis of investment and risk-taking.

The rest of the article is structured as follows. Section 2 presents the model. Section 3 shows the results. Section 4 discusses empirical and policy implications. Section 5 concludes this study.

2. Model

A risk neutral entrepreneur decides an investment I in a project in order to maximize his expected after-tax earnings, which is also the after-tax cash flow. I assume an investment is non-negative, $I \geq 0$. The pre-tax income/loss x is

$$x = I(1+\varepsilon) - I = I\varepsilon, \tag{1}$$

where the random factor ε follows a uniform distribution with a positive mean m and dispersion c, $\varepsilon \sim U(m-c, m+c)$, $E[\varepsilon] = m > 0$, $Var[\varepsilon] = \frac{1}{3}c^2$. I assume c is a large number such that $c > 2m$.

The uncertain revenue is $I(1+\varepsilon)$ and the investment I is expensed. When the revenue is bigger (smaller) than the expense, the entrepreneur has a pre-tax income (loss). A progressive tax is imposed on positive incomes. A progressive tax rate increases in the pre-tax income x (Fellingham and Wolfson 1978 & 1984, Waterson 1985, Taylor 1986),

$$\text{tax rate} = a + bx\,,\ 0 \le a < a\dagger = \frac{4mc}{(m+c)^2} < 1,\ 0 \le b < 1 \tag{2}$$

where a is a proportional tax rate and b represents tax progressiveness. When $b = 0$, the tax rate (2) is purely proportional. When $b > 0$, the tax rate (2) is progressive. The tax liability imposed on a pre-tax income x is [tax rate,(2)]×[pre-tax income, x] = $ax + bx^2$. ax is the proportional tax liability and bx^2 is the progressive tax liability.

I assume that a proportional tax rate a is subject to an upper-bound $a\dagger = \frac{4mc}{(m+c)^2}$ to ensure that the optimal investment is positive. A proportional tax rate a is a base tax rate that is the marginal tax rate of the first dollar pre-tax income. With no loss offset, the after-tax loss (that is also the pre-tax loss) is bigger in magnitude than the after-tax income (see Figure1). When the base tax rate is too high ($a \ge a\dagger$), the expected after-tax income becomes too small compared to the expected after-tax loss and the entrepreneur decides not to invest.

When an entrepreneur has a pre-tax income $x = I\varepsilon > 0$, that is $\varepsilon > 0$, the tax rate (2) is applied and the after-tax income is

$$\begin{aligned}&[\text{after-tax income}] = [\text{pre-tax income}] - [\text{tax rate}] \times [\text{pre-tax income}] \\ &= (1-a)x - bx^2 = (1-a)(I\varepsilon) - b(I\varepsilon)^2\end{aligned} \tag{3}$$

The expected after-tax income is

$$E[\text{after-tax income}] = \int_0^{m+c}\left[(1-a)(I\varepsilon) - b(I\varepsilon)^2\right] f(\varepsilon)\,d\varepsilon\,, \tag{4}$$

where $f(\varepsilon)$ is the uniform probability density of the random factor ε, $f(\varepsilon) = \frac{1}{2c}$.

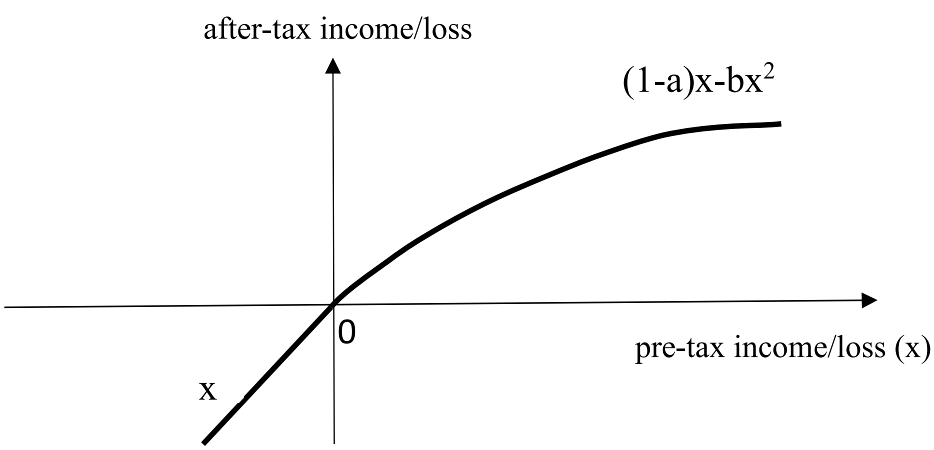

Figure1: Asymmetric tax treatment of losses and incomes

tax rate = a + b × pre-tax income (x)

a = proportional tax rate, b = tax progressiveness.

For incomes, a progressive tax is imposed:

after-tax income = pre-tax income – tax rate × pre-tax income = $(1\text{-a})x\text{-}bx^2$

For losses, no loss offset is available: after-tax loss = pre-tax loss = x

When an entrepreneur incurs a pre-tax loss $x = I\varepsilon < 0$, that is $\varepsilon < 0$, the tax rate (2) is not applied because progressive taxation is not applicable to a loss (see Figure1). A loss can bring about a wide range of loss offsets, depending on various factors including an investor's non-investment incomes, past tax payments, and future estimated tax rates (Shevlin 1990). I assume no loss offset as the entrepreneur is in the first year of operation and no loss carryback is available. I also assume the entrepreneur has no non-investment income. In addition, the assumption of no loss offset allows me to compare the proportional tax liability ax and the progressive tax liability bx^2 in terms of the effects from the lack of corresponding loss offsets. The after-tax loss is

$$\text{[after-tax loss]} = \text{[pre-tax loss]} = x = I\varepsilon \tag{5}$$

The expected after-tax loss is

$$E\text{[after-tax loss]} = \int_{m-c}^{0} (I\varepsilon) f(\varepsilon) d\varepsilon \tag{6}$$

Given (4) and (6), next lemma shows the objective function of an entrepreneur with no loss offset available.

Lemma 1.

A progressive tax liability as well as a proportional tax liability are imposed on incomes while no loss offset is available for losses. The objective function of a risk neutral entrepreneur is

$$\{(1-a)m + a\gamma\} I - b\left(\frac{1}{3}c^2 + m^2 - \eta\right) I^2, \tag{7}$$

where $\gamma \equiv \int_{m-c}^{0} \varepsilon f(\varepsilon) d\varepsilon = -\frac{1}{2c}\frac{1}{2}(m-c)^2 < 0,$ (8)

and $\eta \equiv \int_{m-c}^{0} \varepsilon^2 f(\varepsilon) d\varepsilon = -\frac{1}{2c}\frac{1}{3}(m-c)^3 > 0.$ (9)

(Proof: See Appendix.)

In the objective function (7), $\{(1-a)m + a\gamma\} I$ is the benefit from investment and $b\left(\frac{1}{3}c^2 + m^2 - \eta\right) I^2$ is the burden of progressive taxation caused by investment. The objective function (7) is equivalent to "yield" in dollar amount of Domar and Musgrave (1944)[1]. While Domar and Musgrave (1944) use a finite number of investment outcomes with discrete probabilities, I use a continuous uniform distribution for investment outcomes.

The term $a\gamma$ in the objective function (7) represents the asymmetric proportional tax treatment of losses and incomes and reduces the benefit from investment. Whereas a proportional tax liability ax is imposed on incomes ($x > 0$), there is no loss offset available for losses ($x < 0$), resulting in a negative constant γ (8) (see proof of Lemma 1). γ is equivalent to "risk" of Domar and Musgrave (1944), which is the summation of all possible negative rates of return multiplied by their respective probabilities ($\gamma = \int_{m-c}^{0} \frac{(I\varepsilon)}{I} f(\varepsilon) d\varepsilon$). γ increases in the mean m and decreases in the dispersion c,

[1] Domar and Musgrave (1944) define "yield" and "risk" in the rate of return of investment that is the percentage of incomes/losses relative to investment amounts. As Domar and Musgrave (1944) develop geometric analysis of "yield" and "risk", they define "risk" as a positive variable, that is equivalent to $-\gamma$ of this study.

$$\frac{\partial\gamma}{\partial m} = -\frac{1}{2c}(m-c) > 0 \tag{10}$$

$$\frac{\partial\gamma}{\partial c} = \frac{1}{4c^2}(m-c)(m+c) < 0 \tag{11}$$

The term $b\eta$ in the objective function (7) represents the asymmetric progressive tax treatment of losses and incomes. While a progressive tax liability bx^2 is imposed on incomes ($x > 0$), there is no loss offset available for losses ($x < 0$). If progressive taxation were symmetric for the loss side ($x < 0$), the after-tax loss would be $(1-a)x - bx^2$. The loss offset $-bx^2$ with respect to the progressive tax liability would be punishing rather than helping because given $x < 0$, $-bx^2$ is still negative whereas the loss offset $-ax$ with respect to the proportional tax liability is positive. The asymmetric progressive tax treatment of losses and incomes results in a positive constant η (9) (see proof of Lemma 1). The summation of all possible squared losses multiplied by their respective probabilities results in η ($\int_{m-c}^{0} x^2 f(\varepsilon)d\varepsilon = I^2\eta$). The term $b\eta$ in the objective function (7) reduces the burden of progressive taxation caused by investment as

$$\frac{1}{3}c^2 + m^2 - \eta = \int_{m-c}^{m+c} \varepsilon^2 f(\varepsilon)d\varepsilon - \int_{m-c}^{0} \varepsilon^2 f(\varepsilon)d\varepsilon = \int_{0}^{m+c} \varepsilon^2 f(\varepsilon)d\varepsilon > 0 \tag{12}$$

η decreases in the mean m and increases in the dispersion c,

$$\frac{\partial\eta}{\partial m} = -\frac{1}{2c}(m-c)^2 < 0 \tag{13}$$

$$\frac{\partial\eta}{\partial c} = \frac{1}{6c^2}(m-c)^2(m+2c) > 0 \tag{14}$$

3. Results

A progressive tax induces a risk neutral entrepreneur to optimally invest a finite amount and effectively turn into aversion to risk.

Proposition 1. (Finite optimal investment induced by a progressive tax)

With the assumptions in Lemma 1, a risk neutral entrepreneur is induced to make a finite amount of optimal investment I_N if and only if there is a burden of progressive taxation $b > 0$,

$$I_N = \frac{(1-a)m + a\gamma}{2b\left(\frac{1}{3}c^2 + m^2 - \eta\right)} = \left(\frac{1}{2b}\right)\left(\frac{3}{2}\right)\frac{\left\{4mc - a(m+c)^2\right\}}{(m+c)^3} > 0 \tag{15}$$

The optimal investment I_N (15) decreases in the tax progressiveness b, $\frac{\partial I_N}{\partial b} < 0$, and proportional tax rate a, $\frac{\partial I_N}{\partial a} < 0$.

(Proof: See Appendix.)

In the objective function (7), the incremental benefit from investment is positive ($(1-a)m + a\gamma > 0$) because $a < a^{\dagger} = \frac{4mc}{(m+c)^2} = \frac{m}{m-\gamma}$. The incremental burden of progressive taxation caused by investment is also positive ($2b\left(\frac{1}{3}c^2+m^2-\eta\right)I>0$). The incremental benefit is constant regardless of the investment but the incremental burden grows as the investment gets bigger. The optimal investment I_N (15) is where the incremental burden catches up with the incremental benefit and no more investment is induced.

Next proposition shows that the optimal investment increases in the profitability and decreases in the riskiness of a project. Tax progressiveness affects not only the optimal investment amount but also the responsiveness of an entrepreneur to changes in the profitability and riskiness of investment projects.

Proposition 2. (Effects of tax progressiveness on the responsiveness of an entrepreneur)

[1] The optimal investment I_N (15) increases in the profitability of a project represented by the mean m,

$$\frac{\partial I_N}{\partial m}=\left(\frac{1}{2b}\right)\left(\frac{3}{2}\right)\frac{\left\{4c(c-2m)+a(m+c)^2\right\}}{(m+c)^4}>0 \tag{16}$$

The responsiveness of an entrepreneur to changes in the profitability of a project ($\frac{\partial I_N}{\partial m}$) decreases in the tax progressiveness b and increases in the proportional tax rate a.

[2] The optimal investment I_N (15) decreases in the riskiness of a project represented by the dispersion c,

$$\frac{\partial I_N}{\partial c}=\left(\frac{1}{2b}\right)\left(\frac{3}{2}\right)\frac{\left\{4m(m-2c)+a(m+c)^2\right\}}{(m+c)^4}<0 \tag{17}$$

The responsiveness of an entrepreneur to changes in the riskiness of a project ($-\frac{\partial I_N}{\partial c}$) decreases in the tax progressiveness b and proportional tax rate a.

(Proof: See Appendix.)

The mean m represents the profitability and the dispersion c represents the riskiness of a project. Given the uniform distribution, the dispersion c affects the optimal investment I_N (15) by the variance $Var[\varepsilon]=\frac{1}{3}c^2$ and the probability of incurring a loss $\frac{0-(m-c)}{2c}=\frac{c-m}{2c}$. As the dispersion c gets bigger, both the variance $Var[\varepsilon]=\frac{1}{3}c^2$ and the probability of incurring a loss ($\frac{\partial}{\partial c}\left(\frac{c-m}{2c}\right)=\frac{m}{2c^2}>0$) increase. The variance $Var[\varepsilon]=\frac{1}{3}c^2$ and the probability of incurring a loss together represent the riskiness of a project. [2] of Proposition 2 implies that a risk neutral entrepreneur effectively turns into aversion to risk at the burden of progressive taxation. That is, risk aversion is induced by a progressive tax.

Proposition 2 shows that tax progressiveness makes an entrepreneur less sensitive to changes in investment projects. When a tax becomes less (more) progressive, an entrepreneur becomes more (less) responsive to changes in the profitability and riskiness of a project. On the other hand, as a base tax rate a gets higher, an entrepreneur becomes more responsive to changes in the profitability but less responsive to changes in the riskiness of a project.

Next proposition shows that given no loss offset, the asymmetric progressive tax treatment and the asymmetric proportional tax treatment of losses and incomes have opposite effects on the responsiveness of an entrepreneur.

Proposition 3. (Effects of asymmetric tax treatments of losses and incomes on the responsiveness of an entrepreneur)

The asymmetric *progressive* tax treatment of losses and incomes, represented by $b\eta$, makes an entrepreneur *less* responsive to changes in the profitability (mean m) and riskiness (dispersion c) of a project. On the other hand, the asymmetric *proportional* tax treatment of losses and incomes, represented by $a\gamma$, makes an entrepreneur *more* responsive to changes in the profitability (mean m) and riskiness (dispersion c) of a project.

(Proof: See Appendix.)

The asymmetric progressive tax treatment of losses and incomes, represented by $b\eta$, contributes to the increase of the denominator of I_N (15) with respect to $\frac{\partial I_N}{\partial m} > 0$ and diminishes the increase of the denominator of I_N (15) with respect to $\frac{\partial I_N}{\partial c} < 0$, consequently making the entrepreneur less responsive to changes in the profitability and riskiness of a project. On the other hand, the asymmetric proportional tax treatment of losses and incomes, represented by $a\gamma$, contributes to the increase of the numerator of I_N (15) with respect to $\frac{\partial I_N}{\partial m} > 0$ and causes the decrease of the numerator of I_N (15) with respect to $\frac{\partial I_N}{\partial c} < 0$, consequently making the entrepreneur more responsive to changes in the profitability and riskiness of a project.

4. Empirical and policy implications

Tax reforms often bring about changes in the progressiveness of income taxes. An extreme example is Russia's 2001 flat tax reform to 13% income tax rate, resulting in no progressiveness. For another example, Tax Cuts and Jobs Act of 2017 (TCJA) reduces the progressiveness of individual income taxes while keeping the base tax rate[2]. The results of Proposition 2 imply that TCJA makes entrepreneurs not only increase the amount of investment but also respond more sharply than before to changes in the profitability and riskiness of investment projects. Gale and Haldeman (2021) find the increase in investment during 2018-2019 in the wake of TCJA not related to TCJA but admit the difficulty in empirically singling out the impact of TCJA over the short period.

I emphasize this study is distinguished in that it provides empirical implications on the relations between tax progressiveness and investors' responsiveness to changes in the profitability and riskiness of investment projects, on which no previous studies, to my best knowledge, have conducted empirical tests. In particular, the results of Propositions 2 and 3

[2] For instance, for married filing jointly, 10% for up to $19,050, 12% for $19,050 to $77,400, 22% for $77,400 to $165,000, 24% for $165,000 to $315,000, 32% for $315,000 to $400,000, 35% for $400,000 to $600,000, 37% for $600,000 and up under TCJA which are changed from the prior tax rates, 10% for up to $19,050, 15% for $19,050 to $77,400, 25% for $77,400 to $156,150, 28% for $156,150 to $237,950, 33% for $237,950 to $424,950, 35% for $424,950 to $480,050, 39% for $480,050 and up.

suggest that tax progressiveness and the profitability and riskiness of investment projects have interaction effects. For example, interactions between tax progressiveness and the profitability of investment projects are represented by a regression model on investment as the dependent variable,

$$\underset{(+)}{\beta_1}\text{ Profitability} + \underset{(-)}{\beta_2}\text{ Progressiveness} + \underset{(-)}{\beta_3}\text{ Progressiveness} \times \text{Profitability} \tag{18}$$
$$+ \underset{(-)}{\beta_4}\text{ Progressive tax asymmetry} \times \text{Profitability}$$
$$= \underset{(-)}{\beta_2}\text{ Progressiveness} + \left[\underset{(+)}{\beta_1} + \underset{(-)}{\beta_3}\text{ Progressiveness} + \underset{(-)}{\beta_4}\text{ Progressive tax asymmetry}\right] \times \text{Profitability},$$

where the predicted signs are underneath the slope coefficients β_i, i=1,2,3,4, and "Progressive tax asymmetry" is a dummy variable having the value 1 in the existence of the asymmetric progressive tax treatment of losses and incomes and otherwise having the value 0. (18) illustrates the comprehensive impact of tax progressiveness on investment. Tax progressiveness has indirect effects on investment by lowering the responsiveness to changes in the profitability of investment projects (i.e., $\underset{(-)}{\beta_3}$ and $\underset{(-)}{\beta_4}$) as well as direct effects (i.e., $\underset{(-)}{\beta_2}$). Interactions between tax progressiveness and the riskiness of investment projects can be represented in a symmetric way.

Above results have important implications for governmental officials and legislators who design income tax systems to attract investments in competition with other tax jurisdictions. By adjusting asymmetries and progressiveness of income taxes, tax jurisdictions are able to design income tax systems to induce investors to be more (less) responsive to changes in the profitability and riskiness of investment projects, that is to take a more (less) aggressive "high risk & high return" ("low risk & low return") strategy. Different tax jurisdictions have different strategic strengths and weaknesses and develop diverse preferences over the degree of investors' responsiveness to changes in the profitability and riskiness of investment projects.

5. Conclusion

By employing progressive taxes and considering losses for the analysis of investment decisions, this study shows the comprehensive impact of tax progressiveness on investment. Tax progressiveness makes investors less responsive to changes in investment projects. That is, when a tax becomes less (more) progressive, investors become more (less) responsive to changes in the profitability and riskiness of investment projects. In addition, when there is no loss offset available, the asymmetric progressive tax treatment of losses and incomes makes investors less responsive to changes in investment projects while the asymmetric proportional tax treatment makes investors more responsive.

Appendix: Proofs

Proof of Lemma 1

The objective function is

$$\int_0^{m+c}\left[(1-a)(I\varepsilon)-b(I\varepsilon)^2\right]f(\varepsilon)d\varepsilon + \int_{m-c}^0 (I\varepsilon)f(\varepsilon)d\varepsilon$$
$$= \begin{aligned}&\int_0^{m+c}\left[(1-a)(I\varepsilon)-b(I\varepsilon)^2\right]f(\varepsilon)d\varepsilon + \int_{m-c}^0 (I\varepsilon)f(\varepsilon)d\varepsilon \\ &+\int_{m-c}^0\left[-a(I\varepsilon)-b(I\varepsilon)^2\right]f(\varepsilon)d\varepsilon + \int_{m-c}^0\left[a(I\varepsilon)+b(I\varepsilon)^2\right]f(\varepsilon)d\varepsilon\end{aligned}$$

$$= \int_{m-c}^{m+c} \left[(1-a)(I\varepsilon) - b(I\varepsilon)^2 \right] f(\varepsilon) d\varepsilon + \int_{m-c}^{0} \left[a(I\varepsilon) + b(I\varepsilon)^2 \right] f(\varepsilon) d\varepsilon$$

$$= E\left[(1-a)(I\varepsilon) - b(I\varepsilon)^2 \right] + aI \int_{m-c}^{0} \varepsilon f(\varepsilon) d\varepsilon + bI^2 \int_{m-c}^{0} \varepsilon^2 f(\varepsilon) d\varepsilon ,$$

which is equal to (7) with the definitions of the constants γ (8) and η (9), $E[\varepsilon] = m$, and $E[\varepsilon^2] = Var(\varepsilon) + \{E[\varepsilon]\}^2 = \frac{1}{3}c^2 + m^2$.

Proof of Proposition 1

The first order condition from the objective function (7) is

$$(1-a)m + a\gamma - 2b\left(\frac{1}{3}c^2 + m^2 - \eta \right) I = 0$$

The second order condition is satisfied with (12). The optimal amount of investment (15) results from the first order condition. The optimal investment (15) is positive because $a < a^\dagger = \frac{4mc}{(m+c)^2} = \frac{m}{m-\gamma}$. For the "only if" part, see in the objective function (7) that when there is no progressive tax $b = 0$, the optimal investment is infinite ($(1-a)m + a\gamma > 0$) as the entrepreneur is risk neutral.

Proof of Proposition 2

[1] $c - 2m > 0$ by assumption and $\frac{\partial I_N}{\partial m} > 0$ (16) follows.

[2] With the assumption $a < a^\dagger = \frac{4mc}{(m+c)^2}$,

$$\frac{\partial I_N}{\partial c} = \left(\frac{1}{2b} \right) \left(\frac{3}{2} \right) \frac{\left\{ 4m(m-2c) + a(m+c)^2 \right\}}{(m+c)^4}$$

$$< \left(\frac{1}{2b} \right) \left(\frac{3}{2} \right) \frac{\left\{ 4m(m-2c) + 4mc \right\}}{(m+c)^4}$$

$$= \left(\frac{1}{2b} \right) \left(\frac{3}{2} \right) \frac{4m(m-c)}{(m+c)^4} < 0$$

Proof of Proposition 3

[1] of Proposition 2 shows that the optimal investment I_N (15) increases in the profitability of a project (mean m). The numerator of (15) increases in the mean m,

$$\frac{\partial}{\partial m} \{ (1-a)m + a\gamma \} = (1-a) + a \frac{\partial \gamma}{\partial m} > 0 \text{ (with (10))}$$

As γ increases in the mean m ($\frac{\partial \gamma}{\partial m} > 0$ (10)), the asymmetric proportional tax treatment of losses and incomes, represented by $a\gamma$, makes the entrepreneur more responsive to changes in the profitability of a project.

The denominator of (15) also increases in the mean m. The increase of the denominator of (15) diminishes the increase of the optimal investment I_N,

$$\frac{\partial}{\partial m}\left\{b\left(\frac{1}{3}c^2+m^2-\eta\right)\right\}=2bm-b\frac{\partial \eta}{\partial m}>0 \text{ (with (13))}$$

As η decreases in the mean m (that is, $-\frac{\partial \eta}{\partial m}>0$ with (13)), the asymmetric progressive tax treatment of losses and incomes, represented by $b\eta$, contributes to the increase of the denominator and consequently makes the entrepreneur less responsive to changes in the profitability of a project.

[2] of Proposition 2 shows that the optimal investment I_N (15) decreases in the riskiness of a project (dispersion c). The numerator of (15) decreases in the dispersion c,

$$\frac{\partial}{\partial c}\{(1-a)m+a\gamma\}=a\frac{\partial \gamma}{\partial c}<0 \text{ (with (11))}$$

As γ decreases in the dispersion c ($\frac{\partial \gamma}{\partial c}<0$ (11)), the asymmetric proportional tax treatment of losses and incomes, represented by $a\gamma$, makes the entrepreneur more responsive to changes in the riskiness of a project.

The denominator of (15) increases in the dispersion c. The increase of the denominator of (15) strengthens the decrease of the optimal investment I_N,

$$\frac{\partial}{\partial c}\left\{b\left(\frac{1}{3}c^2+m^2-\eta\right)\right\}=\frac{2}{3}bc-b\frac{\partial \eta}{\partial c}=b\frac{\partial}{\partial c}\int_0^{m+c}\varepsilon^2 f(\varepsilon)d\varepsilon>0 \text{ (with (12))}$$

As the dispersion c gets bigger, the distribution gets wider. The values of ε^2 on the positive support $(0,m+c)$ include positive numbers bigger in magnitude and $\int_0^{m+c}\varepsilon^2 f(\varepsilon)d\varepsilon$ increases. However, η increases in the dispersion c (that is, $-\frac{\partial \eta}{\partial c}<0$ with (14)) and the asymmetric progressive tax treatment of losses and incomes, represented by $b\eta$, diminishes the increase of the denominator and consequently makes the entrepreneur less responsive to changes in the riskiness of a project.

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